Prepzy

CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours Maximum Marks : 43

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **24 questions.** All questions are **compulsory.**
ii. This question paper is divided into **3 sections.**
iii. **Section A** – questions number **1 to 8** are very short answer Each question carries **2 marks.**
iv. **Section B** – questions number **9 to 13** are short answer Each question carries **3 marks.**
v. **Section C** – questions number **14 to 21** are case based questions
vi. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
vii. Use of calculator is NOT allowed.

Section A

Question 1. Solve the quadratic equation for x : $x^2 - 2ax + (4b^2 - a^2) = 0$.

[2 Marks]

Answer: Given the quadratic equation $x^2 - 2ax + (4b^2 - a^2) = 0$, we use the quadratic formula. Here, the coefficient of x^2 is 1, coefficient of x is -2a, and constant term is $(4b^2 - a^2)$. Calculating the discriminant, we get $D = (-2a)^2 - 4 \times 1 \times (4b^2 - a^2) = 4a^2 - 4(4b^2 - a^2) = 4a^2 - 16b^2 + 4a^2 = 8a^2 - 16b^2$. The roots are given by $(2a \pm \sqrt{D}) / 2$. Thus, the two roots are a plus or minus square root of $(2a^2 - 4b^2)$.

Question 2. If the quadratic equation $(1 + a^2) x^2 + 2abx + (b^2 - c^2) = 0$ has equal and real roots, then prove that $b^2 = c^2(1 + a^2)$.

[2 Marks]

Answer: For the quadratic equation $(1 + a^2) x^2 + 2abx + (b^2 - c^2) = 0$ to have equal and real roots, its discriminant must be zero. The discriminant is given by $b^2 - 4ac$. Here, $a = (1 + a^2)$, $b = 2ab$, and $c = (b^2 - c^2)$. So, the discriminant is $(2ab)^2 - 4(1 + a^2)(b^2 - c^2)$. Simplifying, we get $4a^2b^2 - 4(1 + a^2)(b^2 - c^2) = 0$. Dividing both sides by 4, $a^2b^2 = (1 + a^2)(b^2 - c^2)$. Expanding the right side, $a^2b^2 = b^2 + a^2b^2 - c^2 - a^2c^2$. Canceling a^2b^2 from both sides, $0 = b^2 - c^2 - a^2c^2$, which implies $b^2 = c^2(1 + a^2)$. Hence, proved.

Question 3.

Find the sum of first 20 terms of an AP in which $d = 5$ and $a_{20} = 135$.

[2 Marks]

Answer: Given the common difference $d = 5$ and the 20th term $a20 = 135$ of an AP, we first find the first term 'a'. The nth term of an AP is given by $a + (n - 1) d = a20$. So, $a + 19 \times 5 = 135$, which gives $a = 135 - 95 = 40$. Now, the sum of first n terms of an AP is $S_n = (n/2) \times (2a + (n - 1)d)$. For $n = 20$, $S20 = (20/2) \times (2 \times 40 + 19 \times 5) = 10 \times (80 + 95) = 10 \times 175 = 1750$. Therefore, the sum of the first 20 terms is 1750.

Question 4.

Find the mode of the given frequency distribution:

[2 Marks]

Answer: The mode of a frequency distribution is the value or class interval that has the highest frequency. In the given data, the class interval 40 - 55 has the maximum frequency of 7 students. Therefore, the mode of this data is the class interval 40 - 55.

Question 5.

150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, and are completely immersed in water. Find the rise in the level of water in the cylindrical vessel.

[2 Marks]

Answer: First, find the volume of one marble. Diameter = 1.4 cm, so radius = $1.4 \div 2 = 0.7$ cm. Volume of one marble = $4/3 \times \pi \times (0.7)^3 = 1.436$ cm^3 (approximately). For 150 marbles, total volume = $150 \times 1.436 = 215.4$ cm^3. Diameter of the cylinder = 7 cm, so radius = 3.5 cm. The rise in water level, h = volume of marbles ÷ base area of cylinder = $215.4 \div (\pi \times 3.5^2) = 215.4 \div 38.48 = 5.6$ cm approximately. So, the water level rises by about 5.6 cm.

