- Proportional Reasoning

Proportional Reasoning

Proportional reasoning is the ability to understand and work with ratios and proportions, which describe how quantities relate to each other multiplicatively. It is fundamental in solving problems involving scaling, mixing, sharing, and unit conversions.

Concept Explanation

When two quantities change in such a way that their ratio remains constant, they are said to be in proportion. For example, if the length and width of a rectangle increase by the same factor, the shapes remain similar.

Formula Derivation

Consider two ratios $\frac{a}{b}$ and $\frac{c}{d}$. They are proportional if:

$$\frac{a}{b} = \frac{c}{d}$$

Cross-multiplying gives the Rule of Three:

$$a \times d = b \times c$$

This equation allows us to find an unknown term if three terms are known.

Worked Illustrations

Example 1: Check if the ratios 4:6 and 10:15 are proportional.

First, simplify both ratios:

$$4 : 6 = \frac{4}{6} = \frac{2}{3}, \quad 10 : 15 = \frac{10}{15} = \frac{2}{3}$$

Since both simplify to $\frac{2}{3}$, the ratios are proportional.

Solved Examples

Example 2: A recipe requires 3 cups of flour for every 2 cups of sugar. How much sugar is needed for 9 cups of flour?

Set up the proportion:

$$\frac{3}{2} = \frac{9}{x}$$

Cross-multiply:

$$3 \times x = 2 \times 9$$

$$3x = 18$$

Divide both sides by 3:

$$x = \frac{18}{3} = 6$$

So, 6 cups of sugar are needed.

Practice Set

Level 1 – Easy

- Check if 5:10 and 15:30 are proportional.
- Simplify the ratio 12:16.
- Find the missing term: $\frac{7}{x} = \frac{14}{28}$.

Level 2 – Moderate

- A map scale shows 1 cm represents 5 km. How many kilometers does 7 cm represent?
- Divide 60 sweets between two children in the ratio 2:3.
- Convert 3 hours 30 minutes into minutes and express the ratio of hours to minutes.

Level 3 – Challenging

- Two quantities are in the ratio 4:5. If their sum is 81, find the quantities.
- A paint mixture requires red and blue paint in the ratio 3:7. How much blue paint is needed for 12 liters of red paint?
- Prove that if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+c}{b+d}$ lies between $\frac{a}{b}$ and $\frac{c}{d}$.

Answer Key

- Level 1:
  - Yes, $\frac{5}{10} = \frac{1}{2}$ and $\frac{15}{30} = \frac{1}{2}$, so proportional.
  - $12 : 16 = \frac{12}{16} = \frac{3}{4}$.
  - Cross-multiply: $7 \times 28 = 14 \times x \Rightarrow 196 = 14x \Rightarrow x = 14$.
- Level 2:
  - 7 cm represents $7 \times 5 = 35$ km.

    - Total parts = 2 + 3 = 5; each part = $60/5 = 12$. So, children get 24 and 36 sweets respectively.
    - 3 hours 30 minutes = 210 minutes; ratio = $\frac{3}{210} = \frac{1}{70}$.
- Level 3:
    - Sum of parts = 4 + 5 = 9; each part = $81/9 = 9$. Quantities: 36 and 45.
    - Ratio $3:7$ means for 12 liters red paint, blue paint = $\frac{7}{3} \times 12 = 28$ liters.
    - Proof involves algebraic manipulation and ordering of fractions (left as exercise).

Quick Reference

Concept	Formula
Proportion	$\frac{a}{b} = \frac{c}{d}$
Rule of Three	$a \times d = b \times c$
Ratio Simplification	Divide terms by HCF

Glossary

- **Ratio:** A comparison of two quantities of the same kind.
- **Proportion:** An equation stating two ratios are equal.
- **HCF (Highest Common Factor):** The greatest number dividing two or more numbers exactly.
- **Rule of Three:** A method to find an unknown term in a proportion using cross multiplication.
- **Scaling:** Multiplying all dimensions of a shape by the same factor to maintain similarity.