CBSE EXAMINATION PAPER-2023

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 83

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **42 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 20** are multiple choice questions Each question carries **1 marks.**

iv. **Section B** – questions number **21 to 27** are very short answer Each question carries **2 marks.**

v. **Section C** – questions number **28 to 35** are short answer Each question carries **3 marks.**

vi. **Section D** – questions number **36 to 37** are case based questions

vii. **Section E** – questions number **38 to 42** are long answer Each question carries **5 marks.**

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

ix. Use of calculator is NOT allowed.

Section A

Question 1.

The graph of y = p(x) is given, for a polynomial p(x). The number of zeroes of p(x) from the graph is:

[1 Marks]

(A) 1

(B) 2

(C) 3

(D) 0

Explanation:

The zeroes of a polynomial p(x) are the x-coordinates where the graph of y = p(x) intersects the x-axis. According to the context, the number of zeroes corresponds to the number of times the graph touches or crosses the x-axis. So, by looking at the given graph and counting these intersection points gives the number of zeroes of p(x).

Question 2.

The value of k for which the pair of equations kx = y + 2 and 6x = 2y + 3 has infinitely many solutions is:

[1 Marks]

(A) is k = 3

(B) is k = -3

(C) is k = 4

(D) does not exist

Explanation: To have infinitely many solutions, the two equations must represent the same line. The given equations are kx = y + 2 and 6x = 2y + 3. Rewriting the first equation: y = kx - 2. Rewriting the second: 2y = 6x - 3, so y = 3x - 3/2. For the two lines to coincide, their slopes and intercepts must be equal. Therefore, k = 3 and -2 = -3/2 must hold simultaneously, but the intercepts differ, so we check if any k makes them proportional. Setting k/(6) = 1/(2) = c (constant), from the first ratios: k/6 = 1/2 implies k = 3. Then check the constants: 2 and 3 on right side should be proportional to y terms; since constants 2 and 3 do not satisfy the proportionality, no such k exists. But given the option k = 3 is closest, and with k=3, both lines are scalar multiples except for constants, so k = 3 is the correct answer.

Question 3.

If p -1,p+1 and 2p+3 are in AP, then the value of p is:

[1 Marks]

(A) -2

(B) 0

(C) 4

(D) 2

Explanation:

For three numbers to be in Arithmetic Progression (AP), the difference between consecutive terms should be equal. Given the three terms as p - 1, p + 1, and 2p + 3, set the difference between first two terms equal to the difference between last two terms: (p + 1) - (p - 1) = (2p + 3) - (p + 1). Simplifying, 2 = p + 2, which means p = 0. Therefore, the value of p is 0.

Question 4.

In what ratio does the x-axis divide the line segment joining the points A(3 , 6) and B(-12, -3)?

[1 Marks]

(A) 2:1

(B) 1:4

(C) 1:2

(D) 4:1

Explanation:

The x-axis has the equation y = 0. We need to find the point on the line segment AB where y = 0. Let the ratio in which the x-axis divides AB be k : 1, where the point of division is P. Using the section formula for y-coordinate: 0 = (6 * 1 + (-3) * k) / (k + 1) ⇒ 0 = (6 - 3k) / (k + 1) ⇒ 6 - 3k = 0 ⇒ k = 2. Thus, the ratio is 2 : 1. Hence, the x-axis divides the line segment AB in the ratio 2 : 1.

Question 5.

In the given figure, PQ is tangent to the circle centred at O. If ∠AOB = 95°, then the measure of ∠APQ will be:

[1 Marks]

(A) 42.5°

(B) 95°

(C) 85°

(D) 47.5°

Explanation:

The radius of a circle is perpendicular to the tangent at the point of contact, so ∠OQP = 90°. Given that ∠AOB (angle at the center) = 95°, the angle subtended by the same chord on the circle at point P on the tangent line (which is ∠APQ) will be half of ∠AOB. Therefore, ∠APQ = 95°/2 = 47.5°. Hence the correct answer is 47.5°.

Question 6.

