- Quadrilaterals

Quadrilaterals

A quadrilateral is a polygon with four sides and four angles. The sum of the interior angles of any quadrilateral is always 360°. Quadrilaterals include rectangles, squares, parallelograms, rhombuses, trapeziums, and kites, each with unique properties.

Formula Derivation: Sum of Interior Angles

Consider a quadrilateral $ABCD$. Draw a diagonal AC dividing it into two triangles $\triangle ABC$ and $\triangle ADC$. Each triangle has interior angles summing to 180°.

Therefore, sum of interior angles of quadrilateral $ABCD$ is:

$$\angle ABC + \angle BCD + \angle CDA + \angle DAB = 180^\circ + 180^\circ = 360^\circ$$

Worked Illustrations

- **Rectangle:** Four right angles (90°), opposite sides equal and parallel, diagonals equal and bisect each other.
- **Square:** All sides equal, four right angles, diagonals equal, perpendicular, and bisect angles.
- **Parallelogram:** Opposite sides equal and parallel, opposite angles equal, adjacent angles sum to 180°, diagonals bisect each other.
- **Rhombus:** All sides equal, opposite sides parallel, diagonals bisect each other at 90° and bisect angles.
- **Trapezium:** At least one pair of parallel sides; if isosceles, non-parallel sides and base angles are equal.
- **Kite:** Two pairs of adjacent sides equal, one diagonal bisects the other at 90°, bisects a pair of opposite angles.

Solved Example

Example: In a parallelogram, one angle is 70°. Find the other three angles.

Solution:

- Opposite angles are equal, so angle opposite to 70° is 70°.
- Adjacent angles sum to 180°, so each adjacent angle is $180^\circ - 70^\circ = 110^\circ$.
- Opposite angles are equal, so the angle opposite to 110° is 110°.

Thus, the four angles are $70^\circ, 110^\circ, 70^\circ, 110^\circ$.

Practice Set

- **Level 1 – Easy**
  - Find the sum of interior angles of a quadrilateral.
  - Identify the number of right angles in a rectangle.
  - State the properties of a square.

- **Level 2 – Moderate**
  - In a rhombus, if one angle is 60°, find the other three angles.
  - Prove that the diagonals of a rectangle are equal.
  - Calculate the length of diagonals in a parallelogram with given sides and angles.
- **Level 3 – Challenging**
  - Prove that the diagonals of a rhombus bisect each other at right angles.
  - Given a trapezium with one pair of parallel sides, prove the base angles are equal if it is isosceles.
  - Show that a square is both a rectangle and a rhombus using properties.

Answer Key

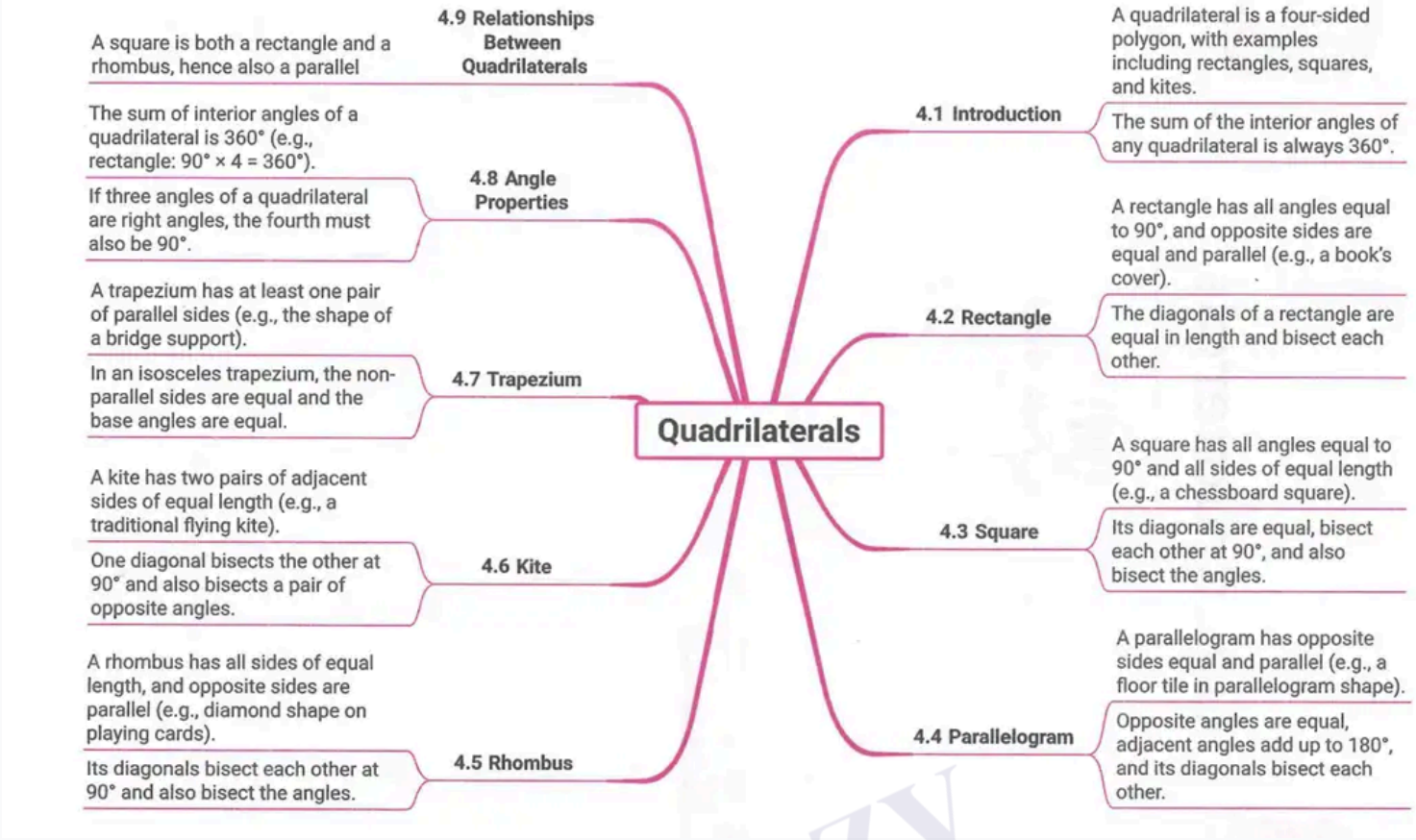
- **Level 1**
  - Sum of interior angles = 360°.
  - Rectangle has 4 right angles.
  - Square has all sides equal and 4 right angles.
- **Level 2**
  - Opposite angles of rhombus are equal, so other angles are $120^\circ, 60^\circ, 120^\circ$.
  - Diagonals of rectangle are equal because it has equal opposite sides and right angles; use Pythagoras theorem.
  - Use cosine rule to calculate diagonals in parallelogram.
- **Level 3**
  - Diagonals of rhombus bisect each other at right angles by definition and can be proved using congruent triangles.
  - In isosceles trapezium, base angles are equal due to symmetry and equal non-parallel sides.
  - Square has all properties of rectangle and rhombus; equal sides, right angles, equal diagonals, perpendicular diagonals.

