Introduction What is Mathematics?

Mathematics is the study of patterns and the reasons behind them, found everywhere in nature and daily life. It combines creativity and logic to explore and explain these patterns, leading to new discoveries and applications.

Understanding patterns helps us solve problems and make predictions, such as the movements of planets or genetic sequences. This chapter focuses on recognizing and analyzing patterns in numbers and shapes.

Concept Explanation

Mathematics identifies recurring structures and relationships, enabling us to describe and predict phenomena.

Formula Derivation

Not applicable for this introductory concept.

Worked Illustrations

Examples include the spiral patterns in sunflowers and the symmetry in snowflakes.

Solved Examples

Example: Recognizing the pattern in the sequence 2, 4, 6, 8, ... as even numbers increasing by 2.

Practice Set

- Level 1: Identify patterns in daily life objects.
- Level 2: Describe the pattern in a given number sequence.
- Level 3: Explain the significance of patterns in scientific discoveries.

Answer Key

- Level 1: Examples include stripes on animals, petals on flowers.
- Level 2: The sequence increases by 2 each time, representing even numbers.
- Level 3: Patterns help formulate laws like gravitation and genetic inheritance.

Quick Reference

Mathematics studies patterns to understand and explain natural and abstract phenomena.

Glossary

- **Pattern:** A repeated or regular arrangement.
- **Mathematics:** The science of numbers, quantities, and shapes.

Patterns in Numbers

Number sequences are ordered lists of numbers following a specific rule. Studying these sequences helps understand mathematical concepts and relationships.

Concept Explanation

Number sequences include counting numbers, odd/even numbers, square numbers, triangular numbers, Fibonacci numbers, and more.

Formula Derivation

Example: Triangular numbers are given by the formula $$ T_n = \frac{n(n+1)}{2} $$

Derivation:

Consider the sum of the first n natural numbers:

$a_1 + a_2 + ... + a_n = 1 + 2 + ... + n$

Pairing terms from start and end:

$a_1 + a_n = 1 + n = n + 1$

$a_2 + a_{n-1} = 2 + (n-1) = n + 1$

There are $\frac{n}{2}$ such pairs if n is even, so sum is:

a_1 + a_2 + … + a_n = \frac{n}{2} (n + 1) = \frac{n(n+1)}{2}

Worked Illustrations

Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, … where each term is the sum of the two preceding terms.

Solved Examples

Example: Find the 7th triangular number.

Solution:

t_7 = \frac{7 \times 8}{2} = 28

Practice Set

- Level 1: Write the first 10 even numbers.
- Level 2: Find the next three terms in the sequence 2, 4, 8, 16, …
- Level 3: Prove that the sum of the first n odd numbers is n^2.

Answer Key

- Level 1: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
- Level 2: 32, 64, 128 (each term doubles)

- Level 3: Sum of first n odd numbers = 1 + 3 + 5 + ... + (2n-1) = n^2 (proved by induction or visual proof)

Quick Reference

Sequence	Formula
Triangular Numbers	$T_n = \frac{n(n+1)}{2}$
Square Numbers	$S_n = n^2$
Fibonacci Numbers	$F_n = F_{n-1} + F_{n-2}$

Glossary

- **Sequence:** An ordered list of numbers.
- **Triangular Number:** Number of dots forming an equilateral triangle.
- **Fibonacci Sequence:** Sequence where each term is sum of two preceding terms.

Visualising Number Sequences

Number sequences can be represented pictorially to better understand their structure and growth.

Concept Explanation

Using dots or shapes arranged in patterns to represent sequences such as triangular numbers, square numbers, cubes, etc.

Formula Derivation

Visual proofs often accompany formulas, such as showing that the sum of odd numbers forms a square.

Worked Illustrations

Triangular numbers shown as dots forming triangles; square numbers as dots forming squares.

Solved Examples

Example: Visualize the 4th square number.

Solution: Arrange 16 dots in a 4 by 4 square.

Practice Set

- Level 1: Draw the first 5 triangular numbers using dots.
- Level 2: Represent the first 4 cube numbers pictorially.
- Level 3: Explain visually why the sum of first n odd numbers equals n^2.

