Geometric Constructions

Geometric constructions involve creating precise figures using only an unmarked ruler and a compass. A fundamental construction is the creation of symmetrical shapes, such as eyes, where arcs are drawn from two centers to form upper and lower arcs with equal radii, ensuring symmetry.

Formula Derivation: For symmetry in the eye shape, the distances satisfy $AX = AY = BX = BY$, where A and B are centers of arcs and X, Y are points on the base line XY.

Worked Illustration: To find points A and B, draw arcs of equal radius from points X and Y above and below the line segment XY. The intersections of these arcs are points A and B.

Solved Example: Construct the perpendicular bisector of a line segment XY:

- Draw arcs of equal radius from X and Y above and below XY.
- Mark the intersection points as A and B.
- Join A and B; this line AB is the perpendicular bisector of XY.

Practice Set:

- Level 1: Construct the perpendicular bisector of a 6 cm line segment.
- Level 2: Construct an eye shape with given base XY and find centers A and B.
- Level 3: Prove that any point equidistant from X and Y lies on the perpendicular bisector of XY.

Answer Key:

- Level 1: Use compass arcs from endpoints; join intersections.
- Level 2: Use equal radius arcs from X and Y; locate A and B.
- Level 3: Use triangle congruence (SAS) to prove the property.

Quick Reference: The perpendicular bisector of a segment is the locus of points equidistant from the segment's endpoints.

Glossary:

- *Perpendicular bisector:* A line that divides a segment into two equal parts at 90°.
- *Congruence:* Equality of shapes and sizes of geometric figures.

Construction of Perpendicular Bisector

To construct the perpendicular bisector of a line segment XY using only a ruler and compass:

1. Choose a radius greater than half of XY.
2. Draw arcs from X and Y above the line; mark intersection A.
3. Draw arcs from X and Y below the line; mark intersection B.
4. Join A and B; this line is the perpendicular bisector of XY.

Worked Illustration: The line AB intersects XY at O, the midpoint, and forms right angles with XY.

Solved Example: Given $XY = 8$ cm, construct its perpendicular bisector.

- Set compass radius > 4 cm.
- Draw arcs from X and Y above and below XY.
- Mark intersections A and B.
- Join A and B; O is midpoint.

Practice Set:

- Level 1: Construct perpendicular bisector of 5 cm segment.
- Level 2: Prove AB is perpendicular bisector using triangle congruence.
- Level 3: Use rope method to construct perpendicular bisector and justify.

Answer Key:

- Level 1: Follow steps above with given length.
- Level 2: Show $\triangle AOX \cong \triangle AOY$ by SAS.
- Level 3: Rope midpoint pulled above and below XY marks A and B; AB is perpendicular bisector.

Quick Reference: The perpendicular bisector passes through the midpoint and is perpendicular to the segment.

Glossary:

- *Midpoint:* Point dividing a segment into two equal parts.
- *Rope method:* Using a rope folded to midpoint to construct perpendicular bisector.

Angle Bisection for a Design

Angle bisection divides an angle into two equal parts using ruler and compass.

Formula Derivation: To bisect $\angle XOY$, construct points A and B on rays OY and OX such that $OA = OB$. Draw arcs from A and B with equal radius intersecting at C. Line OC bisects $\angle XOY$.

Worked Illustration: Steps to bisect an angle:

1. Mark equal points A and B on rays.
2. Draw arcs from A and B with same radius.
3. Mark intersection C.
4. Draw ray OC; it bisects the angle.

Solved Example: Bisect a 90° angle to get 45°.

- Construct 90° angle.
- Bisect it using above steps.
- Resulting angle is 45°.

Practice Set:

- Level 1: Bisect 60° angle.
- Level 2: Construct 30° and 15° angles by successive bisection.
- Level 3: Construct an 8-petalled design by bisecting 360° into 8 equal angles.

Answer Key:

- Level 1: Follow angle bisection steps.
- Level 2: Bisect 60° to get 30°, bisect 30° to get 15°.
- Level 3: Divide 360° by 8 to get 45°; bisect 90° to get 45°.

Quick Reference: Angle bisector divides an angle into two equal angles.

Glossary:

- *Angle bisector:* Ray dividing an angle into two equal parts.

- *Congruent triangles:* Triangles with equal corresponding sides and angles.

Construction of a Line Parallel to the Given Line

To construct a line parallel to a given line m through a point B not on m, use the following method:

1. Draw a transversal l intersecting m at A and passing through B.
2. Copy the angle $\angle mAl$ at point B using angle copying method.
3. The new line through B forming the copied angle is parallel to m.

