Constructing Triangles

To construct a triangle when the lengths of all three sides are given, follow these steps:

1. Draw a line segment equal to one side (e.g., AB).
2. With point A as center and radius equal to the length of the second side (AC), draw an arc.
3. With point B as center and radius equal to the length of the third side (BC), draw another arc intersecting the first.
4. The intersection point of the arcs is point C.
5. Join points A to C and B to C to complete triangle ABC.

Triangle Inequality Theorem: For any triangle with sides a, b, and c, the following must hold:

$$a + b > c, \quad b + c > a, \quad c + a > b$$

This ensures the triangle can exist.

Worked Example: Given sides 3 cm, 4 cm, and 5 cm, verify if a triangle can be formed.

Check sums:

$$3 + 4 = 7 > 5, \quad 4 + 5 = 9 > 3, \quad 5 + 3 = 8 > 4$$

All conditions hold, so the triangle can be constructed.

Practice Set

- Level 1: Construct a triangle with sides 5 cm, 6 cm, and 7 cm.
- Level 2: Verify if sides 2 cm, 2 cm, and 5 cm can form a triangle.
- Level 3: Given sides 7 cm, 10 cm, and 18 cm, determine if a triangle is possible and construct if yes.

Answer Key

- Level 1: Triangle possible; construct using compass arcs as described.
- Level 2: Triangle not possible since 2 + 2 = 4 < 5.
- Level 3: Triangle not possible since 7 + 10 = 17 < 18.

Quick Reference

Condition	Formula
Triangle Inequality	a + b > c, b + c > a, c + a > b

Glossary

- **Vertex:** A corner point of a triangle.
- **Side:** A line segment connecting two vertices.
- **Arc:** Part of a circle used in construction.

Equilateral Triangles

An equilateral triangle has all three sides equal and all three interior angles equal to 60°.

Construction Steps:

1. Draw a line segment AB of the desired length.
2. With A as center and radius AB, draw a circle.
3. With B as center and radius AB, draw another circle intersecting the first at point C.
4. Join points A to C and B to C to form triangle ABC.

Formula: Each angle in an equilateral triangle is 60°.

Worked Example: Construct an equilateral triangle with side length 5 cm.

Follow the construction steps with AB = 5 cm.

Practice Set

- Level 1: Construct an equilateral triangle with side 3 cm.
- Level 2: Prove that all angles in an equilateral triangle are 60° using the angle sum property.
- Level 3: Given an equilateral triangle, find the length of the altitude.

Answer Key

- Level 1: Use compass and straightedge as per construction steps.
- Level 2: Sum of angles = 180°, all equal, so each angle = 180°/3 = 60°.
- Level 3: Altitude $h = \frac{\sqrt{3}}{2} \times \text{side}$.

Quick Reference

Property	Value
Each angle	60°
Altitude	$\frac{\sqrt{3}}{2} \times \text{side}$

Glossary

- **Equilateral Triangle:** Triangle with all sides and angles equal.
- **Altitude:** Perpendicular from a vertex to the opposite side.

Angle Sum Property of Triangles

The sum of the interior angles of any triangle is always 180°.

Proof:

Consider triangle ABC. Draw a line parallel to BC through point A. The alternate interior angles formed are equal to angles at B and C. Adding these with angle at A gives 180°.

Formula:

$$\angle A + \angle B + \angle C = 180^\circ$$

Worked Example: If two angles of a triangle are 50° and 60°, find the third angle.

$$\angle C = 180^\circ - (50^\circ + 60^\circ) = 70^\circ$$

Exterior Angle Property

An exterior angle of a triangle equals the sum of the two opposite interior angles.

If $\angle ACD$ is an exterior angle at vertex C, then:

$$\angle ACD = \angle A + \angle B$$

Practice Set

- Level 1: Find the missing angle if two angles are 40° and 70°.
- Level 2: Prove the exterior angle property for a triangle.
- Level 3: In triangle ABC, if $\angle A = 35^\circ$ and $\angle B = 65^\circ$, find the exterior angle at C.

Answer Key

- Level 1: Missing angle = 180° - (40° + 70°) = 70°.
- Level 2: Use parallel line and alternate interior angles to prove.
- Level 3: Exterior angle at C = 35° + 65° = 100°.

Quick Reference

Property	Formula
Sum of interior angles	180°
Exterior angle	Sum of opposite interior angles

Glossary

- **Interior Angle:** Angle inside the triangle at a vertex.

- **Exterior Angle:** Angle formed outside the triangle by extending a side.