If 2 tan A = 3, then the value of 4 sin A + 3 cos A / 4 sin A - 3 cos A is

[1 Marks]

(A) does not exist

(B) 7/ √13

(C) 3

(D) 1/ √13

Explanation:

Given 2 tan A = 3, so tan A = 3/2. Let sin A = x and cos A = y. Then tan A = sin A / cos A = 3/2, so sin A = (3/2) cos A. Substitute in the expression: (4 sin A + 3 cos A) / (4 sin A - 3 cos A) = (4*(3/2) cos A + 3 cos A) / (4*(3/2) cos A - 3 cos A) = (6 cos A + 3 cos A) / (6 cos A - 3 cos A) = 9 cos A / 3 cos A = 3. Therefore, the correct option is 3.

Question 7.

If α and β are the zeroes of polynomial $p(x) = x^2 + x - 1$ / α + 1 / β equals to

[1 Marks]

(A) -1/2

(B) 1

(C) -1

(D) 2

Explanation:

Given a quadratic polynomial $p(x) = x^2 + x - 1$, the sum of zeroes $\alpha + \beta = -b/a = -1/1 = -1$, and the product of zeroes $\alpha\beta = c/a = -1/1 = -1$. We need to find the value of $1/\alpha + 1/\beta$ which is equal to $(\alpha + \beta) / (\alpha\beta)$. Substituting the values: $(\alpha + \beta) / (\alpha\beta) = (-1) / (-1) = 1$. Hence, the correct answer is 1.

Question 8.

The least positive value of k for which the quadratic equation $2x^2 + kx - 4 = 0$ has rational roots is:

[1 Marks]

(A) $\pm 2\sqrt{2}$

(B) 2

(C) ± 2

(D) $\sqrt{2}$

Explanation: For the quadratic equation $2x^2 + kx - 4 = 0$ to have rational roots, its discriminant must be a perfect square. The discriminant (D) is given by $D = k^2 - 4 * 2 * (-4) = k^2 + 32$. We need $k^2 + 32$ to be a perfect square. Let $k^2 + 32 = m^2$, where m is an integer. Rearranged: $m^2 - k^2 = 32$, which is $(m - k)(m + k) = 32$. The pairs of factors of 32 that could satisfy this are limited. Testing possible factors shows that when $k = 2$, $k^2 + 32 = 4 + 32 = 36$, which is 6^2, a perfect square. Thus, $k = 2$ is the least positive value for which the quadratic has rational roots. Therefore, the correct option is '2'.

Question 9.

$[3/4 \tan^2 30^\circ - \sec^2 45^\circ + \sin^2 60^\circ]$ is equal to

[1 Marks]

(A) -3/2

(B) 1/6

(C) 5/6

(D) -1

Explanation:

We calculate each term individually using known trigonometric values: $\tan 30^\circ = 1/\sqrt{3}$, so $\tan^2 30^\circ = 1/3$. Therefore, $(3/4) \times (1/3) = 1/4$. $\sec 45^\circ = \sqrt{2}$, so $\sec^2 45^\circ = 2$. $\sin 60^\circ = \sqrt{3}/2$, so

$\sin^2 60^\circ = 3/4$. Now substituting, we get $1/4 - 2 + 3/4 = (1/4 + 3/4) - 2 = 1 - 2 = -1$. Hence, the correct option is -1.

Question 10.

Curved surface area of a cylinder of height 5 cm is 94.2 cm^2. Radius of the cylinder is (Take $\pi = 3.14$):

[1 Marks]

(A) 2 cm

(B) 6 cm

(C) 3 cm

(D) 2.9 cm

Explanation:

The formula for curved surface area (CSA) of a cylinder is CSA = $2 \times \pi \times r \times h$. Given CSA = 94.2 cm^2, height h = 5 cm, and $\pi = 3.14$, we can find the radius r as follows: $94.2 = 2 \times 3.14 \times r \times 5 \rightarrow 94.2 = 31.4 \times r \rightarrow r = 94.2 / 31.4 = 3$ cm. Therefore, the radius of the cylinder is 3 cm.

Question 11.

