Introduction to Symmetry

Symmetry is a fundamental geometrical concept observed widely in nature and human creations. It involves balanced and proportionate similarity found in two halves of an object or design. Symmetry is extensively used in art, architecture, engineering, and nature, where objects exhibit balanced patterns.

Line symmetry, also called reflection symmetry, occurs when a figure can be folded along a line (axis of symmetry) such that the two halves coincide exactly. This concept can be explored through activities like creating symmetrical picture albums, ink-dot designs, and paper-cut patterns.

Examples of line symmetry include shapes like the letter "T" with a vertical line of symmetry, an arrow with a horizontal line of symmetry, and the letter "M" with a vertical line of symmetry. The line of symmetry divides the figure into two mirror-image halves.

Worked Illustration

Consider a shape resembling the letter "T" with a vertical dashed line through its center. Folding along this line results in two matching halves, confirming the line as a line of symmetry.

Practice Set

- Identify lines of symmetry in various alphabets and natural objects.
- Draw shapes and mark their lines of symmetry.
- Fold paper shapes to verify symmetry lines physically.

Answer Key

- Vertical line through "T" is a line of symmetry.
- Horizontal line through an arrow shape is a line of symmetry.
- Diagonal line through certain shapes may or may not be a line of symmetry depending on the figure.

Quick Reference

- Line of symmetry divides a figure into two identical halves.
- Folding along the line results in coinciding halves.
- Lines can be vertical, horizontal, or diagonal.

Glossary

- **Symmetry:** Balanced and proportionate similarity between two halves of a figure.
- **Line of Symmetry:** A line dividing a figure into two mirror-image halves.
- **Reflection Symmetry:** Another term for line symmetry.

Lines of Symmetry for Regular Polygons

A polygon is a closed figure made of line segments. A regular polygon has all sides and angles equal. Examples include equilateral triangles, squares, regular pentagons, and regular hexagons.

Concept Explanation

Each regular polygon has as many lines of symmetry as it has sides. These lines pass through vertices and midpoints of opposite sides, dividing the polygon into symmetrical halves.

Formula Derivation

Sum of interior angles of an n-sided polygon:

$$\text{Sum of interior angles} = (n-2) \times 180^\circ$$

Each interior angle of a regular polygon:

$$\text{Each interior angle} = \frac{(n-2) \times 180^\circ}{n}$$

Each exterior angle of a regular polygon:

$$\text{Each exterior angle} = 180^\circ - \text{Each interior angle} = \frac{360^\circ}{n}$$

Worked Illustration

Equilateral triangle (n=3):

$$\text{Each interior angle} = \frac{(3-2)\times 180^\circ}{3} = 60^\circ$$

Lines of symmetry: 3

Square (n=4):

$$\text{Each interior angle} = \frac{(4-2)\times 180^\circ}{4} = 90^\circ$$

Lines of symmetry: 4

Regular pentagon (n=5):

$$\text{Each interior angle} = \frac{(5-2)\times 180^\circ}{5} = 108^\circ$$

Lines of symmetry: 5

Regular hexagon (n=6):

$$\text{Each interior angle} = \frac{(6-2)\times 180^\circ}{6} = 120^\circ$$

Lines of symmetry: 6

Solved Example

Find the number of lines of symmetry in a regular octagon.

Solution:

For an octagon, n = 8.

Number of lines of symmetry = n = 8.

Practice Set

- Calculate interior and exterior angles of regular polygons with sides 7, 8, and 10.
- Draw regular polygons and mark their lines of symmetry.
- Prove that the diagonals of a square bisect each other at right angles.

Answer Key

- Interior angle of heptagon (7 sides): 128.57°
- Exterior angle of octagon (8 sides): 45°
- Lines of symmetry in decagon (10 sides): 10

Quick Reference

- Regular polygon: all sides and angles equal.
- Number of lines of symmetry = number of sides.
- Sum of interior angles = (n-2) × 180°.

