Fractions as Percentages

Percentages represent fractions with denominator 100. The symbol '%' means 'per cent' or 'per hundred'. For example, 25% means 25 out of 100.

To express a percentage as a fraction, write it as $\frac{x}{100}$. For example, 20% = $\frac{20}{100} = \frac{1}{5}$.

To express a fraction as a percentage, convert it to an equivalent fraction with denominator 100 or multiply by 100.

Formula Derivation

Given a fraction $\frac{a}{b}$, to express it as a percentage:

Method 1: Find equivalent fraction with denominator 100:

$$\frac{a}{b} = \frac{x}{100} \implies x = \frac{a}{b} \times 100$$

Method 2: Multiply fraction by 100:

$$\text{Percentage} = \frac{a}{b} \times 100\%$$

Worked Illustration

Express $\frac{3}{4}$ as a percentage.

Method 1:

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

Method 2:

$$\frac{3}{4} \times 100 = 75\%$$

Solved Example

Surya mixes red and yellow paint. Red paint is $\frac{3}{4}$ of the mixture. What percentage is red paint?

$$\frac{3}{4} \times 100 = 75\%$$

So, 75% of the mixture is red paint.

Practice Set

- Express the following fractions as percentages: $\frac{3}{5}$, $\frac{7}{14}$, $\frac{9}{20}$.
- Nandini has 25 marbles, 15 are white. What percentage are white?
- In a school of 80 students, 15 walk to school. What percentage walk?

Answer Key

- $\frac{3}{5} = 60\%$, $\frac{7}{14} = 50\%$, $\frac{9}{20} = 45\%$
- White marbles percentage = $\frac{15}{25} \times 100 = 60\%$
- Walking students percentage = $\frac{15}{80} \times 100 = 18.75\%$

Quick Reference

- Percentage = Fraction × 100
- Fraction = Percentage ÷ 100

Glossary

- **Percentage:** A fraction with denominator 100.
- **Fraction:** A part of a whole expressed as $\frac{a}{b}$.

Expressing Fractions as Percentages

To convert any fraction to a percentage, multiply by 100.

Worked Illustration

Express $\frac{2}{5}$ as a percentage.

Method 1: Find equivalent fraction with denominator 100:

$$\frac{2}{5} = \frac{40}{100} = 40\%$$

Method 2: Multiply by 100:

$$\frac{2}{5} \times 100 = 40\%$$

Solved Example

Surya wants to save $\frac{2}{5}$ of prize money. What percentage is this?

$$\frac{2}{5} \times 100 = 40\%$$

Practice Set

- Express $\frac{1}{3}$ as a percentage.
- Express $\frac{5}{11}$ as a percentage.

Answer Key

- $\frac{1}{3} \times 100 = 33.33\%$
- $\frac{5}{11} \times 100 \approx 45.45\%$

Quick Reference

- Multiply fraction by 100 to get percentage.

Glossary

- **Equivalent fraction:** Fractions representing the same value.

Using Percentages to Compare Proportions

Percentages allow comparison of quantities with different totals by expressing them as parts per hundred.

Solved Example

Eesha scored 42 out of 50 in English and 70 out of 80 in Science. Which is better?

English percentage:

$$\frac{42}{50} \times 100 = 84\%$$

Science percentage:

$$\frac{70}{80} \times 100 = 87.5\%$$

Science score is higher; Eesha did better in Science.

Practice Set

- Compare scores: 35/40 and 45/60.
- Compare sugar content: 9/34 and 13/45.

Answer Key

- $\frac{35}{40} \times 100 = 87.5\%$, $\frac{45}{60} \times 100 = 75\%$. First score is better.
- $\frac{9}{34} \times 100 \approx 26.47\%$, $\frac{13}{45} \times 100 \approx 28.89\%$. Second variety is more sugary.

Quick Reference

- Convert quantities to percentages for comparison.

Glossary

- **Proportion:** Part of a whole expressed as a fraction or percentage.

Percentage Increase and Decrease

Percentage increase or decrease measures how much a quantity changes relative to its original value.

