Correlation

Concept Explanation: Correlation refers to a statistical measure that expresses the extent to which two variables are linearly related. It indicates the degree and direction of association between two data series or variables.

Key Definitions / Features:

- **Correlation:** A casual connection that exists between two series or groups of data.
- **Correlation and Causation:** Correlation indicates a relationship but does not imply cause and effect.
- **Kinds of Correlation:**
  - *Positive Correlation:* Both variables move in the same direction.
  - *Negative Correlation:* Variables move in opposite directions.
  - *Linear Correlation:* Change between variables is in a constant ratio.
  - *Curvilinear Correlation:* Change between variables is not in a constant ratio.
  - *Simple Correlation:* Involves two variables.
  - *Multiple Correlation:* Involves more than two variables.
  - *Partial Correlation:* Relationship between two variables after eliminating effects of others.
- **Degree of Correlation:**
  - *Perfect Correlation:* Changes in variables are in the same ratio.
  - *Absence of Correlation:* No relationship between variables.
  - *Limited Degree of Correlation:* Unequal changes in variables but in same or opposite directions; coefficient value between 0 and 1 or -1 and 0.

Illustrative Examples:

Example of positive correlation: Increase in income and increase in consumption expenditure.

Example of negative correlation: Increase in price and decrease in quantity demanded.

Practice Set:

- *Level 1 – Easy:* Define correlation and explain positive and negative correlation with examples.
- *Level 2 – Moderate:* Differentiate between linear and curvilinear correlation with examples.
- *Level 3 – Challenging:* Explain partial correlation and provide a real-life scenario where it is applicable.

Answer Key:

- Correlation is a measure of association between two variables.
- Positive correlation means variables move together; negative means they move oppositely.
- Linear correlation has constant ratio changes; curvilinear does not.
- Partial correlation studies relationship after removing effects of other variables.

Quick Reference:

- Correlation measures association, not causation.
- Types: Positive, Negative, Linear, Curvilinear, Simple, Multiple, Partial.
- Degrees: Perfect, Limited, Absence.

Glossary:

- **Correlation:** Statistical relationship between two variables.
- **Positive Correlation:** Both variables increase or decrease together.
- **Negative Correlation:** One variable increases while the other decreases.
- **Linear Correlation:** Constant ratio of change between variables.
- **Curvilinear Correlation:** Variable changes not proportional.
- **Partial Correlation:** Correlation between two variables controlling others.

Methods of Estimating Correlation

Concept Explanation: Various methods are used to estimate the degree and direction of correlation between variables.

Key Methods:

- **Scatter Diagram:** A graphical representation plotting paired data points to visualize the relationship.

- **Karl Pearson's Coefficient of Correlation:** A numerical measure of linear correlation between two variables, denoted by r.
- **Spearman's Rank Difference Method:** A non-parametric method using ranks to measure correlation.

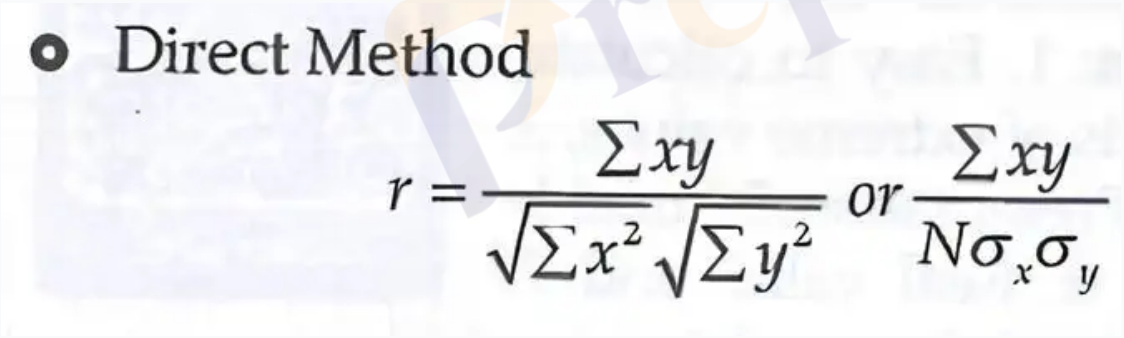
Illustrative Examples:

Scatter Diagram: Plotting heights and weights of individuals to observe association.

Karl Pearson's Coefficient: Calculated using the formula:

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}} = \frac{\sum xy}{\sigma_x \sigma_y}$$

Where x and y are deviations from means.



Short Cut Method: Uses deviations from means dx, dy and sums to calculate r:

$$r = \frac{n\sum dxdy - (\sum dx)(\sum dy)}{\sqrt{[n\sum dx^2 - (\sum dx)^2][n\sum dy^2 - (\sum dy)^2]}}$$

- **Short cut Method**

$$r = \frac{N\Sigma dxdy - (\Sigma dx)(\Sigma dy)}{\sqrt{N\Sigma d^2x - (\Sigma dx)^2}\sqrt{N\Sigma d^2y - (\Sigma dy)^2}}$$

Spearman's Rank Difference Method: Calculate rank differences d, square them d^2, then apply:

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

For tied ranks, adjust formula accordingly.

Practice Set:

- *Level 1 – Easy:* Draw a scatter diagram for given data and interpret.
- *Level 2 – Moderate:* Calculate Karl Pearson's coefficient for a small data set.
- *Level 3 – Challenging:* Use Spearman's rank method to find correlation for ranked data with ties.

Answer Key:

- Scatter diagram shows visual association but no exact value.
- Karl Pearson's coefficient calculated by formula yields value between -1 and +1.
- Spearman's method uses ranks and is suitable for ordinal data.

Quick Reference:

- Scatter Diagram: Visual tool.
- Karl Pearson's Coefficient: Measures linear correlation.
- Spearman's Rank Method: Non-parametric rank correlation.

Glossary:

- **Scatter Diagram:** Graph plotting paired data points.
- **Karl Pearson's Coefficient:** Numerical measure of linear correlation.
- **Spearman's Rank Correlation:** Correlation based on ranks.

Properties and Interpretation of Correlation Coefficient

Concept Explanation: The correlation coefficient r quantifies the strength and direction of a linear relationship between two variables.

Key Properties:

- Range: $-1 \leq r \leq +1$
- $r = +1$: Perfect positive correlation.
- $r = -1$: Perfect negative correlation.
- $r = 0$: No linear correlation.
- Unitless measure: Independent of units of variables.
- Sensitive to outliers.
- Measures only linear relationships.
- Correlation does not imply causation.

Illustrative Examples:

If $r = 0.85$, strong positive linear relationship exists.

If $r = -0.6$, moderate negative linear relationship exists.

Practice Set:

- *Level 1 – Easy:* State the range of correlation coefficient and interpret $r = 0$.
- *Level 2 – Moderate:* Explain why correlation does not imply causation with an example.
- *Level 3 – Challenging:* Discuss the effect of outliers on correlation coefficient with numerical illustration.

Answer Key:

- Correlation coefficient ranges from -1 to +1.
- $r = 0$ means no linear relationship.
- Correlation does not imply causation because other factors may influence variables.

- Outliers can distort r value significantly.

Quick Reference:

- r measures strength and direction of linear relationship.
- Range: -1 to +1.
- Interpretation depends on magnitude and sign.

Glossary:

- **Correlation Coefficient (r):** Numerical measure of linear association.
- **Outliers:** Extreme values affecting statistical measures.
- **Causation:** Cause and effect relationship.