Glossary

- **Regular Polygon:** Polygon with all sides and angles equal.
- **Interior Angle:** Angle inside the polygon at each vertex.

- **Exterior Angle:** Angle between a side and the extension of an adjacent side.
- **Line of Symmetry:** Line dividing a figure into two mirror-image halves.

Rotational Symmetry

Rotational symmetry occurs when a figure can be rotated about a fixed point (center of rotation) by an angle less than 360° and still look exactly the same as the original figure.

Concept Explanation

The angle of rotation is the measure of the turn. A full turn is 360°. The order of rotational symmetry is the number of times the figure matches itself during a full 360° rotation.

For example, a square has rotational symmetry of order 4 because it looks the same after rotations of 90°, 180°, 270°, and 360°.

Formula Derivation

Order of rotational symmetry n relates to the angle of rotation θ by:

$$\theta = \frac{360^\circ}{n}$$

Worked Illustration

Consider a square with center X and a point P at one corner. Rotating the square by 90° clockwise about X moves P to the next corner. After four such rotations (4 × 90° = 360°), the square returns to its original position.

Solved Example

Find the order of rotational symmetry of an equilateral triangle.

Solution:

For an equilateral triangle, the angle of rotation is:

$$\theta = \frac{360^\circ}{3} = 120^\circ$$

Order of rotational symmetry is 3.

Practice Set

- Determine the order of rotational symmetry for regular pentagon and hexagon.
- Identify rotational symmetry in everyday objects like wheels and road signs.
- Draw shapes with rotational symmetry of order 2 and 3.

Answer Key

- Regular pentagon: order 5
- Regular hexagon: order 6
- Road signs with circular shapes often have rotational symmetry of order 2 or more.

Quick Reference

- Rotational symmetry: figure looks the same after rotation by θ.
- Order of rotational symmetry $n = \frac{360^\circ}{\theta}$.
- Center of rotation is the fixed point about which rotation occurs.

Glossary

- **Rotational Symmetry:** Symmetry where a figure matches itself after rotation.
- **Order of Rotational Symmetry:** Number of times a figure matches itself in 360° rotation.
- **Angle of Rotation:** The angle through which a figure is rotated.
- **Center of Rotation:** The fixed point about which rotation occurs.

Line Symmetry and Rotational Symmetry

Some shapes exhibit only line symmetry, some only rotational symmetry, and some both. For example, a square has four lines of symmetry and rotational symmetry of order 4.

The circle is a perfect symmetrical figure with infinite lines of symmetry (every diameter) and rotational symmetry for every angle.

Concept Explanation

Line symmetry involves reflection about a line, while rotational symmetry involves rotation about a point. Understanding both helps in analyzing shapes and patterns.

Worked Illustration

A square has four lines of symmetry: two diagonals and two lines through midpoints of opposite sides. It also has rotational symmetry of order 4 about its center.

Solved Example

Identify the symmetry properties of the letter "H".

Solution:

- Line symmetry: Yes, vertical line of symmetry.
- Rotational symmetry: Yes, order 2 (180° rotation).

Practice Set

- Classify English alphabets based on line and rotational symmetry.
- Draw shapes with both line and rotational symmetry.
- Identify symmetry in natural and man-made objects.

Answer Key

- Letter "E": line symmetry (vertical), no rotational symmetry.
- Letter "I": line symmetry (vertical and horizontal), rotational symmetry order 2.
- Letter "O": infinite lines of symmetry, infinite rotational symmetry.

Quick Reference

- Line symmetry: reflection about a line.
- Rotational symmetry: rotation about a point.
- Some shapes have both symmetries.

Glossary

- **Line Symmetry:** Symmetry about a line dividing a figure into mirror images.
- **Rotational Symmetry:** Symmetry about a point where figure matches after rotation.
- **Order of Rotational Symmetry:** Number of times figure matches in 360° rotation.