Formula

Percentage increase:

$$\frac{\text{Increase}}{\text{Original amount}} \times 100$$

Percentage decrease:

$$\frac{\text{Decrease}}{\text{Original amount}} \times 100$$

Solved Example

Price of tomatoes increased from ₹30 to ₹42.

Increase = 42 - 30 = 12

Percentage increase:

$$\frac{12}{30} \times 100 = 40\%$$

Price increased by 40%.

Practice Set

- Price decreased from ₹160 to ₹100. Find percentage decrease.
- Population increased from 1000 to 1650. Find percentage increase.

Answer Key

- Decrease = 60; Percentage decrease = $\frac{60}{160} \times 100 = 37.5\%$
- Increase = 650; Percentage increase = $\frac{650}{1000} \times 100 = 65\%$

Quick Reference

- Percentage change = (Change / Original) × 100

Glossary

- **Increase:** Amount added to original value.
- **Decrease:** Amount subtracted from original value.

Profit and Loss

Profit and loss are expressed as percentages relative to cost price.

Definitions

- **Cost Price (CP):** Price at which item is bought.
- **Marked Price (MP):** Price tagged on item.
- **Selling Price (SP):** Price at which item is sold.
- **Profit:** When SP > CP.
- **Loss:** When SP < CP.

Formulas

Profit = SP − CP

Loss = CP − SP

Profit % = $\frac{\text{Profit}}{\text{CP}} \times 100$

Loss % = $\frac{\text{Loss}}{\text{CP}} \times 100$

Solved Example

Kishanlal buys sweater at ₹300, sells at ₹430.

Profit = 430 − 300 = 130

Profit % = $\frac{130}{300} \times 100 = 43.3\%$

Practice Set

- Find profit % if CP = ₹36, profit margin = 20%.
- Find loss % if CP = ₹350, SP = ₹300.

Answer Key

- Profit = 20% of 36 = ₹7.2; SP = 36 + 7.2 = ₹43.2
- Loss = 350 - 300 = ₹50; Loss % = $\frac{50}{350} \times 100 = 14.28\%$

Quick Reference

- Profit or loss percentage is always relative to cost price.

Glossary

- **Profit margin:** Percentage of profit on cost price.

Growth and Compounding

Interest is extra money paid on deposits or loans. It can be simple or compound.

Simple Interest

Interest is calculated on original principal only.

Amount after t years:

$$A = P(1 + rt)$$

where P = principal, r = rate (decimal), t = time.

Compound Interest

Interest is added to principal each period; next interest is on increased amount.

Amount after t years:

$$A = P(1 + r)^t$$

Solved Example

Deposit ₹6000 at 10% p.a. for 3 years.

Simple interest:

$$A = 6000(1 + 0.1times3) = 6000times1.3 = 7800$$

Compound interest:

$$A = 6000(1 + 0.1)^3 = 6000times1.331 = 7986$$

Practice Set

- Calculate amount for ₹20,000 at 5% p.a. for 4 years with and without compounding.
- Find interest difference for ₹12,500 at 12% simple and 10% compound for 3 years.

Answer Key

- Without compounding: $20000(1+0.05times4)=24000$
- With compounding: $20000(1+0.05)^4=20000times1.2155=24310$
- Simple interest: $12500times0.12times3=4500$
- Compound interest: $12500(1+0.10)^3-12500=12500times1.331-12500=4137.5$
- Difference = 4500 - 4137.5 = 362.5

Quick Reference

- Simple interest grows linearly.
- Compound interest grows exponentially.

Glossary

- **Principal:** Initial amount deposited or borrowed.
- **Rate:** Interest rate per period as decimal.
- **Time:** Number of periods.

Decline and Depreciation

Depreciation is reduction in value over time.

Solved Example

TV bought at ₹21,000 depreciates 5% after 1 year.

Depreciation amount:

$$0.05times21000 = 1050$$

Value after 1 year:

$$21000 - 1050 = 19950$$

Practice Set

- Population reduces by 10% every decade. Current population 1250. Find population after 3 decades.

Answer Key

Population after 3 decades:

$$1250times0.9^3 = 1250times0.729 = 911.25$$

Quick Reference

- Value after depreciation = Original value × (1 - depreciation rate)

Glossary

- **Depreciation:** Decrease in value over time.