The Distributive Property

The distributive property explains how multiplication distributes over addition. It states that multiplying a sum by a number is equivalent to multiplying each addend by the number and then adding the products. Formally, for any numbers a, b, and c,

$$a(b+c) = ab + ac$$

Formula Derivation

Consider the expression $a(b+c)$. By definition, $b+c$ is the sum of two numbers. Multiplying a by this sum means adding a to itself $b+c$ times, which is the same as adding a b times and then a c times:

$$a(b+c) = a \times b + a \times c = ab + ac$$

Worked Illustration

Calculate $3(4+5)$ using the distributive property.

Step 1: Apply the distributive property:

$$3(4+5) = 3 \times 4 + 3 \times 5$$

Step 2: Multiply:

$$= 12 + 15$$

Step 3: Add:

$$= 27$$

Solved Examples

Example 1: Simplify $5(x+7)$.

Step 1: Apply distributive property:

$$5(x+7)=5\times x+5\times 7$$

Step 2: Multiply:

$$=5x+35$$

Example 2: Expand and simplify $2(3a+4b)$.

Step 1: Apply distributive property:

$$2(3a+4b)=2\times 3a+2\times 4b$$

Step 2: Multiply:

$$=6a+8b$$

Practice Set

- **Level 1 – Easy**
- Simplify $4(2+3)$.
- Expand $3(x+5)$.
- **Level 2 – Moderate**
- Expand and simplify $5(2a+3b)$.
- Calculate $7(4+y)$ and simplify.
- **Level 3 – Challenging**
- Expand $(x+3)(2+y)$ using distributive property.
- Simplify $3(2x+4)+5(x+1)$.

Answer Key

- **Level 1**
- $4(2+3)=4\times 2+4\times 3=8+12=20$
- $3(x+5)=3x+15$

- **Level 2**
- $5(2a + 3b) = 10a + 15b$
- $7(4 + y) = 28 + 7y$
- **Level 3**
- $(x + 3)(2 + y) = x \times 2 + x \times y + 3 \times 2 + 3 \times y = 2x + xy + 6 + 3y$
- $3(2x + 4) + 5(x + 1) = 6x + 12 + 5x + 5 = 11x + 17$

Quick Reference

Expression	Expanded Form
$a(b + c)$	$ab + ac$
$(x + m)(y + n)$	$xy + xn + my + mn$

Glossary

- **Distributive Property:** A property that allows multiplication to be distributed over addition.
- **Addend:** A number involved in addition.
- **Expand:** To multiply out expressions to remove parentheses.
- **Like Terms:** Terms with the same variable raised to the same power.

Quick Calculation Tricks

Quick calculation tricks use distributive property and number patterns to multiply numbers rapidly without a calculator.

Formula Derivation

Multiplying by 11, for example, can be done by adding adjacent digits:

For a two-digit number ab,

$$ab \times 11 = a(a + b)b$$

where a and b are digits, and the middle digit is the sum $a + b$. Carry over if needed.

Worked Illustration

Calculate 23×11 quickly.

Step 1: Add digits: $2 + 3 = 5$

Step 2: Place the sum between the digits: 253

Answer: 253

Solved Examples

Example 1: Multiply 54×11.

Step 1: Add digits: $5 + 4 = 9$

Step 2: Place sum between digits: 594

Answer: 594

Example 2: Multiply 79×11.

Step 1: Add digits: $7 + 9 = 16$

Step 2: Place 6 between digits and add 1 to 7: 869

Answer: 869

Practice Set

- **Level 1 – Easy**
- Calculate 31×11.
- Calculate 42×11.
- **Level 2 – Moderate**
- Calculate 68×11.
- Calculate 95×11.
- **Level 3 – Challenging**
- Multiply 123×11 using the pattern.
- Multiply 456×11 using the pattern.

Answer Key

- **Level 1**
- $31 \times 11 = 3(3+1)1 = 341$
- $42 \times 11 = 4(4+2)2 = 462$
- **Level 2**
- $68 \times 11 = 6(6+8)8 = 748$
- $95 \times 11 = 9(9+5)5 = 1045$ (carry over applied)
- **Level 3**
- $123 \times 11 = 1353$ (add digits pairwise: 1, (1+2)=3, (2+3)=5, 3)
- $456 \times 11 = 49116 = 5016$ (carry over applied)

Quick Reference

Multiplication	Quick Method

$ab \times 11$	$a(a+b)b$ with carry over
$abc \times 11$	Add adjacent digits from right to left with carry over

Glossary

- **Carry Over:** When a sum exceeds 9, the extra digit is carried to the next place value.
- **Adjacent Digits:** Digits next to each other in a number.
- **Pattern Recognition:** Identifying regularities to simplify calculations.