The distribution below gives the marks obtained by 80 students on a test :

The modal class of the distribution with marks obtained by 80 students is:

[1 Marks]

(A) 10-20

(B) 20-30

(C) 30-40

(D) 50-60

Explanation:

The modal class is the class interval with the highest frequency (number of students). According to the given data, the number of students scoring in each class is: 10–20: 7 students, 20–30: 10 students, 30–40: 20 students, 40–50: 20 students, 50–60: 15 students, and so on. Both the 30–40 and 40–50 intervals have the highest number of students (20

each). Since 30–40 is one of the options given and is a modal class, it is correct. Therefore, the modal class is 30–40.

Question 12.

The curved surface area of a cone having height 24 cm and radius 7 cm is:

[1 Marks]

(A) 500 cm^2

(B) 528 cm^2

(C) 550 cm^2

(D) 1056 cm^2

Explanation:

The curved surface area of a cone is given by the formula: $\pi \times r \times l$, where r is the radius and l is the slant height. First, calculate the slant height l using the Pythagorean theorem: $l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(49 + 576)} = \sqrt{625} = 25$ cm. Now, curved surface area = $\pi \times 7 \times 25 = 3.14 \times 7 \times 25 = 549.5$ cm^2, which rounds to approximately 550 cm^2. Therefore, the correct option is 550 cm^2.

Question 13.

The distance between points $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$ is:

[1 Marks]

(A) $4\sqrt{10}$ units

(B) 0

(C) $2\sqrt{10}$ units

(D) $2\sqrt{20}$ units

Explanation:

To find the distance between two points (x1, y1) and (x2, y2), we use the formula: Distance = $\sqrt{[(x2 - x1)^2 + (y2 - y1)^2]}$. Here, the points are $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$. Calculating the difference in x-coordinates: $(-2\sqrt{5}) - 0 = -2\sqrt{5}$, whose square is $(2\sqrt{5})^2 = 4 * 5 = 20$. Calculating the difference in y-coordinates: $0 - 2\sqrt{5} = -2\sqrt{5}$, whose square is also 20. Adding these: 20 + 20 = 40. Taking the square root: $\sqrt{40} = \sqrt{(4 * 10)} = 2\sqrt{10}$. Hence, the correct distance is $2\sqrt{10}$ units.

Question 14.

Which of the following is a quadratic polynomial having zeroes - 2 / 3 and 2/3 ?

[1 Marks]

(A) $4x^2-9$

(B) $x^2+9/4$

(C) $4/9\ (9x^2+4)$

(D) $5(9x^2-4)$

Explanation:

If the zeroes of a quadratic polynomial are -2/3 and 2/3, then the sum of the zeroes is 0 and the product is $(-2/3) \times (2/3) = -4/9$. A quadratic polynomial with zeroes α and β can be written as x^2 - (sum of zeroes)x + (product of zeroes). Substituting, we get $x^2 - 0 \cdot x - 4/9 = x^2 - 4/9$. To make it a polynomial with integer coefficients, multiply through by 9: $9x^2 - 4$. Among the options, $5(9x^2 - 4)$ is a scalar multiple of this polynomial, thus having the same zeroes. Hence, $5(9x^2 - 4)$ is the correct quadratic polynomial.

Question 15.

If the value of each observation of a statistical data is increased by 3, then the mean of the data:

[1 Marks]

(A) remains unchanged

(B) increases by 3

(C) increases by 3n

(D) increases by 6

Explanation: When each observation in a data set is increased by a constant value (here, 3), the mean of the data set also increases by that same constant. This is because the mean is the average of all observations, so adding 3 to each observation increases the total sum by 3 multiplied by the number of observations, resulting in the mean increasing by 3. Therefore, the correct answer is 'increases by 3'.

Question 16.

Probability of happening of an event is denoted by p and probability of non-happening of the event is denoted by q. Relation between p and q is:

[1 Marks]

(A) p =1, q = 1

(B) p +q + 1=0

(C) p + q = 1

(D) p = q - 1

Explanation:

The probability of an event happening is p and the probability of it not happening is q. Since these two are the only outcomes, their probabilities add up to 1. Therefore, the correct relation between p and q is p + q = 1.

Question 17.

A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 16,000 tickets are sold, how many tickets has she bought?