Answer Key

- Level 1: Triangles with 1, 3, 6, 10, 15 dots respectively.
- Level 2: Cubes with 1, 8, 27, 64 dots arranged in 3D.
- Level 3: Visualize adding successive odd layers around a square.

Quick Reference

Visual models help understand numeric growth and relationships.

Glossary

- **Visualisation:** Representation of data or concepts using images.
- **Cube Number:** Number of dots forming a cube, n^3.

Relationship among Number Sequences

Number sequences often relate to each other through addition, multiplication, or geometric arrangements.

Concept Explanation

For example, the sum of the first n odd numbers equals the nth square number.

Formula Derivation

Sum of first n odd numbers:

a_1 + a_2 + ... + a_n = 1 + 3 + 5 + ... + (2n-1)

Number of terms = n

Sum = n^2

Proof by induction or visual arrangement of dots forming squares.

Worked Illustrations

Adding odd numbers stepwise to form perfect squares.

Solved Examples

Example: Calculate 1 + 3 + 5 + 7 + 9.

Solution:

the sum is 25, which is

5^2

.

Practice Set

- Level 1: Verify sum of first 3 odd numbers equals 9.
- Level 2: Show that sum of numbers 1 + 2 + 3 + 2 + 1 equals 9.
- Level 3: Prove the sum of first n odd numbers equals n^2 using induction.

Answer Key

- Level 1: 1 + 3 + 5 = 9 = 3^2
- Level 2: 1 + 2 + 3 + 2 + 1 = 9
- Level 3: Inductive proof showing base case and inductive step.

Quick Reference

Sum of first n odd numbers = n^2

Glossary

- **Induction:** A method of mathematical proof.
- **Odd Number:** Integer not divisible by 2.

Patterns in Shapes

Geometry studies patterns in shapes, including polygons, graphs, fractals, and stacked shapes.

Concept Explanation

Shape sequences involve regular polygons, complete graphs, stacked squares and triangles, and fractal patterns like the Koch snowflake.

Formula Derivation

Number of sides in regular polygons follows counting numbers: triangle (3), quadrilateral (4), pentagon (5), etc.

Worked Illustrations

Examples of polygons and their properties, graphs connecting vertices, and fractal iterations.

Solved Examples

Example: Identify the next shape in the sequence of polygons with sides 3, 4, 5, 6.

Solution: The next polygon has 7 sides, a heptagon.

Practice Set

- Level 1: Name polygons with 3 to 6 sides.
- Level 2: Describe the pattern in the number of edges in complete graphs K2 to K6.
- Level 3: Explain the iterative process of creating the Koch snowflake.

Answer Key

- Level 1: Triangle, quadrilateral, pentagon, hexagon.
- Level 2: Number of edges increases as $\frac{n(n-1)}{2}$ for Kn.
- Level 3: Each iteration adds smaller triangles to each side, increasing complexity.

Quick Reference

Shape	Number of Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6

Glossary

- **Polygon:** A closed shape with straight sides.
- **Fractal:** A complex pattern created by repeating a simple process.
- **Koch Snowflake:** A fractal formed by iterative addition of triangles.

Relationship to Number Sequences

Shape sequences relate to number sequences through properties like the number of sides corresponding to counting numbers.

Concept Explanation

Regular polygons have sides counted by natural numbers, linking shape patterns to number sequences.

Formula Derivation

Number of sides in polygons: 3, 4, 5, 6, ... corresponds to counting numbers.

Worked Illustrations

Examples of polygons and their side counts.

Solved Examples

Example: Find the next two terms in the sequence 243, 81, 27, 9, ...

Solution: Each term is divided by 3, so next terms are 3 and 1.

Practice Set

- Level 1: Identify the number of sides in a pentagon and hexagon.
- Level 2: Find the next three terms in the sequence 81, 27, 9, 3, ...

- Level 3: Explain the relationship between polygon sides and counting numbers.

Answer Key

- Level 1: Pentagon has 5 sides, hexagon has 6 sides.
- Level 2: Next terms are 1, $\frac{1}{3}$, $\frac{1}{9}$ if continued dividing by 3.
- Level 3: Polygon sides increase by 1, matching natural numbers.

Quick Reference

Polygon sides correspond to counting numbers.

Glossary

- **Counting Numbers:** Natural numbers starting from 1.
- **Sequence:** Ordered list of numbers following a rule.