# Geometric Twins

Geometric twins refer to figures that are exact replicas of each other in shape and size, known as congruent figures. To recreate a symbol or figure exactly, measurements such as side lengths and angles are essential.

## Concept Explanation

To recreate a figure like the checkmark inside a square on a signboard, tracing is one method but impractical for large symbols. Instead, taking precise measurements of the figure's components allows exact reconstruction.

For example, naming corner points A, B, and C, and measuring the lengths of arms AB and BC alone is insufficient to fix the figure uniquely, as multiple figures can have the same arm lengths but different angles.

Including the measure of the angle $\angle ABC$ along with the arm lengths $AB$ and $BC$ fixes the shape and size uniquely, enabling exact replication.

## Formula Derivation

Given two sides and the included angle, the figure is fixed uniquely. This is the basis of the SAS (Side-Angle-Side) condition for congruence.

## Worked Illustration

Given $AB = 4$ cm, $BC = 8$ cm, and $\angle ABC = 80^\circ$, the figure can be constructed by:

- Drawing segment $AB$ of length 4 cm.
- At point B, constructing an angle of 80°.
- Drawing segment $BC$ of length 8 cm along the angle.

## Solved Examples

**Example:** Construct a figure with $AB = 4$ cm, $BC = 8$ cm, and $\angle ABC = 80^\circ$.

**Solution:**

1. Draw segment $AB$ of length 4 cm.
2. At point B, use a protractor to construct an angle of 80°.
3. From point B, draw segment $BC$ of length 8 cm along the angle.
4. Connect points A and C to complete the figure.

## Practice Set

### Level 1 – Easy

- Draw a figure with $AB = 3$ cm, $BC = 5$ cm, and $\angle ABC = 60^\circ$.
- Identify if two figures with $AB = 6$ cm, $BC = 6$ cm, and $\angle ABC = 90^\circ$ are congruent.

### Level 2 – Moderate

- Given $AB = 7$ cm, $BC = 9$ cm, and $\angle ABC = 45^\circ$, construct the figure and verify congruence with a given figure.
- Explain why arm lengths alone are insufficient to guarantee congruence.

### Level 3 – Challenging

- Prove that two figures with the same arm lengths but different angles are not congruent.
- Given multiple figures with the same arm lengths, determine which are congruent by measuring angles.

## Answer Key

- Level 1: Figures with given measurements can be constructed using ruler and protractor; congruence is confirmed by matching side lengths and angles.
- Level 2: Construction steps as above; arm lengths alone do not fix the angle, so multiple non-congruent figures exist.
- Level 3: Figures with different angles but same arm lengths differ in shape; congruence requires equal corresponding sides and angles.

## Quick Reference

| Measurement | Determines |
|---|---|
| Two sides only | Multiple figures possible (not unique) |
| Two sides + included angle (SAS) | Unique figure (congruent) |

## Glossary

- **Congruent Figures:** Figures that have the same shape and size.
- **Included Angle:** The angle formed between two given sides.

- **Arm Lengths:** Lengths of the sides forming an angle.
- **SAS Condition:** Side-Angle-Side condition for congruence.

## Congruence of Triangles

Two triangles are congruent if they have exactly the same size and shape, meaning all corresponding sides and angles are equal. Various conditions guarantee triangle congruence.

### Concept Explanation

To replicate a triangular frame exactly, measuring all three side lengths is sufficient. Angles need not be measured separately because side lengths determine the triangle uniquely.

However, measuring only angles is insufficient to guarantee congruence, as triangles with the same angles can be similar but differ in size.

### Formula Derivation

Conditions for triangle congruence include:

- **SSS (Side Side Side):** All three sides equal.
- **SAS (Side Angle Side):** Two sides and the included angle equal.
- **ASA (Angle Side Angle):** Two angles and the included side equal.
- **AAS (Angle Angle Side):** Two angles and a non-included side equal.
- **RHS (Right angle Hypotenuse Side):** In right triangles, hypotenuse and one side equal.

