

- A Quick Recap of Integers
- Multiplication of Integers
- Patterns in Integer Multiplication
- Division of Integers
- Expressions Using Integers

A Quick Recap of Integers

This section focuses on understanding integers through practical problem-solving and visualization using number lines and token models.

Concept Explanation

Given two numbers, their sum and difference can be used to find the numbers themselves by solving simple equations or by trial and error. The difference is defined as the first number minus the second number.

Formula Derivation

Let the two numbers be x and y . Given:

$$x + y = S$$

$$x - y = D$$

Adding the two equations:

$$(x + y) + (x - y) = S + D$$

$$2x = S + D$$

$$x = \frac{S + D}{2}$$

Subtracting the second from the first:

$$(x + y) - (x - y) = S - D$$

$$2y = S - D$$

$$y = \frac{S - D}{2}$$

Worked Illustration

For sum = 25 and difference = 11:

$$x = \frac{25 + 11}{2} = \frac{36}{2} = 18$$

$$y = \frac{25 - 11}{2} = \frac{14}{2} = 7$$

Thus, the numbers are 18 and 7.

Solved Examples

Find two numbers whose sum is 27 and difference is 9.

Solution:

$$x = \frac{27 + 9}{2} = 18$$

$$y = \frac{27 - 9}{2} = 9$$

Numbers are 18 and 9.

Practice Set

- Level 1: Find two numbers with sum 20 and difference 4.

- Level 2: Find two numbers with sum -10 and difference 6.
- Level 3: Find two numbers with sum 0 and difference -14.

Answer Key

- Level 1: 12 and 8
- Level 2: -2 and -8
- Level 3: 7 and -7

Quick Reference

Use formulas:

$$x = \frac{S + D}{2}, \quad y = \frac{S - D}{2}$$

Glossary

- **Sum:** The result of addition.
- **Difference:** The result of subtraction.
- **Integer:** Whole numbers including negatives, zero, and positives.

Multiplication of Integers

This section explains multiplication of integers using token models and explores the rules governing signs in multiplication.

Concept Explanation

Multiplication can be visualized as repeated addition. Tokens represent positive and negative integers: green tokens for positive (+1) and red tokens for negative (-1).

Formula Derivation

Multiplying two integers a and b follows these rules:

- Positive \times Positive = Positive
- Positive \times Negative = Negative
- Negative \times Positive = Negative
- Negative \times Negative = Positive

These can be derived using token addition and removal models.

Worked Illustration

Example: $4 \times (-2)$

Interpret as placing 2 negative tokens 4 times, total tokens = 8 negatives, so product = -8.

Solved Examples

Calculate $(-4) \times (-2)$:

Removing 2 negative tokens 4 times results in 8 positive tokens, so product = +8.

Practice Set

- Level 1: Find $3 \times (-2)$
- Level 2: Find $(-5) \times (-2)$
- Level 3: Find $(-7) \times 3$

Answer Key

- Level 1: -6
- Level 2: 10
- Level 3: -21

Quick Reference

Multiplication sign rules:

Multiplier	Multiplicand	Product Sign
+	+	+
+	-	-
-	+	-
-	-	+

Glossary

- **Multiplier:** The number of times to add.
- **Multiplicand:** The number being multiplied.
- **Token Model:** Visual representation using positive and negative tokens.

Patterns in Integer Multiplication

This section explores patterns observed in multiplication involving positive and negative integers.

Concept Explanation

Multiplying a fixed integer by decreasing integers shows a pattern in the product values, increasing or decreasing by the multiplicand.

Formula Derivation

For positive multiplicand m and multiplier n , decreasing n by 1 decreases product by m :

$$m \times n - m = m \times (n - 1)$$

For negative multiplicand $-m$, decreasing n by 1 increases product by m :

$$(-m) \times n + m = (-m) \times (n - 1)$$

Worked Illustration

Multiplying 3 by 4, 3, 2, 1, 0:

12, 9, 6, 3, 0 (decreasing by 3 each step)

Multiplying -3 by 4, 3, 2, 1, 0:

-12, -9, -6, -3, 0 (increasing by 3 each step)

Solved Examples

Find the product of -3 and -2 :

Using pattern, product is 6.