Question 6. Three cubes of side 6 cm each are joined as shown in Figure 1. Find the total surface area of the resulting cuboid.

[2 Marks]

Answer: Each cube has a side of 6 cm, so the area of one face is 6 cm × 6 cm = 36 cm^2. The total surface area of one cube is 6 × 36 = 216 cm^2. When three cubes are joined in a line, the combined shape forms a cuboid with dimensions 18 cm (length), 6 cm (width), and 6 cm (height). The total surface area of this cuboid is 2 × (length × width + width × height + height × length) = 2 × (18 × 6 + 6 × 6 + 6 × 18) = 2 × (108 + 36 + 108) = 2 × 252 = 504 cm^2.

Question 7.

For what value of 'n', are the n^{th} terms of the APs 9, 7, 5, ... and 15, 12, 9, ... the same?

[2 Marks]

Answer: The first AP has first term a_1 = 9 and common difference d = 7 - 9 = -2. Its n^{th} term is given by a_n = 9 + (n - 1)(-2) = 9 - 2n + 2 = 11 - 2n. The second AP has first term 15 and common difference 12 - 15 = -3. Its n^{th} term is a_n = 15 + (n - 1)(-3) = 15 - 3n + 3 = 18 - 3n. To find n for which the n^{th} terms are equal, set 11 - 2n = 18 - 3n. Rearranging, we get n = 7. Therefore, the n^{th} terms of both APs are the same when n = 7.

Question 8. In Figure 2, PQ and PR are tangents to the circle centred at O. If ∠OPR = 45°, prove that ORPQ is a square.

[2 Marks]

Answer: Given that PQ and PR are tangents to the circle from point P, and ∠OPR = 45°. Since OP and OR are radii to the points of tangency R and Q, angles OQP and ORP are right angles (90°) because the radius is perpendicular to the tangent at the point of contact. In triangle OPR, angle OPR is 45°, and angle ORP is 90°, so the third angle POR equals 45° (because the sum of angles in a triangle is 180°). Since OP = OR (radii of the same circle) and angles at P and R are 90° and 45°, the quadrilateral ORPQ has four right angles, and all sides equal, thus it forms a square.

Section B

Question 9. Draw a line segment AB of length 8 cm and locate a point P on AB such that AP : PB = 1 : 5.

[3 Marks]

Answer: To draw a line segment AB of length 8 cm, use a ruler to draw a straight line and mark points A and B such that the distance between them is 8 cm. Next, to locate point P on AB so that AP is to PB as 1 is to 5, divide the segment AB into 6 equal parts (because 1 +

5 = 6 parts). Each part will be 8 cm divided by 6, which is approximately 1.33 cm. Starting from point A, measure 1.33 cm along the segment AB and mark that point as P. This ensures that AP is 1 part and PB is 5 parts, maintaining the ratio 1:5.

Question 10. Draw a circle of radius 3 cm. From a point P lying outside the circle at a distance of 6 cm from its centre, construct two tangents PA and PB to the circle.

[3 Marks]

Answer: To draw a circle of radius 3 cm with centre O, use a compass set to 3 cm and draw the circle. Mark a point P outside the circle, such that the distance between P and O is 6 cm. To construct tangents from P to the circle, follow these steps: First, join points P and O. With O as the centre and radius 3 cm, the given circle is already drawn. Now, find the midpoint M of the line segment PO. Using M as the centre and MO as radius, draw a circle. This new circle will intersect the given circle at points A and B. Join points P to A and P to B. The line segments PA and PB are the required tangents to the circle from point P. These tangents touch the circle at exactly one point each, and their lengths from P to A and P to B are equal.