### Worked Illustrations

**SSS Condition:** Triangles with sides 4 cm, 6 cm, and 8 cm are congruent.

**SAS Condition:** Triangles with two sides 6 cm and 5 cm and included angle 30° are congruent.

**ASA Condition:** Triangles with two angles 50° and 30°, and included side 5 cm are congruent.

**AAS Condition:** Triangles with two angles 35° and 75°, and a non-included side 4 cm are congruent.

**RHS Condition:** Right triangles with hypotenuse 5 cm and one side 4 cm are congruent.

## Solved Examples

**Example 1:** Given $\triangle ABC$ with sides 40 cm, 60 cm, and 80 cm, construct a congruent triangle.

**Solution:** Draw side 60 cm, then draw arcs of radii 40 cm and 80 cm from endpoints to locate the third vertex.

**Example 2:** Are triangles with sides 3.5 cm, 5 cm, and 6 cm congruent if arranged differently?

**Solution:** Check side correspondence; if all sides match in order, triangles are congruent.

## Practice Set

### Level 1 – Easy

- Construct triangles with sides 5 cm, 7 cm, and 9 cm.
- Identify congruent triangles from given side lengths.

### Level 2 – Moderate

- Prove congruence using SAS and ASA conditions for given triangles.
- Determine if triangles with two equal sides and a non-included angle are congruent.

### Level 3 – Challenging

- Prove that SSA condition does not guarantee congruence with examples.
- Use RHS condition to prove congruence of right triangles in complex figures.

## Answer Key

- Level 1: Triangles constructed with given sides are congruent by SSS.
- Level 2: SAS and ASA conditions guarantee congruence; SSA does not always.
- Level 3: SSA counterexamples exist; RHS condition applies to right triangles.

## Quick Reference

| Condition | Description |
|---|---|
| SSS | All three sides equal |
| SAS | Two sides and included angle equal |
| ASA | Two angles and included side equal |
| AAS | Two angles and non-included side equal |
| RHS | Right angle, hypotenuse, and one side equal |

## Glossary

- **Congruent Triangles:** Triangles with equal corresponding sides and angles.
- **Included Angle:** Angle between two given sides.
- **Hypotenuse:** Side opposite the right angle in a right triangle.
- **SSS, SAS, ASA, AAS, RHS:** Conditions for triangle congruence.

## Angles of Isosceles and Equilateral Triangles

Using congruence, we can deduce important properties of isosceles and equilateral triangles.

### Concept Explanation

In an isosceles triangle, two sides are equal. The angles opposite these equal sides are also equal.

In an equilateral triangle, all three sides are equal, so all three angles are equal.

### Formula Derivation

For isosceles triangle $\triangle ABC$ with $AB = AC$:

- Construct altitude $AD$ perpendicular to $BC$.
- Triangles $\triangle ADB$ and $\triangle ADC$ are congruent by RHS condition.
- Therefore, $\angle B = \angle C$.

For equilateral triangle, since all sides are equal, all angles are equal and sum to 180°, so each angle is 60°.

### Worked Illustrations

Given $\triangle ABC$ is isosceles with $AB = AC$ and $\angle A = 80^\circ$, find $\angle B$ and $\angle C$.

Sum of angles in triangle is 180°, so $\angle B + \angle C = 100^\circ$. Since $\angle B = \angle C$, each is 50°.

## Solved Examples

**Example:** Prove that in an equilateral triangle, each angle is 60°.

**Solution:** Since all sides are equal, all angles are equal. Sum of angles is 180°, so each angle is $\frac{180^\circ}{3} = 60^\circ$.

## Practice Set

### Level 1 – Easy

- Find the base angles of an isosceles triangle with vertex angle 40°.
- Verify that all angles in an equilateral triangle are 60°.

### Level 2 – Moderate

- Prove that the altitude in an isosceles triangle bisects the base.
- Construct an equilateral triangle and measure its angles.

### Level 3 – Challenging

- Use congruence to prove properties of isosceles triangles in complex figures.
- Explore real-life structures using congruent triangles.

## Answer Key

- Level 1: Base angles are 70°; equilateral triangle angles are 60° each.
- Level 2: Altitude bisects base by congruence; construction confirms equal angles.
- Level 3: Use RHS and SSS conditions to prove properties; identify congruent triangles in structures.

## Quick Reference

| Triangle Type | Equal Sides | Equal Angles |
|---|---|---|
| Isosceles | Two sides | Angles opposite equal sides |
| Equilateral | All sides | All angles (60° each) |

## Glossary

- **Isosceles Triangle:** Triangle with two equal sides.
- **Equilateral Triangle:** Triangle with all sides equal.
- **Altitude:** Perpendicular segment from a vertex to the opposite side.
- **Base Angles:** Angles opposite the equal sides in an isosceles triangle.