Practice Set

- Level 1: Find products of 5×3 , 5×2 , 5×1
- Level 2: Find products of -4×3 , -4×2 , -4×1
- Level 3: Find products of -6×-2 , -6×-3 , -6×-4

Answer Key

- Level 1: 15, 10, 5
- Level 2: -12 , -8 , -4
- Level 3: 12, 18, 24

Quick Reference

Multiplication patterns depend on signs and magnitude of multiplicand and multiplier.

Glossary

- **Multiplicand:** Number being multiplied.
- **Multiplier:** Number of times multiplicand is added.

Division of Integers

This section explains division of integers by relating it to multiplication and sign rules.

Concept Explanation

Division is the inverse operation of multiplication. To divide integers, find the number which when multiplied by the divisor gives the dividend.

Formula Derivation

For integers a and b ($b \neq 0$):

- $a \div b = c$ if and only if $b \times c = a$
- Sign rules for division are similar to multiplication:
 - Positive \div Positive = Positive
 - Positive \div Negative = Negative
 - Negative \div Positive = Negative
 - Negative \div Negative = Positive

Worked Illustration

Calculate $(-100) \div 25$:

Find c such that $25 \times c = -100$. Since $25 \times (-4) = -100$, $c = -4$.

Solved Examples

Calculate $(-100) \div (-4)$:

Find c such that $(-4) \times c = -100$. Since $(-4) \times 25 = -100$, $c = 25$.

Practice Set

- Level 1: Find $36 \div (-18)$
- Level 2: Find $(-46) \div (-23)$
- Level 3: Find $50 \div (-25)$

Answer Key

- Level 1: -2
- Level 2: 2
- Level 3: -2

Quick Reference

Division sign rules mirror multiplication sign rules.

Glossary

- **Dividend:** Number being divided.
- **Divisor:** Number by which dividend is divided.
- **Quotient:** Result of division.

Expressions Using Integers

This section discusses properties of integer multiplication and distributive property over addition.

Concept Explanation

Integer multiplication is associative and commutative. The distributive property holds for integers.

Formula Derivation

Associative property:

$$a \times (b \times c) = (a \times b) \times c$$

Commutative property:

$$a \times b = b \times a$$

Distributive property:

$$a \times (b + c) = a \times b + a \times c$$

Worked Illustration

Calculate $5 \times (-3) \times 4$ in different groupings:

$$(5 \times -3) \times 4 = -15 \times 4 = -60$$

$$5 \times (-3 \times 4) = 5 \times -12 = -60$$

$$(5 \times 4) \times -3 = 20 \times -3 = -60$$

Solved Examples

Check distributive property for $(-2) \times (4 + (-3))$:

$$(-2) \times 1 = -2$$

$$(-2) \times 4 + (-2) \times (-3) = -8 + 6 = -2$$

Practice Set

- Level 1: Calculate $(-5) \times (18 + (-3))$
- Level 2: Calculate $(-7) \times 4 \times (-1)$
- Level 3: Calculate $(-2) \times (-1) \times (-5) \times (-3)$

Answer Key

- Level 1: -75
- Level 2: 28
- Level 3: 30

Quick Reference

Integer multiplication properties:

- Associative: $a \times (b \times c) = (a \times b) \times c$
- Commutative: $a \times b = b \times a$
- Distributive: $a \times (b + c) = a \times b + a \times c$

Glossary

- **Associative Property:** Grouping of factors does not affect product.
- **Commutative Property:** Order of factors does not affect product.
- **Distributive Property:** Multiplication distributes over addition.

Prepzy