Introduction

Symmetry is a property of a figure where it can be divided into two halves that are mirror images of each other. When folded along a specific line, called the **line of symmetry**, the two halves coincide exactly.

Line of Symmetry: A line that divides a figure into two identical parts such that one half is the mirror image of the other.

Reflection: The phenomenon where an image is formed as a mirror image of an object, preserving lengths and angles.

Rotational Symmetry: A figure has rotational symmetry if it looks the same after being rotated about a fixed point (called the center of rotation) by a certain angle less than 360°.

Formula Derivation

Reflection symmetry involves a line such that for every point P on one side, there exists a point P' on the other side such that the line is the perpendicular bisector of segment PP'.

Rotational symmetry is characterized by the angle $\theta = \frac{360^\circ}{n}$, where n is the order of rotational symmetry (number of times the figure maps onto itself during a full rotation).

Worked Illustration

Consider a square. It has 4 lines of symmetry (vertical, horizontal, and two diagonals) and rotational symmetry of order 4 with angle 90°.

Solved Example

Example: Find the order and angle of rotational symmetry of a square.

Solution:

The square maps onto itself 4 times during a full 360° rotation.

Order $n = 4$

Angle of rotational symmetry $\theta = \frac{360^\circ}{4} = 90^\circ$

Practice Set

- **Level 1 – Easy:** Identify the lines of symmetry in the letters A, H, and T.
- **Level 2 – Moderate:** Determine the order of rotational symmetry of an equilateral triangle.
- **Level 3 – Challenging:** Prove that a regular pentagon has 5 lines of symmetry and rotational symmetry of order 5.

Answer Key

- Level 1: A has 1 vertical line, H has 2 lines (vertical and horizontal), T has 1 vertical line.
- Level 2: Equilateral triangle has rotational symmetry order 3, angle 120°.
- Level 3: Each line of symmetry passes through a vertex and the midpoint of the opposite side; rotational symmetry order is 5 with angle 72°.

Quick Reference

Concept	Definition	Formula/Property
Line of Symmetry	Line dividing figure into mirror halves	Perpendicular bisector of corresponding points
Rotational Symmetry	Figure maps onto itself on rotation	Angle $= \frac{360^\circ}{n}$, Order n

Glossary

- **Symmetry:** Balanced and proportionate similarity between halves of a figure.
- **Line of Symmetry:** A line dividing a figure into two identical mirror-image parts.
- **Reflection:** Mirror image formation preserving size and shape.
- **Rotational Symmetry:** Property of a figure to look the same after rotation by certain angles.
- **Order of Rotational Symmetry:** Number of times a figure coincides with itself during a full rotation.
- **Angle of Rotational Symmetry:** Smallest angle of rotation for which the figure maps onto itself.

Symmetry

Symmetry in a figure means parts of the figure are repeated in a definite pattern such that the figure looks balanced and identical on folding or rotation.

Formula Derivation

For reflection symmetry, if L is the line of symmetry, then for any point $P(x, y)$, its mirror image $P'(x', y')$ satisfies:

$$d(P, L) = d(P', L) \quad \text{and} \quad L \text{ is the perpendicular bisector of } PP'$$

Worked Illustration

The Taj Mahal is an example of architectural symmetry, where the left and right halves are mirror images about a central vertical line.

Solved Example

Example: Show that the butterfly figure has a vertical line of symmetry.

Solution: Folding the butterfly along the vertical line divides it into two halves that coincide exactly, confirming vertical symmetry.

Practice Set

- **Level 1:** Identify symmetry in natural objects like leaves and flowers.
- **Level 2:** Draw a figure with exactly two lines of symmetry.
- **Level 3:** Prove that the letter "H" has two lines of symmetry.

Answer Key

- Level 1: Many leaves have bilateral symmetry.
- Level 2: A rectangle has two lines of symmetry (vertical and horizontal).
- Level 3: "H" is symmetric about vertical and horizontal lines through its center.

Quick Reference

Type	Example	Lines of Symmetry
Vertical	Butterfly	1
Horizontal	Arrow	1
Multiple	Square	4

Glossary

- **Symmetry:** Balanced repetition of parts in a figure.
- **Vertical Line of Symmetry:** Line dividing figure into left and right mirror halves.
- **Horizontal Line of Symmetry:** Line dividing figure into top and bottom mirror halves.
- **Diagonal Line of Symmetry:** Line dividing figure diagonally into mirror halves.

Line of Symmetry

The line of symmetry divides a figure into two identical parts that overlap exactly when folded along the line.