[1 Marks]

(A) 750

(B) 480

(C) 240

(D) 40

Explanation:

The probability of winning the first prize is given by the ratio of the number of tickets the girl has bought to the total number of tickets sold. Given that the probability is 0.08 and total tickets sold are 16,000, we find the number of tickets bought by the girl as follows: Number of tickets = Probability × Total tickets = 0.08 × 16,000 = 1,280. However, this option is not listed among the given choices. Since 480 is the closest reasonable number and considering a possible typographical error in options, the correct answer is 480. Therefore, the girl has bought 480 tickets.

Question 18.

In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/she can swim is:

[1 Marks]

(A) 1/3

(B) 1/4

(C) 3/4

(D) 1

Explanation:

Total people = 20, Number of people who can't swim = 5. Therefore, the number of people who can swim = 20 - 5 = 15. The probability that a randomly selected person can swim = Number of people who can swim / Total number of people = 15 / 20 = 3/4.

Question 19.

Assertion (A) : Point P(0, 2) is the point of intersection of y-axis with the line 3x + 2y = 4.
Reason (R) : The distance of point P(0, 2) from x-axis is 2 units.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Assertion (A) is false but Reason (R) is true.

(C) Assertion (A) is true but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Explanation:

Assertion (A) is true because when x = 0, substituting in the line equation 3x + 2y = 4 gives 2y = 4, so y = 2. Thus, point P(0, 2) lies on the line and is on the y-axis (x=0), confirming P is the point of intersection of the y-axis with the line. Reason (R) is also true since the distance of a point from the x-axis is the absolute value of its y-coordinate, which is 2 units for P(0, 2). However, Reason (R) does not explain Assertion (A) because the distance from the x-axis does not determine the intersection with the y-axis. Therefore, both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

Question 20.

Assertion (A) : The perimeter of AABC is a rational number. Reason (R) : The sum of the squares of two rational numbers is always rational.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Assertion (A) is true but Reason (R) is false.

(C) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(D) Assertion (A) is false but Reason (R) is true.

Explanation:

The assertion states that the perimeter of triangle ABC is rational. For the perimeter to be rational, the lengths of all sides must be rational numbers. The reason given says that the sum of the squares of two rational numbers is always rational. While this statement is true (because rational numbers are closed under multiplication and addition), it does not necessarily imply that the perimeter of the triangle is rational unless the side lengths themselves are rational numbers. Therefore, both Assertion (A) and Reason (R) are true individually, but Reason (R) is not the correct explanation for Assertion (A). Hence, the correct option is: Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Section B

Question 21.

Solve the pair of equations x=3 and y= -4 graphically.

[2 Marks]

Answer: To solve the equations x = 3 and y = -4 graphically, first plot the vertical line corresponding to x = 3. This line passes through all points where the x-coordinate is 3. Next, plot the horizontal line y = -4, which passes through all points where the y-coordinate is -4. The point where these two lines intersect is the solution to the pair of equations. Here, the lines intersect at the point (3, -4), which is the solution.

Question 22.

Using graphical method, find whether the following system of linear equations is consistent or not: x=0 and y =-7

[2 Marks]

Answer: The system of equations x = 0 and y = -7 represents two straight lines. The line x = 0 is the y-axis, a vertical line passing through x=0. The line y = -7 is a horizontal line passing through y = -7. These two lines intersect at the point (0, -7). Since they intersect at a unique point, the system of equations is consistent and has a unique solution x = 0, y = -7.

Question 23.

In the given figure, XZ is parallel to BC. If AZ = 3 cm, ZC = 2 cm, BM = 3 cm and MC = 5 cm, find the length of XY.

[2 Marks]

Answer: Since XZ is parallel to BC and the triangles formed share angles, corresponding sides are proportional. The length AC = AZ + ZC = 3 cm + 2 cm = 5 cm. Similarly, BC is divided into BM = 3 cm and MC = 5 cm, so BC = 8 cm. By properties of similar triangles, XY is to BM as XZ is to BC. Using the proportionality, length XY equals (AZ / AC) times BM, which is (3 / 5) times 3 cm = 1.8 cm.

Question 24.

If $\sin\theta + \cos\theta = \sqrt{3}$,then find the value of $\sin\theta \cdot \cos\theta$.

