

- Power Play

## Power Play

**Concept Explanation:** The concept of powers and exponents involves repeated multiplication of the same number, called the base, raised to a certain exponent which indicates how many times the base is multiplied by itself. This concept is fundamental in understanding exponential growth, scientific notation, and operations involving powers.

Exponential growth is a pattern where a quantity multiplies by the same factor repeatedly over equal intervals. For example, folding a sheet of paper doubles its thickness each time, demonstrating exponential growth.

### Formula Derivation

Let the base be  $a$  and the exponent be  $n$ , where  $n$  is a positive integer. Then, the power is defined as:

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n \text{ times}}$$

Key properties of exponents include:

- Multiplying powers with the same base:  $a^m \times a^n = a^{m+n}$
- Dividing powers with the same base:  $\frac{a^m}{a^n} = a^{m-n}$
- Power of a power:  $(a^m)^n = a^{m \times n}$

- Multiplying powers with the same exponent:  $a^n \times b^n = (a \times b)^n$
- Zero exponent:  $a^0 = 1$  for  $a \neq 0$
- Negative exponent:  $a^{-n} = \frac{1}{a^n}$

## Worked Illustrations

**Example 1:** Calculate  $2^4$ .

Solution:

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

**Example 2:** Simplify  $3^5 \times 3^2$ .

Solution:

Using the multiplication property of exponents:

$$3^5 \times 3^2 = 3^{5+2} = 3^7 = 2187$$

**Example 3:** Simplify  $\frac{5^6}{5^3}$ .

Solution:

Using the division property of exponents:

$$\frac{5^6}{5^3} = 5^{6-3} = 5^3 = 125$$

## Solved Examples

**Example 4:** Find the thickness of a sheet of paper after 30 folds if the initial thickness is 0.001 cm.

Solution:

Each fold doubles the thickness, so after  $n$  folds, thickness  $T_n = T_0 \times 2^n$ .

Given  $T_0 = 0.001$  cm and  $n = 30$ ,

$$T_{30} = 0.001 \times 2^{30}$$

Calculate  $2^{30}$ :

$$2^{10} = 1024 \approx 10^3 \Rightarrow 2^{30} = (2^{10})^3 = (10^3)^3 = 10^9$$

So,

$$T_{30} = 0.001 \times 10^9 = 10^6 \text{ cm} = 10,000 \text{ meters} = 10 \text{ km}$$

Thus, after 30 folds, the paper thickness is approximately 10 km.

## Practice Set

### Level 1 – Easy

- Calculate  $4^3$ .
- Simplify  $7^2 \times 7^3$ .
- Find  $5^0$ .

### Level 2 – Moderate

- Simplify  $\frac{9^5}{9^2}$ .
- Calculate  $(2^3)^4$ .
- Find the thickness of a paper after 20 folds if initial thickness is 0.002 cm.

### Level 3 – Challenging

- Simplify  $3^4 \times 5^4$  using exponent rules.
- Calculate the thickness of a paper after 46 folds if initial thickness is 0.001 cm and express the answer in kilometers.
- Prove that  $a^m \times a^n = a^{m+n}$  using the definition of exponents.

## Answer Key

- $4^3 = 64$
- $7^2 \times 7^3 = 7^{2+3} = 7^5 = 16807$
- $5^0 = 1$
- $\frac{9^5}{9^2} = 9^{5-2} = 9^3 = 729$
- $(2^3)^4 = 2^{3 \times 4} = 2^{12} = 4096$
- Thickness after 20 folds:  
 $0.002 \times 2^{20} = 0.002 \times 1,048,576 = 2097.152 \text{ cm} = 20.97152 \text{ meters}$
- $3^4 \times 5^4 = (3 \times 5)^4 = 15^4 = 50625$
- Thickness after 46 folds:  
 $0.001 \times 2^{46} \approx 0.001 \times 7.04 \times 10^{13} = 7.04 \times 10^{10} \text{ cm} = 7.04 \times 10^5 \text{ km}$

- Proof of  $a^m \times a^n = a^{m+n}$ : By definition,  $a^m = \underbrace{a \times a \times \dots \times a}_{m \text{ times}}$  and  $a^n = \underbrace{a \times a \times \dots \times a}_{n \text{ times}}$ .

Multiplying these gives  $\underbrace{a \times a \times \dots \times a}_{m+n \text{ times}} = a^{m+n}$ .

## Quick Reference

Operation	Rule	Example
Multiplying powers with same base	$a^m \times a^n = a^{m+n}$	$2^3 \times 2^4 = 2^7 = 128$
Dividing powers with same base	$\frac{a^m}{a^n} = a^{m-n}$	$5^5/5^2 = 5^3 = 125$
Power of a power	$(a^m)^n = a^{m \times n}$	$(3^2)^3 = 3^6 = 729$
Multiplying powers with same exponent	$a^n \times b^n = (a \times b)^n$	$2^3 \times 5^3 = (2 \times 5)^3 = 10^3 = 1000$
Zero exponent	$a^0 = 1$	$7^0 = 1$
Negative exponent	$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

## Glossary

- **Exponential Growth:** Growth pattern where a quantity multiplies by the same factor repeatedly over equal intervals.
- **Linear Growth:** Growth pattern where a quantity increases by the same amount in each step.
- **Exponential Notation:** Shorthand for repeated multiplication of the same number, written as  $a^n$  where  $a$  is the base and  $n$  is the exponent.
- **Exponent/Power:** The number that indicates how many times the base is multiplied by itself.