Question 11. The tops of two poles of heights 20 m and 28 m are connected with a wire. The wire is inclined to the horizontal at an angle of 30°. Find the length of the wire and the distance between the two poles.

[3 Marks]

Answer: Given two poles of heights 20 meters and 28 meters, connected by a wire inclined at 30° to the horizontal, we need to find the length of the wire and the distance between the poles. First, find the vertical difference between the poles: 28 m - 20 m = 8 m. Consider the triangle formed by the wire, the horizontal distance between the poles, and the vertical difference. The wire acts as the hypotenuse of this right triangle with a vertical side of 8 m and an angle of 30° with the horizontal. Using trigonometric relations, the vertical height (opposite side) = 8 m = wire length × sin 30°. Since sin 30° = 1/2, wire length = 8 ÷ (1/2) = 16 meters. Next, find the horizontal distance between the poles, which is adjacent side = wire length × cos 30°. Cos 30° = √3/2 ≈ 0.866, so horizontal distance = 16 × 0.866 = 13.856 meters approximately. Thus, the length of the wire is 16 meters, and the distance between the two poles is about 13.86 meters.

Question 12.

The weights (in kg) of 50 wild animals of a National Park were recorded and the following data was obtained:

Find the mean weight (in kg) using assumed mean method.

[3 Marks]

Answer: To find the mean weight using the assumed mean method, first select an assumed mean (A) from the mid-points of the given class intervals of weights. Then, calculate the deviation (d) of each mid-point from the assumed mean by subtracting A from each mid-point. Multiply each deviation by the frequency (f) of that class to get f × d. Sum all these products to get Σ(f × d). Finally, use the formula: Mean = assumed mean + (Σ(f × d) / total frequency). Here, the total number of animals (N) is 50. Substitute the values into the formula to calculate the mean weight.

Question 13.

For the following frequency distribution, find the median:

[3 Marks]

Answer:

To find the median of a grouped frequency distribution, follow these steps:

1. Calculate the total number of observations (N) by adding all the frequencies.

2. Find N/2, which indicates the median position in the cumulative frequency.

3. Create a cumulative frequency column by successively adding the frequencies.

4. Identify the median class, which is the class interval where the cumulative frequency is just greater than or equal to N/2.

5. Use the median formula:

Median = L + ((N/2 - F) / f) × h

where:

- L = lower boundary of the median class
- F = cumulative frequency before the median class
- f = frequency of the median class
- h = class width of the median class

Substitute the values into the formula to calculate the median.

This method ensures that the median accurately represents the middle value of the grouped data.

Section C

Question 14. In the picture given below, one can see a rectangular in-ground swimming pool installed by a family in their backyard. There is a concrete sidewalk around the pool of width x m. The outside edges of the sidewalk measure 7 m and 12 m. The area of the pool is 36 sq. m.

(1) Based on the information given above, form a quadratic equation in terms of x.

[2 Marks]

Answer: Let the length and width of the pool be L and W respectively. We know the area of the pool is 36 sq. m, so $L \times W = 36$. The sidewalk is x meters wide and surrounds the pool, so the outer dimensions including the sidewalk are $(L + 2x)$ and $(W + 2x)$. Given the outside edges measure 7 m and 12 m, so $(L + 2x) = 12$ and $(W + 2x) = 7$. From these we get $L = 12 - 2x$ and $W = 7 - 2x$. Since area $L \times W = 36$, we substitute to get $(12 - 2x)(7 - 2x) = 36$. Expanding, $84 - 24x - 14x + 4x^2 = 36$. Simplify: $4x^2 - 38x + 84 = 36$. Bring 36 to the left: $4x^2 - 38x + 48 = 0$. This is the quadratic equation in terms of x.

Key Points: Define variables for length and width of the pool-Express pool area as length × width = 36-Express outer dimensions including sidewalk as (length + 2x) and (width + 2x)-Use given dimensions of outside edges (7 m and 12 m)-Substitute length and width in terms of x into the area equation-Expand and simplify to form quadratic equation

(2)

Find the width of the sidewalk around the pool.

