Introduction to Triangles

A triangle is a simple closed figure formed by three line segments. It has three vertices, three sides, and three angles. Consider $\triangle ABC$ with vertices A, B, and C.

- **Sides:** AB, BC, CA
- **Angles:** $\angle BAC, \angle ABC, \angle BCA$
- **Vertices:** A, B, C

The sum of the interior angles of any triangle is always 180°:

$$\angle A + \angle B + \angle C = 180^\circ$$

Triangles can be classified based on sides and angles:

- **Based on Sides:** Equilateral (all sides equal), Isosceles (two sides equal), Scalene (all sides different)

- **Based on Angles:** Acute-angled (all angles < 90°), Right-angled (one angle = 90°), Obtuse-angled (one angle > 90°)

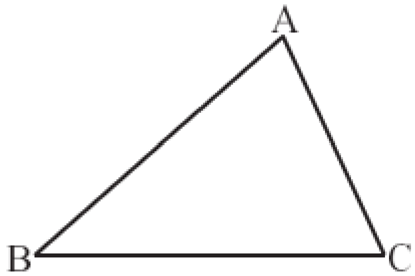


Fig 6.1

Worked Example

Identify the sides opposite to each vertex in $\triangle ABC$:

- Side opposite to vertex A is BC
- Side opposite to vertex B is AC
- Side opposite to vertex C is AB

Practice Set

- Write the six elements (3 sides and 3 angles) of $\triangle ABC$.
- For $\triangle PQR$, name the side opposite to vertex Q.
- For $\triangle LMN$, name the angle opposite to side LM.
- For $\triangle RST$, name the vertex opposite to side RT.

Answer Key

- Sides: AB, BC, CA; Angles: $\angle BAC, \angle ABC, \angle BCA$
- Side opposite to Q is PR
- Angle opposite to LM is $\angle N$
- Vertex opposite to RT is S

Quick Reference

Triangle naming convention: $\triangle ABC$ with sides AB, BC, CA and angles at vertices A, B, C.

Glossary

- **Vertex:** Corner point of a triangle
- **Side:** Line segment joining two vertices
- **Angle:** Space between two sides at a vertex

Medians of a Triangle

A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side.

In $\triangle ABC$, if D is the midpoint of BC, then AD is a median.

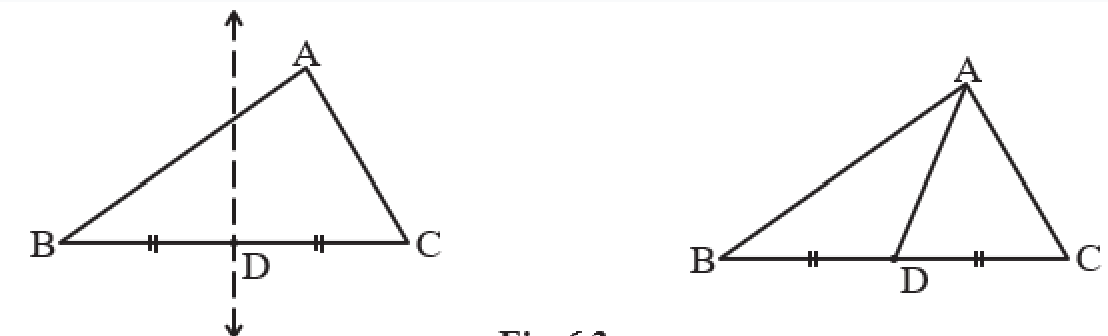


Fig 6.3

Properties

- Each triangle has three medians.
- A median divides the opposite side into two equal parts.
- Medians intersect at the centroid of the triangle.

Worked Example

Find the medians of $\triangle ABC$ given coordinates of vertices A, B, and C.

Step 1: Find midpoints of sides BC, CA, and AB.

Step 2: Draw line segments from each vertex to the midpoint of the opposite side.

Practice Set

- Draw $\triangle XYZ$ and find the medians.
- Prove that medians intersect at a single point.
- Find the centroid of a triangle with given vertices.

Answer Key

- Medians are line segments from vertices to midpoints of opposite sides.
- Centroid is the point of concurrency of medians.

Quick Reference

Median: Vertex to midpoint of opposite side.

Glossary

- **Median:** Segment joining vertex to midpoint of opposite side
- **Centroid:** Point where medians intersect

Altitudes of a Triangle

An altitude of a triangle is a perpendicular segment from a vertex to the line containing the opposite side.

In $\triangle ABC$, the altitude from vertex A is the perpendicular segment AL to side BC.

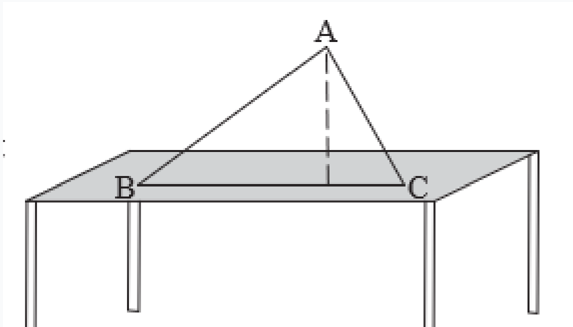


Fig 6.4

Properties

- Each triangle has three altitudes.
- Altitudes may lie inside or outside the triangle depending on the type of triangle.
- Altitudes intersect at the orthocenter.

