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The Greatest of All

To find the largest square tile that can cover a rectangular floor without gaps or overlaps, the side length of the tile must be a common factor of the room's length and breadth. The largest such tile corresponds to the Highest Common Factor (HCF) of the two dimensions.

Formula Derivation

Given two numbers a and b, their HCF is the greatest number that divides both a and b exactly.

For example, for a room of dimensions 12 ft by 16 ft, the tile side length s must satisfy:

$$s \mid 12 \quad \text{and} \quad s \mid 16$$

where s divides both 12 and 16.

Worked Illustration

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 16: 1, 2, 4, 8, 16

Common factors: 1, 2, 4

Largest common factor (HCF) = 4 ft

Number of tiles needed = $\frac{12 \times 16}{4 \times 4} = \frac{192}{16} = 12$

Solved Example

Sameeksha wants to tile a 12 ft by 16 ft room with square tiles of whole number side length, using the fewest tiles possible. Find the tile size and number of tiles.

Solution:

Step 1: Find HCF of 12 and 16.

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 16: 1, 2, 4, 8, 16

HCF = 4

Step 2: Tile side length = 4 ft

Step 3: Number of tiles = $\frac{12}{4} \times \frac{16}{4} = 3 \times 4 = 12$

Practice Set

- **Level 1 – Easy:** Find the HCF of 8 and 12.
- **Level 2 – Moderate:** A rectangular garden is 18 m by 24 m. Find the largest square tile size to cover the garden without gaps.
- **Level 3 – Challenging:** A farmer has two fields measuring 84 m and 108 m in length. He wants to fence them with equal length sections. Find the length of each section to minimize the number of sections.

Answer Key

- HCF(8,12) = 4
- HCF(18,24) = 6 m
- HCF(84,108) = 12 m

Quick Reference

The HCF of two numbers is the greatest number that divides both exactly. It can be found by listing factors or using prime factorisation.

Glossary

- **Factor:** A number that divides another number exactly.
- **Highest Common Factor (HCF):** The greatest common factor of two or more numbers.
- **Prime Factorisation:** Expressing a number as a product of prime numbers.

Prime Factorisation

Prime factorisation is the process of expressing a composite number as a product of prime numbers. This method simplifies finding factors, HCF, and LCM.

Procedure for Prime Factorisation

Use a factor tree or division method:

- Start with the number.
- Divide by the smallest prime number possible.
- Repeat the process with the quotient until all factors are prime.

Worked Illustration

Find prime factorisation of 90:

$$90 = 2 \times 45 = 2 \times 3 \times 15 = 2 \times 3 \times 3 \times 5$$

Solved Example

Find prime factorisation of 225:

Divide 225 by 5: 225 ÷ 5 = 45

Divide 45 by 5: 45 ÷ 5 = 9

Divide 9 by 3: 9 ÷ 3 = 3

Divide 3 by 3: 3 ÷ 3 = 1

Prime factors: 5, 5, 3, 3

So, $225 = 3 \times 3 \times 5 \times 5$

Practice Set

- **Level 1 – Easy:** Prime factorise 60.
- **Level 2 – Moderate:** Prime factorise 180 and 210.
- **Level 3 – Challenging:** Prime factorise 840 and list all its factors.

Answer Key

- 60 = 2 × 2 × 3 × 5
- 180 = 2 × 2 × 3 × 3 × 5, 210 = 2 × 3 × 5 × 7
- 840 = 2 × 2 × 2 × 3 × 5 × 7; factors include 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84, 105, 120, 140, 168, 210, 280, 420, 840

Quick Reference

Prime factorisation breaks down numbers into prime components, aiding in finding HCF and LCM.

Glossary

- **Prime Number:** A number greater than 1 with only two factors: 1 and itself.
- **Composite Number:** A number with more than two factors.
- **Factor Tree:** A diagram used to break down numbers into prime factors.

Finding the HCF Using Prime Factorisation

The HCF of two or more numbers is found by multiplying the common prime factors with the smallest powers.

