

- Find the Unknowns
- Solving Equations Systematically
- Generating Equations
- Mind the Mistake, Mend the Mistake
- A Pinch of History

Find the Unknowns

In problems involving unknown weights or values, we use algebraic expressions to represent unknown quantities and solve for them systematically. The key is to set up equations based on the balance or total given, then solve for the unknowns step-by-step.

Concept Explanation

When objects or weights are balanced or summed, the total on one side equals the total on the other. We represent unknown weights by variables (like x , y , or e) and write equations expressing these equalities.

Formula Derivation

For example, if two objects each weigh e and balance a total weight of 6, then:

$$2e = 6$$

Solving for e :

$$e = \frac{6}{2} = 3$$

Similarly, for multiple unknowns, set up equations based on the total weights and solve using addition, subtraction, multiplication, or division.

Worked Illustrations

- Two characters each weighing 2 balance a bar labeled 4: $2 + 2 = 4$.
- Characters weighing 4 and 3 balance a bar labeled 7: $4 + 3 = 7$.
- For sacks on scales, if two sacks balance 14 kg ($10 \text{ kg} + 4 \text{ kg}$), then one sack weighs $\frac{14}{2} = 7 \text{ kg}$.

Solved Examples

Example: On a balance scale, 5 bags balance 2 blocks of 10 kg, 1 block of 1 kg, and 2 bags. Find the weight of one bag.

Let the weight of one bag be b kg.

$$5b = 2 \times 10 + 1 + 2b$$

$$5b - 2b = 21$$

$$3b = 21$$

$$b = \frac{21}{3} = 7 \text{ kg}$$

Practice Set

Level 1 – Easy

- Two weights of 3 kg each balance a bar. What is the total weight?
- One sack weighs 5 kg. How much do 4 sacks weigh?

Level 2 – Moderate

- On a scale, 3 bags and 2 blocks of 4 kg balance 22 kg. Find the weight of one bag.
- Two unknown weights x and y sum to 15 kg. If $x = 2y$, find x and y .

Level 3 – Challenging

- On a scale, 90 sacks and a 50 kg block balance 60 sacks and a 500 kg block. Find the weight of one sack.
- Five bags balance 2 blocks of 10 kg, 1 block of 1 kg, and 2 bags. Find the weight of one bag.

Answer Key

- Level 1: 6 kg; 20 kg
- Level 2: Bag weighs 3 kg; $x = 10$, $y = 5$
- Level 3: Sack weighs 30 kg; Bag weighs 7 kg

Quick Reference

- Set up equations based on balance or total weight.
- Use algebraic operations to isolate unknowns.
- Check solutions by substituting back into original equations.

Glossary

- **Unknown:** A quantity represented by a variable whose value is to be found.
- **Equation:** A mathematical statement showing equality between two expressions.
- **Balance Scale:** A device showing equality of weights on both sides.

Solving Equations Systematically

Solving equations involves finding the value of unknown variables that make the equation true. Systematic methods use algebraic operations to isolate the unknown step-by-step.

Concept Explanation

Equations have two sides: Left Hand Side (LHS) and Right Hand Side (RHS). The goal is to perform the same operation on both sides to keep the equality and isolate the unknown.

Formula Derivation

Given an equation $Ax + B = Cx + D$, rearranging terms gives:

$$Ax - Cx = D - B$$

$$(A - C)x = D - B$$

$$x = \frac{D - B}{A - C}$$

This formula directly gives the solution for x .

Worked Illustrations

Example: Solve $5x - 4 = 7$.

$$5x - 4 + 4 = 7 + 4$$

$$5x = 11$$

$$x = \frac{11}{5}$$

Solved Examples

Example: Solve $6y + 7 = 4y + 21$.

$$6y + 7 - 4y = 4y + 21 - 4y$$

$$2y + 7 = 21$$

$$2y = 14$$

$$y = 7$$

Level 1 – Easy

- Solve $3x - 10 = 35$.
- Solve $5s = 3s$.

Level 2 – Moderate

- Solve $4(m + 6) - 8 = 2m - 4$.
- Solve $11y + (-5) = 61$.

Level 3 – Challenging

- Solve $2x + 1 = 6 - (2x - 3)$.
- Solve $10 - 5x = 3(x - 4) - 2(x - 7)$.

Answer Key

- Level 1: $x = 15$; $s = 0$
- Level 2: $m = 3$; $y = 6$
- Level 3: $x = 2$; $x = 3$

Quick Reference

- Perform inverse operations to remove constants and coefficients.
- Always perform the same operation on both sides.
- Check solutions by substitution.

Glossary

- **Inverse Operation:** An operation that reverses another (e.g., addition and subtraction).
- **Coefficient:** The numerical factor multiplying a variable.
- **Isolate:** To get the variable alone on one side of the equation.

Generating Equations

Generating equations involves writing algebraic expressions or equations that represent a given situation or satisfy a given solution.

Concept Explanation

Given a solution, we can create equations by reversing the solving process or by modeling real-life situations with variables and constants.

Formula Derivation

For a solution $y = 5$, possible equations include:

$$y + 1 = 6$$

$$3y = 15$$

These equations satisfy the solution $y = 5$.