Worked Example

Find the length of the altitude from vertex A to side BC in $\triangle ABC$ with given coordinates.

Step 1: Find equation of line BC.

Step 2: Find perpendicular distance from A to BC.

Practice Set

- Draw altitudes in different types of triangles.
- Find the orthocenter of a triangle.
- Prove properties of altitudes in isosceles and equilateral triangles.

Answer Key

- Altitude is perpendicular from vertex to opposite side.
- Orthocenter is concurrency point of altitudes.

Quick Reference

Altitude: Perpendicular from vertex to opposite side.

Glossary

- **Altitude:** Perpendicular segment from vertex to opposite side
- **Orthocenter:** Point where altitudes intersect

Exterior Angle Property of a Triangle

An exterior angle of a triangle is equal to the sum of the two interior opposite angles.

Given $\triangle ABC$ with exterior angle $\angle ACD$ at vertex C,

$$\angle ACD = \angle A + \angle B$$

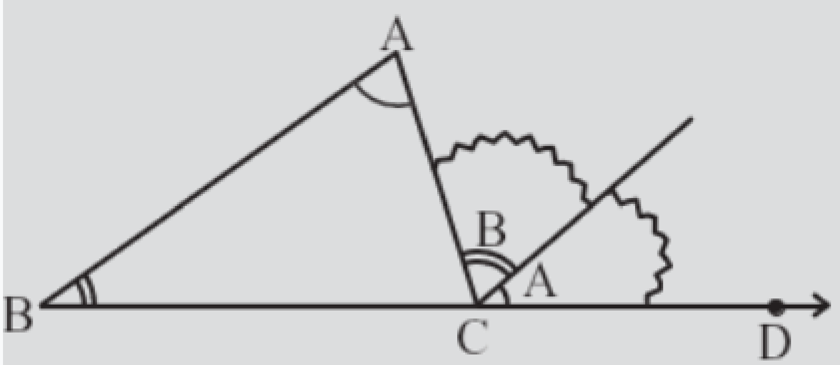


Fig 6.8

Proof Outline

- Draw line CE parallel to BA through C.
- Use alternate interior and corresponding angles to show $\angle ACD = \angle A + \angle B$.

Worked Example

Find angle x if $\angle A = 50^\circ$, exterior angle $\angle ACD = 110^\circ$:

$$110^\circ = 50^\circ + x \implies x = 60^\circ$$

Practice Set

- Find unknown angles using exterior angle property.
- Prove exterior angle property for different triangles.
- Apply exterior angle property in word problems.

Answer Key

- Exterior angle equals sum of opposite interior angles.
- Use to find unknown angles.

Quick Reference

Exterior angle = sum of two opposite interior angles.

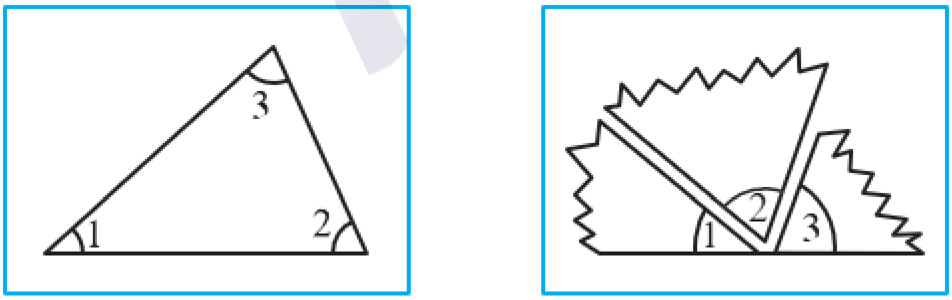
Glossary

- **Exterior Angle:** Angle formed by extending a side of a triangle
- **Interior Opposite Angles:** Two angles inside the triangle opposite to the exterior angle

Angle Sum Property of a Triangle

The sum of the three interior angles of a triangle is always 180°.

$$\angle A + \angle B + \angle C = 180^\circ$$



(i)

(ii) **Fig 6.13**

Proof Using Exterior Angle Property

- Extend side BC to D.
- Use exterior angle property: $\angle ACD = \angle A + \angle B$.
- Since $\angle ACD + \angle C = 180^\circ$, add $\angle C$ to both sides.
- Conclude $\angle A + \angle B + \angle C = 180^\circ$.

Worked Example

Find the third angle of $\triangle PQR$ if $\angle Q = 47^\circ$ and $\angle R = 52^\circ$:

$$\angle P = 180^\circ - 47^\circ - 52^\circ = 81^\circ$$

Practice Set

- Find missing angles in triangles.
- Classify triangles based on angle measures.
- Prove angle sum property using different methods.

Answer Key

- Sum of interior angles is 180°.
- Use subtraction to find unknown angles.

Quick Reference

Sum of interior angles of a triangle = 180°.