Altitudes of Triangles

An altitude of a triangle is a perpendicular segment from a vertex to the line containing the opposite side.

Every triangle has three altitudes, one from each vertex.

Construction Steps:

1. Draw triangle ABC.
2. To construct altitude from vertex A, draw a perpendicular from A to side BC, meeting at D.
3. Repeat for vertices B and C.

Properties:

- Altitudes may intersect inside or outside the triangle.
- The point of concurrency of altitudes is called the orthocenter.

Worked Example: Construct altitudes of triangle ABC and locate the orthocenter.

Use set square or compass to draw perpendiculars from each vertex to opposite sides.

Practice Set

- Level 1: Construct altitude from vertex A in triangle ABC.
- Level 2: Prove that altitudes intersect at a single point.

- Level 3: Given coordinates of triangle vertices, find equations of altitudes and their intersection point.

Answer Key

- Level 1: Use perpendicular construction methods.
- Level 2: Use geometric proofs involving perpendiculars and concurrency.
- Level 3: Solve simultaneous equations of altitudes.

Quick Reference

Term	Definition
Altitude	Perpendicular from vertex to opposite side
Orthocenter	Point where altitudes meet

Glossary

- **Altitude:** Perpendicular segment from vertex to opposite side.
- **Orthocenter:** Intersection point of altitudes.

Types of Triangles

Triangles are classified based on sides and angles.

Based on Sides

- **Equilateral:** All sides equal; all angles 60°.
- **Isosceles:** Two sides equal; angles opposite equal sides equal.
- **Scalene:** All sides and angles different.

Based on Angles

- **Acute:** All angles less than 90°.
- **Right:** One angle exactly 90°; side opposite is hypotenuse.
- **Obtuse:** One angle greater than 90°.

Worked Example: Classify a triangle with sides 5 cm, 5 cm, and 8 cm.

Two sides equal → Isosceles.

Check angles to determine angle-based type.

Practice Set

- Level 1: Identify type of triangle with sides 3 cm, 4 cm, 5 cm.
- Level 2: Classify triangle with angles 40°, 90°, 50°.
- Level 3: Prove properties of isosceles triangle angles.

Answer Key

- Level 1: Scalene and right-angled.
- Level 2: Right-angled triangle.
- Level 3: Angles opposite equal sides are equal by congruence.

Quick Reference

Type	Property
Equilateral	All sides and angles equal
Isosceles	Two sides and two angles equal

Scalene	All sides and angles different
Acute	All angles < 90°
Right	One angle = 90°
Obtuse	One angle > 90°

Glossary

- **Hypotenuse:** Side opposite right angle in right triangle.
- **Congruence:** Equality of sides and angles.

Circle Touching and Intersection Cases

Two circles can relate in the following ways based on their centers and radii:

- **Externally touching:** Circles touch at exactly one point outside.
- **Internally touching:** One circle lies inside the other, touching at one point.
- **Intersecting:** Circles intersect at two points.
- **Non-touching:** Circles neither touch nor intersect.

Condition for two circles with centers A and B and radii r_1 and r_2:

Let distance between centers be d = |AB|.

- Touch externally if $d = r_1 + r_2$
- Touch internally if $d = |r_1 - r_2|$
- Intersect if $|r_1 - r_2| < d < r_1 + r_2$
- No intersection if $d > r_1 + r_2$ or $d < |r_1 - r_2|$

Worked Example: Two circles with radii 3 cm and 4 cm have centers 7 cm apart. Determine their relation.

Check:

$$|3 - 4| = 1 < 7 < 3 + 4 = 7$$

Since 7 equals 7, circles touch externally.

Practice Set

- Level 1: For circles with radii 5 cm and 2 cm, find distance for external touching.
- Level 2: Determine if circles with radii 6 cm and 4 cm and center distance 1 cm intersect.
- Level 3: Prove the conditions for circle intersection using distance and radii.

Answer Key

- Level 1: Distance = 5 + 2 = 7 cm.
- Level 2: Since $1 < |6 - 4| = 2$, circles do not intersect.
- Level 3: Use triangle inequality and circle definitions.

Quick Reference

Condition	Relation
$d = r_1 + r_2$	Touch externally
$d = \|r_1 - r_2\|$	Touch internally
$\|r_1 - r_2\| < d < r_1 + r_2$	Intersect
$d > r_1 + r_2$ or $d < \|r_1 - r_2\|$	No intersection

Glossary

- **Radius:** Distance from center to any point on circle.
- **Center:** Fixed point equidistant from all points on circle.
- **Chord:** Line segment joining two points on circle.