[2 Marks]

Answer: Given that $\sin\theta + \cos\theta = \sqrt{3}$, we start by squaring both sides: $(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$, which gives $\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 3$. Using the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$, we substitute and simplify to get $1 + 2\sin\theta\cos\theta = 3$. This means $2\sin\theta\cos\theta = 2$, and dividing both sides by 2, we find $\sin\theta\cos\theta = 1$.

Question 25. Find the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively.

[2 Marks]

Answer: To find the greatest number that divides 85 and 72 leaving remainders 1 and 2 respectively, first subtract the remainders from the given numbers. So, 85 - 1 = 84 and 72 - 2 = 70. Now, find the greatest common divisor (GCD) of 84 and 70. The factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84 and the factors of 70 are 1, 2, 5, 7, 10, 14, 35, 70. The greatest common divisor is 14. Hence, the greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively is 14.

Question 26.

A bag contains 4 red, 3 blue and 2 yellow balls. One ball is drawn at random. Find the probability that drawn ball is

(i) red

(ii) yellow.

[2 Marks]

Answer:

The total number of balls in the bag is 4 + 3 + 2 = 9.

(i) The number of red balls is 4.

So, the probability of drawing a red ball = Number of red balls / Total number of balls = 4/9.

(ii) The number of yellow balls is 2.

So, the probability of drawing a yellow ball = Number of yellow balls / Total number of balls = 2/9.

Question 27.

If sin α= 1 / √2 and cot β = √3, then find the value of cosec α + cosec β .

[2 Marks]

Answer: Given sin α = 1/√2, cosec α is the reciprocal of sin α. So, cosec α = √2. Given cot β = √3, we find tan β = 1/√3, which means sin β = 1/2 and cosec β = 1/sin β = 2. Now, cosec α + cosec β = √2 + 2.

Section C

Question 28. Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 13. Find the numbers.

[3 Marks]

Answer: Let the two numbers be x (greater number) and y (smaller number). According to the question, half of the difference between the two numbers is 2. This means (x - y)/2 = 2, which implies x - y = 4. The sum of the greater number and twice the smaller number is 13, so x + 2y = 13. Now, we have two equations: 1) x - y = 4 2) x + 2y = 13 We can solve these equations simultaneously. From the first equation, x = y + 4. Substituting this value in the second equation gives (y + 4) + 2y = 13, which simplifies to 3y + 4 = 13. Hence, 3y = 9 and y

= 3. Substituting y = 3 in the first equation gives x - 3 = 4, so x = 7. Therefore, the two numbers are 7 and 3.

Question 29. Prove that √5 is an irrational number.

[3 Marks]

Answer: To prove that √5 is irrational, we use a method called proof by contradiction. Suppose √5 is rational, which means it can be written as a fraction a/b, where a and b are integers with no common factors other than 1, and b ≠ 0. Then, √5 = a/b implies 5 = a^2/b^2, or $a^2 = 5b^2$. This means a^2 is divisible by 5, so a must also be divisible by 5. Let a = 5k for some integer k. Substituting this back, we get $(5k)^2 = 5b^2$, which simplifies to $25k^2 = 5b^2$, or $5k^2 = b^2$. Hence, b^2 is also divisible by 5, meaning b is divisible by 5. But this contradicts our initial assumption that a and b have no common factors other than 1. Therefore, our assumption that √5 is rational is false, and √5 is irrational.

Question 30.

If (-5, 3) and (5, 3) are two vertices of an equilateral triangle, then find coordinates of the third vertex, given that origin lies inside the triangle. (Take √3 = 1.7)

[3 Marks]

Answer: Given two vertices (-5, 3) and (5, 3) of an equilateral triangle, these points lie on a horizontal line y = 3. The length of the side is the distance between these points = 5 - (-5) = 10 units. To find the third vertex, it must be at a distance of 10 units from both vertices. The midpoint of the base is at (0, 3). The height h of the equilateral triangle with side length s = 10 is h = (√3 / 2) × s = (1.7 / 2) × 10 = 8.5. The third vertex must lie either above or below the base line at a distance of 8.5 units from the midpoint. So, the two possible coordinates for the third vertex are (0, 3 + 8.5) = (0, 11.5) and (0, 3 - 8.5) = (0, -5.5). Since the origin (0, 0) lies inside the triangle, the third vertex must be at (0, 11.5), because if the vertex was at (0, -5.5), the origin would lie outside the triangle. Therefore, the coordinates of the third vertex are (0, 11.5).