Quick Reference

Quadrilateral	Properties
Rectangle	4 right angles, opposite sides equal, diagonals equal and bisect each other
Square	All sides equal, 4 right angles, diagonals equal, perpendicular, bisect angles
Parallelogram	Opposite sides equal and parallel, opposite angles equal, diagonals bisect each other
Rhombus	All sides equal, opposite sides parallel, diagonals bisect at 90° and bisect angles
Trapezium	At least one pair of parallel sides; isosceles trapezium has equal non-parallel sides and base angles
Kite	Two pairs of adjacent sides equal, one diagonal bisects other at 90°, bisects opposite angles

Glossary

- **Quadrilateral:** A polygon with four sides and four angles.
- **Diagonal:** A line segment joining two non-adjacent vertices.
- **Parallelogram:** A quadrilateral with opposite sides parallel and equal.
- **Rhombus:** A parallelogram with all sides equal.
- **Rectangle:** A parallelogram with four right angles.
- **Square:** A rectangle with all sides equal.
- **Trapezium:** A quadrilateral with at least one pair of parallel sides.
- **Kite:** A quadrilateral with two pairs of adjacent equal sides.


Quadrilaterals
4.1 Introduction
A quadrilateral is a four-sided polygon, with examples including rectangles, squares, and kites.
The sum of the interior angles of any quadrilateral is always 360°.
4.2 Rectangle
A rectangle has all angles equal to 90°, and opposite sides are equal and parallel (e.g., a book's cover).
The diagonals of a rectangle are equal in length and bisect each other.
4.3 Square
A square has all angles equal to 90° and all sides of equal length (e.g., a chessboard square).
Its diagonals are equal, bisect each other at 90°, and also bisect the angles.
4.4 Parallelogram
A parallelogram has opposite sides equal and parallel (e.g., a floor tile in parallelogram shape).
Opposite angles are equal, adjacent angles add up to 180°, and its diagonals bisect each other.
4.5 Rhombus
A rhombus has all sides of equal length, and opposite sides are parallel (e.g., diamond shape on playing cards).
Its diagonals bisect each other at 90° and also bisect the angles.
4.6 Kite
A kite has two pairs of adjacent sides of equal length (e.g., a traditional flying kite).
One diagonal bisects the other at 90° and also bisects a pair of opposite angles.
4.7 Trapezium
A trapezium has at least one pair of parallel sides (e.g., the shape of a bridge support).
In an isosceles trapezium, the non-parallel sides are equal and the base angles are equal.
4.8 Angle Properties
The sum of interior angles of a quadrilateral is 360° (e.g., rectangle: 90° × 4 = 360°).
If three angles of a quadrilateral are right angles, the fourth must also be 90°.
4.9 Relationships Between Quadrilaterals
A square is both a rectangle and a rhombus, hence also a parallel