Worked Illustrations

Example: Write an equation with solution $x = -2$.

$$x + 2 = 0$$

$$4x = -8$$

Solved Examples

Example: Model the situation: "A taxi charges ₹800 fixed plus ₹20 per km. Total cost is ₹2200. Find km traveled."

$$800 + 20k = 2200$$

$$20k = 1400$$

$$k = 70$$

Practice Set

Level 1 – Easy

- Write 3 equations with solution $x = 3$.
- Write an equation for "Sum of two numbers is 76, one is thrice the other."

Level 2 – Moderate

- Write an equation for "A number increased by 36 equals ten times itself."

- Write an equation for "A brick weighs 1 kg more than half its weight."

Level 3 – Challenging

- Write equations to model the problem: "Two friends save money monthly with different starting amounts and rates."
- Write an equation for the number of tiles in a pattern given by $3k + 1$.

Answer Key

- Level 1: $x - 3 = 0$, $2x = 6$, $x/2 = 1.5$
- Level 2: $x + 36 = 10x$, $w = \frac{1}{2}w + 1$
- Level 3: $4000 + 650m = 5050 + 500m$, $3k + 1 = n$

Quick Reference

- Identify variables and constants in the problem.
- Translate words into algebraic expressions.
- Use known formulas or patterns to write equations.

Glossary

- **Modeling:** Representing real-world situations with mathematical expressions.
- **Variable:** A symbol representing an unknown quantity.
- **Constant:** A fixed value in an expression or equation.

Mind the Mistake, Mend the Mistake

Careful step-by-step solving of equations is essential. Mistakes often occur in arithmetic or algebraic manipulation. Identifying and correcting these mistakes helps strengthen

understanding.

Concept Explanation

Check each step for correct operations: distributing multiplication, combining like terms, moving terms across the equal sign, and dividing or multiplying correctly.

Formula Derivation

For example, in solving $4(4q + 2) = 50$, distribute 4:

$$16q + 8 = 50$$

Then subtract 8 from both sides:

$$16q = 42$$

Divide both sides by 16:

$$q = \frac{42}{16} = \frac{21}{8}$$

Worked Illustrations

Check for errors such as incorrect subtraction or division, and correct them accordingly.

Solved Examples

Example: Solve $-2(3 - 4x) = 14$.

$$-6 + 8x = 14$$

$$8x = 14 + 6$$

$$8x = 20$$

$$x = \frac{20}{8} = \frac{5}{2}$$

Practice Set

Level 1 – Easy

- Identify mistakes in $6x + 9 = 66$ solved as $x + 9 = 11$.

Level 2 – Moderate

- Correct errors in solving $14y + 24 = 36$ given as $7y + 12 = 18$.

Level 3 – Challenging

- Find and fix mistakes in solving $4x - 5 = 9x + 8$ with incorrect steps.

Answer Key

- Level 1: Correct step is subtracting 9, not dividing.
- Level 2: Divide entire equation by 2, not partial terms.
- Level 3: Move all terms correctly and divide by coefficient.

Quick Reference

- Distribute multiplication correctly.
- Perform inverse operations on both sides.
- Check arithmetic carefully.

Glossary

- **Distribute:** Multiply a term across terms inside parentheses.
- **Inverse Operation:** Operation that reverses another.
- **Coefficient:** Number multiplying a variable.

A Pinch of History

Algebra, known as *bījagaṇita* in ancient India, has a rich history dating back to mathematicians like Brahmagupta and Bhāskarāchārya. The use of symbols to represent unknowns and systematic methods to solve equations were developed centuries ago.

Concept Explanation

Brahmagupta introduced methods to add, subtract, and multiply unknowns using letters. The term algebra comes from the Arabic word *al-jabr*, influenced by Indian mathematics.

Formula Derivation

General linear equations of the form $Ax + B = Cx + D$ were solved using the formula:

$$x = \frac{D - B}{A - C}$$

Worked Illustrations

Example from Bhāskarāchārya:

$$300 + 6x = 10x - 100$$

$$400 + 6x = 10x$$

$$400 = 4x$$

$$x = 100$$

Solved Examples

Using Brahmagupta's formula, solve $2x + 3 = 4x + 5$:

$$x = \frac{5 - 3}{2 - 4} = \frac{2}{-2} = -1$$

Practice Set

Level 1 – Easy

- Write equations using ancient notation for $2x + 1$.

Level 2 – Moderate

- Use Brahmagupta's formula to solve $5x + 4 = 3x + 8$.

Level 3 – Challenging

- Explain the historical significance of bījagaṇita and its influence on modern algebra.

Answer Key

- Level 1: $yā\ 2\ rū\ 1$
- Level 2: $x = \frac{8-4}{5-3} = 2$
- Level 3: Essay-style explanation.

Quick Reference

- Bījagaṇita means algebra in Sanskrit.
- Brahmagupta and Bhāskarāchārya contributed foundational methods.
- Algebra spread from India to the Arab world and Europe.

Glossary

- **Bījagaṇita:** Ancient Indian algebra.
- **Al-jabr:** Arabic term meaning "restoring" or "balancing," origin of the word algebra.
- **Equation:** A statement of equality between two expressions.

Prepzy