Glossary

- **Interior Angles:** Angles inside the triangle

Special Triangles: Equilateral and Isosceles

Equilateral Triangle: All three sides are equal, and all three angles measure 60°.

Isosceles Triangle: Two sides are equal, and the base angles opposite these sides are equal.

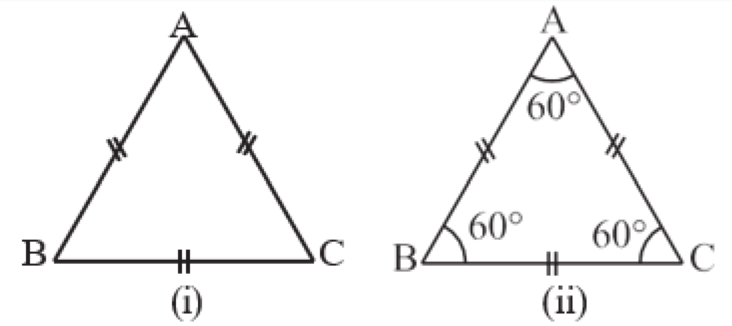


Fig 6.19

Properties

- Equilateral triangle is also equiangular.
- In isosceles triangle, altitude from vertex opposite equal sides bisects the base and angles.

Worked Example

Find the base angles of an isosceles triangle with vertex angle 40°:

$$2x + 40^\circ = 180^\circ \implies 2x = 140^\circ \implies x = 70^\circ$$

Practice Set

- Find unknown angles in isosceles triangles.
- Prove properties of equilateral triangles.
- Classify triangles based on side lengths.

Answer Key

- Base angles in isosceles triangle are equal.
- Equilateral triangle has all sides and angles equal.

Quick Reference

Equilateral: all sides and angles equal; Isosceles: two sides and base angles equal.

Glossary

- **Equilateral Triangle:** Triangle with all sides equal
- **Isosceles Triangle:** Triangle with two sides equal
- **Base Angles:** Angles opposite equal sides in isosceles triangle

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

- $AB + BC > AC$
- $BC + CA > AB$
- $CA + AB > BC$

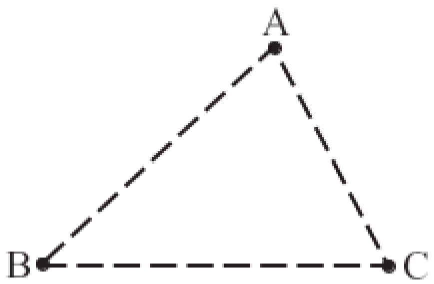


Fig 6.21

Worked Example

Check if a triangle with sides 10.2 cm, 5.8 cm, and 4.5 cm is possible:

- 4.5 + 5.8 = 10.3 > 10.2 (True)
- 5.8 + 10.2 = 16 > 4.5 (True)
- 10.2 + 4.5 = 14.7 > 5.8 (True)

All conditions hold, so the triangle is possible.

Practice Set

- Given two sides, find possible range for third side.
- Verify triangle inequality for given side lengths.
- Form triangles with given sticks and check validity.

Answer Key

- Third side length must be greater than difference and less than sum of other two sides.

Quick Reference

For sides a, b, c: $|a - b| < c < a + b$

Glossary

- **Triangle Inequality:** Sum of lengths of any two sides > third side

Right-Angled Triangles and Pythagorean Theorem

In a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides.

For $\triangle ABC$ right-angled at B, with sides AB, BC, and hypotenuse AC:

$$AC^2 = AB^2 + BC^2$$

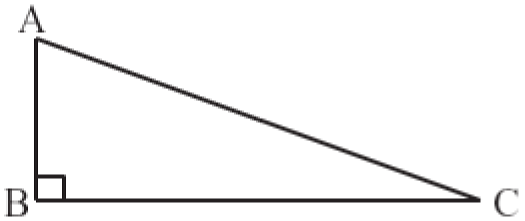


Fig 6.23

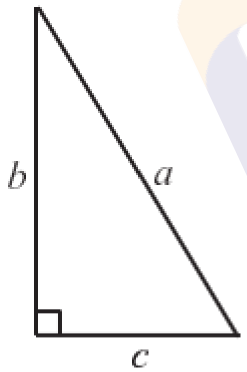


Fig 6.24

Worked Example

Find the hypotenuse AB if AC = 5 cm, BC = 12 cm in right-angled $\triangle ABC$ at C:

$$AB^2 = AC^2 + BC^2 = 5^2 + 12^2 = 25 + 144 = 169$$

$$AB = \sqrt{169} = 13 \text{ cm}$$

Practice Set

- Find unknown sides in right triangles using Pythagorean theorem.
- Verify if given sides form a right triangle.
- Apply theorem in word problems.

Answer Key

- Use $c^2 = a^2 + b^2$ to find unknown sides.
- Check if $a^2 + b^2 = c^2$ to confirm right triangle.

Quick Reference

Pythagorean theorem: $c^2 = a^2 + b^2$ where c is hypotenuse.

Glossary

- **Hypotenuse:** Side opposite right angle, longest side
- **Legs:** Other two sides forming right angle