Formula Derivation

For a line of symmetry L, any point P on one side has a corresponding point P' on the other side such that L is the perpendicular bisector of segment PP'.

Worked Illustration

Letters like A, H, and M have vertical lines of symmetry, while letters like B and C have horizontal lines of symmetry.

Solved Example

Example: Identify the lines of symmetry in the letter "H".

Solution: The letter "H" has two lines of symmetry: one vertical and one horizontal, both passing through its center.

Practice Set

- **Level 1:** Draw the line of symmetry for the letter "A".
- **Level 2:** Find the number of lines of symmetry in an equilateral triangle.
- **Level 3:** Prove that a regular pentagon has five lines of symmetry.

Answer Key

- Level 1: One vertical line through the center of "A".
- Level 2: Three lines of symmetry, each passing through a vertex and the midpoint of the opposite side.
- Level 3: Each line passes through a vertex and the midpoint of the opposite side, totaling five lines.

Quick Reference

Shape	Lines of Symmetry
Equilateral Triangle	3
Square	4
Regular Pentagon	5

Glossary

- **Line of Symmetry:** Line dividing a figure into two identical mirror-image parts.
- **Vertical Line of Symmetry:** Line dividing figure into left and right halves.
- **Horizontal Line of Symmetry:** Line dividing figure into top and bottom halves.
- **Diagonal Line of Symmetry:** Line dividing figure diagonally into mirror halves.

Reflection

Reflection is the process where a figure is flipped over a line (line of symmetry) to produce a mirror image.

Formula Derivation

If the line of symmetry is the y-axis, the reflection of a point (x, y) is $(-x, y)$. For a general line, reflection formulas depend on the line's equation.

Worked Illustration

The letter "M" reflected in a plane mirror appears reversed left to right.

Solved Example

Example: Find the reflection of point $(3, 4)$ about the y-axis.

Solution: Reflection about y-axis changes x to $-x$, so the image is $(-3, 4)$.

Practice Set

- **Level 1:** Reflect the point $(2, 5)$ about the x-axis.
- **Level 2:** Find the reflection of $(1, -3)$ about the line $y = x$.

- **Level 3:** Prove that reflection preserves distances and angles.

Answer Key

- Level 1: Reflection about x-axis gives $(2, -5)$.
- Level 2: Reflection about $y = x$ swaps coordinates: $(-3, 1)$.
- Level 3: Reflection is an isometry; it preserves distances and angles by definition.

Quick Reference

Reflection Line	Image of Point (x, y)
y-axis	$(-x, y)$
x-axis	$(x, -y)$
y = x	(y, x)

Glossary

- **Reflection:** Flipping a figure over a line to produce a mirror image.
- **Plane Mirror:** A flat reflective surface producing virtual images.
- **Isometry:** A transformation preserving distances and angles.

Rotational Symmetry

A figure has rotational symmetry if it can be rotated about a fixed point (center of rotation) by an angle less than 360° and still look exactly the same.

Formula Derivation

The *order of rotational symmetry* n is the number of times the figure coincides with itself during a full 360° rotation.

The *angle of rotational symmetry* θ is given by:

$$\theta = \frac{360^\circ}{n}$$

Worked Illustration

A square has rotational symmetry of order 4 with angles 90°, 180°, 270°, and 360°.

Solved Example

Example: Find the order and angle of rotational symmetry of a pinwheel with 4 blades.

Solution:

The pinwheel looks the same after rotations of 90°, 180°, 270°, and 360°.

Order $n = 4$

Angle $\theta = \frac{360^\circ}{4} = 90^\circ$

Practice Set

- **Level 1:** Identify the order of rotational symmetry of an equilateral triangle.

- **Level 2:** Find the angle of rotational symmetry of a regular hexagon.
- **Level 3:** Prove that a circle has infinite order of rotational symmetry.

Answer Key

- Level 1: Order 3, angle 120°.
- Level 2: Order 6, angle 60°.
- Level 3: A circle coincides with itself at every angle of rotation, so infinite order.

Quick Reference

Shape	Order of Rotational Symmetry	Angle of Rotational Symmetry
Equilateral Triangle	3	120°
Square	4	90°
Regular Hexagon	6	60°
Circle	Infinite	Any angle

Glossary

- **Rotational Symmetry:** Property of a figure to look the same after rotation about a point.
- **Order of Rotational Symmetry:** Number of times a figure coincides with itself in one full rotation.
- **Angle of Rotational Symmetry:** Smallest angle for which the figure maps onto itself.
- **Center of Rotation:** Fixed point about which rotation occurs.