Special Algebraic Identities

Special algebraic identities are formulas that simplify the expansion of certain algebraic expressions.

Formula Derivation

1. Square of a sum:

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

2. Square of a difference:

$$(a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$$

3. Product of sum and difference:

$$(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$$

Worked Illustration

Expand $(x+3)^2$.

Step 1: Apply square of sum formula:

$$(x+3)^2 = x^2 + 2 \times x \times 3 + 3^2 = x^2 + 6x + 9$$

Solved Examples

Example 1: Expand $(5+y)^2$.

$$(5+y)^2 = 5^2 + 2\times 5\times y + y^2 = 25 + 10y + y^2$$

Example 2: Expand $(a-4)^2$.

$$(a-4)^2 = a^2 - 2\times a\times 4 + 4^2 = a^2 - 8a + 16$$

Example 3: Simplify $(x+7)(x-7)$.

$$(x+7)(x-7) = x^2 - 7^2 = x^2 - 49$$

Practice Set

- **Level 1 – Easy**
- Expand $(x+2)^2$.
- Expand $(3-y)^2$.
- **Level 2 – Moderate**
- Expand $(2a+5)^2$.
- Simplify $(m+4)(m-4)$.
- **Level 3 – Challenging**
- Expand and simplify $(x+3)(x-5)+(x-3)^2$.
- Prove that $(a+b)^2-(a-b)^2=4ab$.

Answer Key

- **Level 1**
- $(x+2)^2 = x^2+4x+4$
- $(3-y)^2 = 9-6y+y^2$
- **Level 2**
- $(2a+5)^2 = 4a^2+20a+25$
- $(m+4)(m-4) = m^2-16$
- **Level 3**
- $(x+3)(x-5)+(x-3)^2 = (x^2-5x+3x-15)+(x^2-6x+9) = (x^2-2x-15)+(x^2-6x+9) = 2x^2-8x-6$
- Proof:
  $(a+b)^2-(a-b)^2 = (a^2+2ab+b^2)-(a^2-2ab+b^2) = 4ab$

Quick Reference

Identity	Formula
Square of sum	$(a+b)^2 = a^2+2ab+b^2$
Square of difference	$(a-b)^2 = a^2-2ab+b^2$
Product of sum and difference	$(a+b)(a-b) = a^2-b^2$

Glossary

- **Algebraic Identity:** An equation true for all values of the variables.
- **Square of a Binomial:** The product of a binomial multiplied by itself.
- **Difference of Squares:** The product of the sum and difference of the same two terms.

Multiple Problem Solving Approaches

Mathematical problems can often be solved in various ways. Developing flexibility in approach enhances understanding and efficiency.

Concept Explanation

For example, to expand $(x+2)(x+3)$, one can use distributive property or FOIL method:

- Distributive property: $x(x+3)+2(x+3)=x^2+3x+2x+6=x^2+5x+6$
- FOIL method: First, Outer, Inner, Last multiplication leading to the same result.

Worked Illustration

Calculate $(a+4)(a+5)$ using two methods.

Method 1 (Distributive):

$$a(a+5)+4(a+5)=a^2+5a+4a+20=a^2+9a+20$$

Method 2 (FOIL):

$$a\times a+a\times 5+4\times a+4\times 5=a^2+5a+4a+20=a^2+9a+20$$

Practice Set

- **Level 1 – Easy**
- Expand $(x+1)(x+2)$ using two methods.
- **Level 2 – Moderate**
- Expand $(2x+3)(x+4)$ using distributive property and FOIL.
- **Level 3 – Challenging**
- Prove that $(x+y)^2=x^2+2xy+y^2$ using two different approaches.

Answer Key

- **Level 1**
- $(x+1)(x+2)=x^2+2x+x+2=x^2+3x+2$

- **Level 2**
- $(2x + 3)(x + 4) = 2x^2 + 8x + 3x + 12 = 2x^2 + 11x + 12$
- **Level 3**
- Proof 1: Expand directly using distributive property.
  Proof 2: Use geometric interpretation or FOIL method.

Quick Reference

Method	Description
Distributive Property	Multiply each term in the first bracket by each term in the second.
FOIL Method	Multiply First, Outer, Inner, Last terms and add.

Glossary

- **FOIL:** Acronym for First, Outer, Inner, Last multiplication in binomials.
- **Flexibility:** Ability to use different methods to solve problems.