Formula Derivation

Given numbers a and b with prime factorisations:

$$a = p_1^{e_1} \times p_2^{e_2} \times \cdots \quad b = p_1^{f_1} \times p_2^{f_2} \times \cdots$$

The HCF is:

$$\text{HCF}(a, b) = p_1^{\min(e_1, f_1)} \times p_2^{\min(e_2, f_2)} \times \cdots$$

Worked Illustration

Find HCF of 45 and 75:

Prime factorisation:

45 = $3^2 \times 5$

75 = 3×5^2

Common prime factors with minimum powers: $3^{\min(2,1)} = 3^1$, $5^{\min(1,2)} = 5^1$

HCF = $3 \times 5 = 15$

Solved Example

Find HCF of 112 and 84:

Prime factorisation:

112 = $2^4 \times 7$

84 = $2^2 \times 3 \times 7$

Common prime factors with minimum powers: 2^2, 7^1

HCF = $2^2 \times 7 = 4 \times 7 = 28$

Practice Set

- **Level 1 – Easy:** Find HCF of 24 and 36.
- **Level 2 – Moderate:** Find HCF of 60, 72, and 90.
- **Level 3 – Challenging:** Find HCF of 225 and 750.

Answer Key

- HCF(24,36) = 12
- HCF(60,72,90) = 6
- HCF(225,750) = 75

Quick Reference

HCF is the product of common prime factors with the smallest exponents.

Glossary

- **Minimum Power:** The smaller exponent of a prime factor in the factorisations.

Least, but not Last

The Lowest Common Multiple (LCM) of two or more numbers is the smallest number that is a multiple of all the numbers.

Formula Derivation

Given numbers a and b with prime factorisations:

$$a = p_1^{e_1} \times p_2^{e_2} \times \cdots \quad b = p_1^{f_1} \times p_2^{f_2} \times \cdots$$

The LCM is:

$$\text{LCM}(a, b) = p_1^{\max(e_1, f_1)} \times p_2^{\max(e_2, f_2)} \times \cdots$$

Worked Illustration

Find LCM of 14 and 35:

Prime factorisation:

14 = 2×7

35 = 5×7

LCM includes all prime factors with highest powers: $2^1, 5^1, 7^1$

LCM = $2 \times 5 \times 7 = 70$

Solved Example

Find LCM of 96 and 360:

Prime factorisation:

96 = $2^5 \times 3$

360 = $2^3 \times 3^2 \times 5$

LCM includes 2^5 (max of 5 and 3), 3^2 (max of 1 and 2), and 5^1

LCM = $2^5 \times 3^2 \times 5 = 32 \times 9 \times 5 = 1440$

Practice Set

- **Level 1 – Easy:** Find LCM of 4 and 6.
- **Level 2 – Moderate:** Find LCM of 30 and 72.
- **Level 3 – Challenging:** Find LCM of 105, 195, and 65.

Answer Key

- LCM(4,6) = 12
- LCM(30,72) = 360
- LCM(105,195,65) = 2145

Quick Reference

LCM is the product of all prime factors with the highest powers appearing in any number.

Glossary

- **Maximum Power:** The larger exponent of a prime factor in the factorisations.

Patterns, Properties, and a Pretty Procedure

When one number is a factor of another, the HCF is the smaller number, and the LCM is the larger number.

For numbers n and kn where k is a positive integer:

$$\text{HCF}(n, kn) = n$$

$$\text{LCM}(n, kn) = kn$$

General Statements

- If two numbers are consecutive, their HCF is 1.
- If two numbers are co-prime, their HCF is 1.
- If both numbers are multiples of a number m, their HCF is at least m.

Efficient Procedure for HCF and LCM

Use the division method:

- Divide both numbers by their common prime factors stepwise.
- Multiply all common prime factors to get HCF.
- Multiply all prime factors used in division (including those dividing only one number) to get LCM.

Property Involving HCF and LCM

For any two numbers a and b:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Practice Set

- **Level 1 – Easy:** Find HCF and LCM of 18 and 24.
- **Level 2 – Moderate:** Verify the property $\text{HCF} \times \text{LCM} = a \times b$ for 45 and 75.
- **Level 3 – Challenging:** Find the number of cows if they pass through gates of 3, 5, and 7 equally, with less than 200 cows.

Answer Key

- HCF(18,24) = 6, LCM(18,24) = 72
- HCF(45,75) = 15, LCM(45,75) = 225, and $15 \times 225 = 45 \times 75 = 3375$
- Number of cows = 105 (LCM of 3, 5, and 7)

Quick Reference

HCF and LCM are related by the product of the two numbers.

Glossary

- **Co-prime Numbers:** Two numbers whose HCF is 1.
- **Consecutive Numbers:** Numbers that follow each other in order.
- **Generalisation:** A statement that holds true for all cases of a pattern.