Worked Illustration: Use compass to copy arcs and distances to replicate the angle at B.

Solved Example: Construct a line parallel to m through B using compass and ruler.

- Draw arcs from A and B with same radius.
- Measure arc length between intersections on m.
- Transfer this length to arc at B.
- Draw line through B and new point.

Practice Set:

- Level 1: Construct parallel line through given point.
- Level 2: Construct multiple pairs of parallel lines at different orientations.
- Level 3: Construct a 12-pointed star using parallel lines and angle constructions.

Answer Key:

- Level 1: Follow angle copying steps.
- Level 2: Repeat construction at various points.

- Level 3: Use angle division and parallel line construction.

Quick Reference: Parallel lines have equal corresponding angles with a transversal.

Glossary:

- *Parallel lines:* Lines in the same plane that never intersect.
- *Transversal:* A line that intersects two or more lines.

Regular Hexagons

A regular hexagon has six equal sides and six equal angles of 120° each.

Formula Derivation: A regular hexagon can be divided into six equilateral triangles, each with angles 60°.

Worked Illustration: Construct a regular hexagon by:

1. Drawing a circle with center O and radius equal to side length.
2. Marking six points on the circle by stepping off the radius length around the circumference.
3. Joining these points consecutively to form the hexagon.

Solved Example: Construct a regular hexagon with side length 4 cm.

- Draw circle with radius 4 cm.
- Mark points every 60° on circumference.
- Join points to form hexagon.

Practice Set:

- Level 1: Construct regular hexagon with side 3 cm.
- Level 2: Construct 30°, 60°, and 120° angles using hexagon properties.
- Level 3: Construct a 6-pointed star using overlapping equilateral triangles.

Answer Key:

- Level 1: Use compass and ruler as above.
- Level 2: Use equilateral triangle and hexagon angle properties.
- Level 3: Overlap two equilateral triangles rotated 60°.

Quick Reference: Regular hexagon angles are 120°, sides equal; can be constructed from equilateral triangles.

Glossary:

- *Regular polygon:* Polygon with all sides and angles equal.
- *Equilateral triangle:* Triangle with all sides equal and angles 60°.

Tiling

Tiling is covering a plane or region completely with shapes without gaps or overlaps.

Concept Explanation: Using shapes like squares, triangles, or hexagons to cover surfaces.

Worked Illustration: Tiling a rectangular grid with 2 × 1 tiles:

- For even dimensions, tiles can be arranged vertically or horizontally to cover the grid.

- For odd dimensions, tiling may not be possible due to area mismatch.

Solved Example: Can a 4 × 6 grid be tiled with 2 × 1 tiles?

- Yes, since total squares = 24 (even), arrange tiles in columns or rows.

Practice Set:

- Level 1: Tile a 4 × 8 grid with 2 × 1 tiles.
- Level 2: Determine if a 5 × 7 grid can be tiled with 2 × 1 tiles.
- Level 3: Prove using coloring argument why some grids are not tileable.

Answer Key:

- Level 1: Yes, arrange tiles vertically.
- Level 2: No, total squares odd (35), impossible.
- Level 3: Use checkerboard coloring; unequal black and white squares prevent tiling.

Quick Reference: Tiling requires total area divisible by tile area and compatible shape arrangement.

Glossary:

- *Tiling:* Covering a surface with shapes without gaps or overlaps.
- *Checkerboard coloring:* Coloring squares alternately to analyze tiling possibilities.

Tiling the Entire Plane

Tiling the entire plane involves repeating shapes infinitely without gaps or overlaps.

Concept Explanation: Regular polygons like squares, equilateral triangles, and regular hexagons can tile the plane.

Worked Illustration: Examples include:

- Squares arranged in a grid.
- Equilateral triangles forming tessellations.
- Regular hexagons fitting together like honeycombs.

Solved Example: Show that regular hexagons tile the plane:

- Each hexagon has internal angles 120°.
- Three hexagons meet at a point, angles sum to 360°.
- They fit without gaps or overlaps.

Practice Set:

- Level 1: Draw a tessellation using squares.
- Level 2: Create a tessellation with equilateral triangles.
- Level 3: Explore tessellations with irregular polygons or multiple shapes.

Answer Key:

- Level 1: Arrange squares in rows and columns.
- Level 2: Arrange triangles edge to edge.
- Level 3: Use known tessellation patterns or design new ones.

Quick Reference: Only certain regular polygons tile the plane: triangles, squares, hexagons.

Glossary:

- *Tessellation:* Repeating pattern of shapes covering a plane without gaps.
- *Regular polygon:* Polygon with equal sides and angles.