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Introduction to Rational Numbers

We extend the number system from natural numbers to whole numbers, then to integers, and now to rational numbers. Rational numbers include all numbers that can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$. This includes fractions, integers, and decimals that can be converted into fractions.

Concept Explanation

Rational numbers are numbers that can be expressed as a ratio of two integers. They include all fractions and integers.

Formula Derivation

A rational number is defined as $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$.

Worked Illustrations

Examples of rational numbers: $\frac{4}{5}$, $-\frac{3}{4}$, 0 , $2 = \frac{2}{1}$.

Solved Examples

Is $\frac{2}{3}$ rational? Yes, since numerator and denominator are integers and denominator is not zero.

Practice Set

- List ten rational numbers.
- Is 0.5 a rational number? Express it as a fraction.

Answer Key

- Examples: $\frac{1}{2}$, $-\frac{3}{7}$, 0 , 5 , -2 , $\frac{7}{3}$, $-\frac{4}{9}$, 1.25 , -1.5 , 0.75
- Yes, $0.5 = \frac{5}{10} = \frac{1}{2}$

Quick Reference

Rational numbers = numbers expressible as $\frac{p}{q}$, $p, q \in \mathbb{Z}$, $q \neq 0$.

Glossary

- **Rational Number:** Number expressible as a ratio of two integers.
- **Numerator:** Top part of a fraction.
- **Denominator:** Bottom part of a fraction, non-zero.

Equivalent Rational Numbers

Equivalent rational numbers represent the same value but have different numerators and denominators. They are obtained by multiplying or dividing numerator and denominator by the same non-zero integer.

Concept Explanation

Two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if $a \times d = b \times c$.

Formula Derivation

Given $\frac{a}{b}$, multiplying numerator and denominator by $k \neq 0$ gives $\frac{a \times k}{b \times k}$, which is equivalent.

Worked Illustrations

$$-\frac{2}{3} = -\frac{4}{6} = \frac{10}{-15} = -\frac{10}{15}$$

Solved Examples

Show that $\frac{5}{4} = \frac{20}{16} = \frac{25}{20} = \frac{-15}{-12}$.

Practice Set

- Find three equivalent rational numbers for $\frac{3}{7}$.
- Simplify $\frac{10}{-15}$ to its standard form.

Answer Key

- Examples: $\frac{6}{14}$, $\frac{9}{21}$, $\frac{12}{28}$

- $\frac{10}{-15} = -\frac{2}{3}$

Quick Reference

Equivalent rational numbers: multiply/divide numerator and denominator by same non-zero integer.

Glossary

- **Equivalent Rational Numbers:** Different fractions representing the same number.

Positive and Negative Rational Numbers

Rational numbers can be positive or negative depending on the signs of numerator and denominator.

Concept Explanation

If numerator and denominator have the same sign, the rational number is positive; if signs differ, it is negative.

Formula Derivation

For $\frac{p}{q}$, sign is positive if $p \times q > 0$, negative if $p \times q < 0$.

Worked Illustrations

$$\frac{3}{8} > 0, -\frac{3}{8} < 0, \frac{8}{-3} = -\frac{8}{3} < 0, -\frac{3}{-5} = \frac{3}{5} > 0.$$

Solved Examples

Identify sign of $\frac{-2}{3}$, $\frac{5}{-7}$, $\frac{-2}{-9}$.

Practice Set

- List five positive rational numbers.
- List five negative rational numbers.

Answer Key

- Positive: $\frac{1}{2}$, $\frac{3}{4}$, 2, 5, $\frac{7}{8}$
- Negative: $-\frac{1}{2}$, $-\frac{3}{4}$, -2, -5, $-\frac{7}{8}$

Quick Reference

Sign of rational number depends on signs of numerator and denominator.

Glossary

- **Positive Rational Number:** Both numerator and denominator have same sign.
- **Negative Rational Number:** Numerator and denominator have opposite signs.

Rational Numbers on a Number Line

Rational numbers can be represented on a number line, with positive numbers to the right of zero and negative numbers to the left.

Concept Explanation

Mark rational numbers by dividing intervals between integers into equal parts according to the denominator.

Formula Derivation

Distance from zero for $\frac{p}{q}$ is $\frac{|p|}{q}$ units; sign determines side of zero.

Worked Illustrations

Mark $\frac{1}{2}$ at midpoint between 0 and 1; $-\frac{1}{2}$ at midpoint between 0 and -1.

Solved Examples

Mark $-\frac{3}{2}$ on number line; it lies left of zero, 1.5 units away.

Practice Set

- Mark $\frac{3}{4}$ and $-\frac{3}{4}$ on number line.
- Mark $\frac{5}{3}$ and $-\frac{5}{3}$.

Answer Key

- $\frac{3}{4}$ is three-quarters to the right of zero; $-\frac{3}{4}$ is three-quarters to the left.
- $\frac{5}{3}$ is 1 and $\frac{2}{3}$ units right; $-\frac{5}{3}$ is same distance left.

Quick Reference

Positive rational numbers lie right of zero; negative lie left.

Glossary

- **Number Line:** Visual representation of numbers on a line.

Standard Form of Rational Numbers

A rational number is in standard form if its denominator is positive and numerator and denominator have no common factors other than 1.

Concept Explanation

Reduce rational numbers by dividing numerator and denominator by their highest common factor (HCF).

Formula Derivation

Standard form: $\frac{p}{q}$ with $q > 0$ and $\text{gcd}(p, q) = 1$.

Worked Illustrations

Reduce $\frac{-45}{30}$ by dividing numerator and denominator by 15 to get $\frac{-3}{2}$.