[2 Marks]

Answer: Let the width of the sidewalk be x meters. The dimensions of the pool are given such that its area is 36 sq. m. Since the sidewalk surrounds the pool, the total outside dimensions become (length of pool + 2x) and (width of pool + 2x). According to the question, the outside dimensions are 12 m and 7 m. So, the area of the pool is 36 sq. m, which means length × width = 36. We have length + 2x = 12 and width + 2x = 7. Let the length be L and width be W. Then $L \times W = 36$, $L + 2x = 12$, and $W + 2x = 7$. From the last two equations, $L = 12 - 2x$ and $W = 7 - 2x$. Substitute these into the area equation: $(12 - 2x)(7 - 2x) = 36$. Expanding, $84 - 24x - 14x + 4x^2 = 36$, which simplifies to $4x^2 - 38x + 84 = 36$. Subtract 36 from both sides: $4x^2 - 38x + 48 = 0$. Divide by 2: $2x^2 - 19x + 24 = 0$. Solving this quadratic equation for x, we find $x = 3$ or $x = 4$. Since x must be positive and less than half of the smaller outside dimension, $x = 1.5$ m is the correct width of the sidewalk. Thus, the width of the sidewalk is 1.5 meters.

Key Points: Identify the variables and relationships between pool dimensions and sidewalk width-Use given outside dimensions to express pool dimensions in terms of x-Set up area equation for the pool using the expressions for length and width-Formulate and solve quadratic equation to find the width x-Select the correct positive value for x as width of the sidewalk

Question 15. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 16. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 17. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 18. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 19. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 20. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 21.

John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions :

Cake : Cylindrical shape with diameter 24 cm and height 14 cm.

Cap : Conical shape with base circumference 44 cm and height 24cm.

Based on the above information, answer the following questions :

(1) How many square cm paper would be used to make 4 such caps ?

[2 Marks]

Answer: The caps are in the shape of a cone with base circumference 44 cm and height 24 cm. First, find the radius of the base using circumference = $2 \times \pi \times r$. So, r = circumference / $(2 \times \pi) = 44 / (2 \times 22/7) = 7$ cm. Now, find the slant height l of the cone using Pythagoras theorem: $l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(49 + 576)} = \sqrt{625} = 25$ cm. The curved surface area (CSA) of one cone = $\pi \times r \times l = 22/7 \times 7 \times 25 = 550$ cm^2. The paper required to make 4 caps = $4 \times 550 = 2200$ cm^2. Therefore, 2200 square cm of paper will be needed to make 4 caps.

Key Points: Identify shape of the cap as cone - Find radius using circumference formula - Calculate slant height using Pythagoras theorem - Find curved surface area of one cap - Multiply by 4 for four caps

(2)

The bakery shop sells cakes by weight (0·5 kg, 1 kg, 1·5 kg, etc.). To have the required dimensions, how much cake should they order, if 650 cm3 equals 100 g of cake ?

[2 Marks]

Answer: First, find the volume of the cake which is a cylinder. The radius of the cake is half of its diameter, so radius = 24 cm ÷ 2 = 12 cm. Volume of cylinder = $\pi \times \text{radius}^2 \times \text{height} = 3.14 \times 12 \times 12 \times 14 = 3.14 \times 144 \times 14 = 6325.44$ cm^3. Now, 650 cm^3 of cake weighs

100 g, so the weight of 1 cm^3 cake = 100 g ÷ 650 = 0.1538 g. Hence, weight of cake needed = 6325.44 × 0.1538 ≈ 972.5 g ≈ 1 kg. Therefore, they should order a 1 kg cake.

Key Points: Calculate radius from diameter-Calculate volume of cylinder using formula $\pi r^2 h$-Use given conversion 650 cm^3 = 100 g to find weight per cm^3-Calculate total weight by multiplying volume by weight per cm^3-Round weight to nearest 0.5 kg increment for ordering