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# Measures of Central Tendency: Arithmetic Mean

**Concept Explanation:** Arithmetic Mean is the average of a set of numbers, calculated by dividing the sum of all observations by the total number of observations. It represents a central value of the data set.

## Key Definitions / Features:

- **Mean:**  $\bar{x} = \frac{\sum x}{N}$ , where  $x$  are observations and  $N$  is the number of observations.
- **Objectives of Statistical Average:** To summarize data, facilitate comparison, infer about population from sample, trace mathematical relationships, and aid decision making.
- **Essentials of a Good Average:** Rigidly defined, representative, easy to understand, stable against sampling fluctuations, absolute number, algebraically treatable, unaffected by extreme values, and easy to compute.
- **Kinds of Statistical Average:** Mathematical Averages and Positional Averages.
- **Types of Arithmetic Mean:** Simple Arithmetic Mean, Weighted Arithmetic Mean, Combined Mean.

## Methods of Calculating Simple Arithmetic Mean:

*For Individual Data Series:*

- **Direct Method:**  $\bar{x} = \frac{\sum x}{N}$
- **Shortcut Method:** Choose an assumed mean  $A$ , calculate deviations  $dx = x - A$ , then  $\bar{x} = A + \frac{\sum dx}{N}$
- **Step Deviation Method:** Choose a common factor  $C$ , calculate  $d' = \frac{x-A}{C}$ , then  $\bar{x} = A + \frac{\sum d'}{N} \times C$

*For Discrete Series:*

- **Direct Method:**  $\bar{x} = \frac{\sum fx}{\sum f}$ , where  $f$  is frequency.
- **Shortcut Method:**  $\bar{x} = A + \frac{\sum fdx}{\sum f}$
- **Step Deviation Method:**  $\bar{x} = A + \frac{\sum fd'}{\sum f} \times C$

*For Continuous Series:*

- Calculate mid-values of class intervals.
- Apply Direct, Shortcut, or Step Deviation methods similarly using mid-values and frequencies.

**Calculation in Cumulative Frequency Distribution:** Convert cumulative frequency to normal frequency distribution and apply any of the above methods.

**Combined Arithmetic Mean:**

For two groups with means  $\bar{x}_1, \bar{x}_2$  and sizes  $N_1, N_2$ :

$$\bar{x} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

**Weighted Arithmetic Mean:**

When items have different importance, weights  $W$  are assigned:

$$\bar{x}_w = \frac{\sum Wx}{\sum W}$$

## Properties of Arithmetic Mean:

- Mean changes proportionally if all observations are increased, decreased, multiplied, or divided by a constant.
- Sum of deviations from the mean is zero.
- Sum of squared deviations from the mean is minimum compared to any other value.

## Illustrative Example:

Calculate the mean of data: 5, 7, 9, 10, 12 using Direct Method.

$$\bar{x} = \frac{5 + 7 + 9 + 10 + 12}{5} = \frac{43}{5} = 8.6$$

## Practice Set:

- *Level 1:* Find the arithmetic mean of 3, 6, 9, 12.
- *Level 2:* Calculate the weighted mean of marks where weights are credits.
- *Level 3:* Given two groups with means and sizes, find combined mean.

## Answer Key:

- Level 1: Mean =  $(3+6+9+12)/4 = 7.5$
- Level 2: Use formula  $\bar{x}_w = \frac{\sum Wx}{\sum W}$
- Level 3: Use combined mean formula.

**Quick Reference:** Arithmetic Mean formulas and methods summarized above.

## Glossary:

- **Mean:** Average value.
- **Frequency (f):** Number of occurrences.
- **Assumed Mean (A):** Reference value for shortcut methods.
- **Step Deviation (d')**: Scaled deviation for simplification.

## Measures of Central Tendency: Median (M)

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**Concept Explanation:** Median is the middle value in an ordered data set that divides the data into two equal halves.

### Key Definitions / Features:

- **Median:** The value at position  $\frac{N+1}{2}$  for odd  $N$ , or average of values at positions  $\frac{N}{2}$  and  $\frac{N}{2} + 1$  for even  $N$ .
- **Merits:** Easy to calculate, unaffected by extreme values, certain, suitable for qualitative data, graphical representation possible.
- **Demerits:** Requires data ordering, lacks algebraic treatment, less representative, affected by sampling.

### Calculation Methods:

*Individual Series:* Arrange data in order and find middle value(s) as per odd/even count.

*Discrete Series:* Use cumulative frequency and formula:

$$\text{Median} = L + \left( \frac{\frac{N}{2} - F}{f} \right) \times h$$

Where:

- $L$  = lower boundary of median class
- $N$  = total frequency
- $F$  = cumulative frequency before median class
- $f$  = frequency of median class
- $h$  = class width

*Continuous Series:* Convert inclusive to exclusive series, find cumulative frequency, then apply above formula.

**Graphic Method:** Median can be found using ogive (cumulative frequency curve) by locating  $\frac{N}{2}$  on frequency axis and projecting to data axis.

**Other Partition Values:** Quartiles, deciles, and percentiles divide data into more parts.

**Illustrative Example:**

Find median of data: 3, 5, 7, 9, 11.

Ordered data count  $N = 5$  (odd), median position =  $\frac{5+1}{2} = 3^{rd}$  value = 7.

**Practice Set:**

- *Level 1:* Find median of 2, 4, 6, 8, 10.
- *Level 2:* Calculate median for grouped data using formula.
- *Level 3:* Determine median graphically from ogive.

**Answer Key:**

- Level 1: Median = 6
- Level 2: Apply formula with given frequencies.
- Level 3: Use ogive to find median value.

**Quick Reference:** Median formulas and steps summarized above.

**Glossary:**

- **Median Class:** Class interval containing median.
- **Cumulative Frequency (F):** Sum of frequencies up to a class.
- **Class Width (h):** Difference between upper and lower class boundaries.

## Measures of Central Tendency: Mode (Z)

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**Concept Explanation:** Mode is the value that occurs most frequently in a data set.

**Key Definitions / Features:**

- **Mode:** Observation with highest frequency.
- Calculated by inspection in individual series.
- For discrete and continuous series, use Grouping and Analysis Table to identify modal class.

**Grouping and Analysis Table:** Six columns are prepared to analyze frequencies in pairs and groups to identify modal class.

**Mode Formula for Continuous Series:**

$$\text{Mode} = L + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Where:

- $L$  = lower limit of modal class
- $f_1$  = frequency of modal class
- $f_0$  = frequency of class preceding modal class
- $f_2$  = frequency of class succeeding modal class
- $h$  = class width

**Illustrative Example:**

Given class intervals and frequencies, identify modal class and calculate mode using formula.

**Practice Set:**

- *Level 1:* Identify mode from individual data.
- *Level 2:* Use grouping table to find modal class.
- *Level 3:* Calculate mode for grouped data using formula.

**Answer Key:**

- Level 1: Mode is the value with highest frequency.
- Level 2: Prepare grouping table and identify modal class.
- Level 3: Apply mode formula with given frequencies.

**Quick Reference:** Mode calculation steps and formula summarized above.

## Glossary:

- **Modal Class:** Class interval with highest frequency.
- **Grouping Table:** Frequency analysis tool to identify mode.

## Key Words

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- **Average:** A single value representing the entire data set, located centrally.
- **Arithmetic Mean:** Sum of observations divided by number of observations.
- **Median:** Middle value dividing data into two equal parts.
- **Quartiles:** Values dividing data into four equal parts.
- **Mode:** Most frequently occurring value in data.

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