Solved Examples

Reduce $\frac{36}{-24}$ and $\frac{-3}{-15}$ to standard form.

Practice Set

- Reduce $\frac{-18}{45}$ to standard form.
- Reduce $\frac{-12}{18}$ to standard form.

Answer Key

- $\frac{-18}{45} = \frac{-2}{5}$
- $\frac{-12}{18} = \frac{-2}{3}$

Quick Reference

Divide numerator and denominator by HCF; denominator positive.

Glossary

- **Standard Form:** Simplest form of a rational number with positive denominator.
- **HCF:** Highest common factor of two integers.

Comparison of Rational Numbers

Rational numbers can be compared by converting to common denominators or using number line positions.

Concept Explanation

For positive rational numbers, larger numerator with same denominator means larger number. For negative rational numbers, the order reverses.

Formula Derivation

To compare $\frac{a}{b}$ and $\frac{c}{d}$, find common denominator bd and compare ad and cb .

Worked Illustrations

Compare $-\frac{1}{2}$ and $-\frac{1}{5}$: $-\frac{1}{5} > -\frac{1}{2}$ because $\frac{1}{5} < \frac{1}{2}$.

Solved Examples

Compare $-\frac{7}{5}$ and $-\frac{5}{3}$ by comparing $\frac{7}{5}$ and $\frac{5}{3}$ and reversing inequality.

Practice Set

- Compare $\frac{3}{4}$ and $\frac{2}{3}$.
- Compare $-\frac{3}{7}$ and $-\frac{2}{5}$.

Answer Key

- $\frac{3}{4} > \frac{2}{3}$
- $-\frac{3}{7} < -\frac{2}{5}$

Quick Reference

Positive rational numbers: larger numerator means larger number; negative rational numbers: reverse order.

Glossary

- **Comparison:** Determining which number is greater or smaller.

Rational Numbers Between Two Rational Numbers

Between any two rational numbers, there are infinitely many rational numbers.

Concept Explanation

By finding equivalent fractions with common denominators, we can list rational numbers between two given rational numbers.

Formula Derivation

Between $\frac{a}{b}$ and $\frac{c}{d}$, convert to $\frac{a'}{n}$ and $\frac{c'}{n}$ with common denominator n , then list fractions with numerators between a' and c' .

Worked Illustrations

Between $-\frac{3}{5} = -\frac{9}{15}$ and $-\frac{1}{3} = -\frac{5}{15}$, rational numbers include $-\frac{8}{15}$, $-\frac{7}{15}$, $-\frac{6}{15}$.

Solved Examples

List three rational numbers between -2 and -1 by expressing as $\frac{-10}{5}$ and $\frac{-5}{5}$ and choosing intermediate numerators.

Practice Set

- Find five rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

- Find three rational numbers between $-\frac{5}{7}$ and $-\frac{3}{8}$.

Answer Key

- Examples: $\frac{3}{8}, \frac{2}{5}, \frac{5}{12}, \frac{3}{7}, \frac{7}{16}$
- Examples: $-\frac{4}{7}, -\frac{13}{28}, -\frac{3}{6}$

Quick Reference

Infinite rational numbers exist between any two rational numbers.

Glossary

- **Density Property:** Between any two rational numbers, there are infinitely many rational numbers.

Operations on Rational Numbers

Addition

To add rational numbers with the same denominator, add numerators and keep denominator same:

$$\frac{a}{q} + \frac{b}{q} = \frac{a + b}{q}$$

For different denominators, find LCM m , convert to equivalent fractions, then add.

Subtraction

Subtraction is addition of additive inverse:

$$\frac{a}{q} - \frac{b}{r} = \frac{a}{q} + \left(-\frac{b}{r}\right)$$

Find LCM of denominators, convert, then add.

Multiplication

Multiply numerators and denominators:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Multiplying by integer n is $\frac{a}{b} \times n = \frac{a \times n}{b}$.

Division

Divide by multiplying by reciprocal:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \quad (c \neq 0)$$

Worked Illustrations

Addition: $\frac{7}{3} + \left(-\frac{5}{3}\right) = \frac{2}{3}$

Subtraction: $\frac{5}{7} - \frac{3}{8} = \frac{19}{56}$

Multiplication: $-\frac{3}{5} \times 2 = -\frac{6}{5}$

Division: $\frac{4}{9} \div \left(-\frac{5}{7}\right) = -\frac{28}{45}$

Solved Examples

Find $\frac{-11}{5} + \frac{7}{5} = \frac{-4}{5}$

Find $\frac{5}{7} - \frac{3}{8} = \frac{19}{56}$

Find $\frac{-2}{9} \times (-5) = \frac{10}{9}$

Find $\frac{4}{9} \div \left(-\frac{5}{7}\right) = -\frac{28}{45}$

Practice Set

- Calculate $-\frac{13}{7} + \frac{6}{7}$.
- Calculate $\frac{7}{9} - \frac{2}{5}$.
- Calculate $\frac{-3}{5} \times 7$.
- Calculate $\frac{-6}{5} \div \left(-\frac{2}{3}\right)$.

Answer Key

- $-\frac{7}{7} = -1$
- $\frac{29}{45}$
- $-\frac{21}{5}$
- $\frac{18}{10} = \frac{9}{5}$

Quick Reference

Operations on rational numbers follow fraction rules with attention to signs and denominators.

Glossary

- **Additive Inverse:** Number which when added to original number gives zero.
- **Reciprocal:** Number which when multiplied by original number gives one.