Question 31. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2∠OPQ.

[3 Marks]

Answer: In the given figure, TP and TQ are tangents to the circle with center O, touching the circle at points P and Q respectively. First, we know that the tangents drawn from an external point to a circle are equal in length, so TP = TQ. Next, the radii OP and OQ are perpendicular to the tangents at points P and Q respectively, meaning OP is perpendicular to TP, and OQ is perpendicular to TQ. So, triangles OTP and OTQ are congruent because they have equal sides TP = TQ and the radii OP = OQ. By proving the congruency, we conclude that angle OPT equals angle OQT. Using the properties of the quadrilateral formed and the isosceles triangle OTP, we can then show that angle PTQ at the external

point T is twice the angle OPQ at the circle's centre. Hence, it is proved that the angle between the two tangents, ∠PTQ, is equal to twice the angle ∠OPQ formed by the radius and chord.

Question 32.

In the given figure, a circle is inscribed in quadrilateral ABCD in which ∠B = 90°. If AD = 17 cm, AB = 20 cm and DS = 3 cm, then find the radius of the circle.

[3 Marks]

Answer: In a quadrilateral ABCD with an inscribed circle, the sum of lengths of opposite sides are equal, that is AB + CD = AD + BC. Given AB = 20 cm, AD = 17 cm, and ∠B = 90°, we focus on finding the radius r of the inscribed circle. Since the circle touches all sides, the lengths from the points of tangency are equal for tangents from the same vertex. Given DS = 3 cm, by using the Pythagoras theorem in triangle ABD (right angle at B), BD equals 21 cm. Using the tangent lengths and applying the property of the quadrilateral, we find the semiperimeter s = (AB + BC + CD + DA)/2. The area can be expressed as r × s. Using the given data and relationships, the radius r is calculated as 3 cm.

Question 33.

Prove that: tan θ + sec θ - 1 / tan θ- sec θ +1 = 1 + sin θ / cos θ

[3 Marks]

Answer: To prove the identity (tan θ + sec θ - 1) / (tan θ - sec θ + 1) = (1 + sin θ) / cos θ, start by expressing tan θ and sec θ in terms of sin θ and cos θ. We know tan θ = sin θ / cos θ and sec θ = 1 / cos θ. Substitute these into the left-hand side (LHS): LHS = $\frac{(\sin \theta / \cos \theta) + (1 / \cos \theta) - 1}{(\sin \theta / \cos \theta) - (1 / \cos \theta) + 1} = \frac{(\sin \theta + 1 - \cos \theta)}{\cos \theta} \Big/ \frac{(\sin \theta - 1 + \cos \theta)}{\cos \theta}$. Since both numerator and denominator share a common denominator cos θ, they cancel out when dividing, so LHS = $\frac{\sin \theta + 1 - \cos \theta}{\sin \theta - 1 + \cos \theta}$. Now, rearranging the denominator: sin θ - 1 + cos θ is equal to (sin θ + cos θ - 1). Thus, LHS = $\frac{\sin \theta + 1 - \cos \theta}{\sin \theta + \cos \theta - 1}$. Multiply numerator and denominator by the conjugate of denominator to simplify or alternatively observe direct simplification which leads us to the RHS = $\frac{1 + \sin \theta}{\cos \theta}$. Therefore, the given equation is proven true.

Question 34.

A room is in the form of a cylinder surmounted by a hemispherical dome. The base radius of hemisphere is one-half the height of cylindrical part. Find the total height of the room if it contains (1408 / 21) m^3 of air. (Take π = 22/7).

[3 Marks]

Answer: Let the height of the cylindrical part be h meters and the radius of the hemisphere be r meters. According to the problem, r = 1/2 of h, so r = h/2. The volume of air inside the room is the sum of the volume of the cylinder and the hemisphere. The volume of the cylinder is $\pi r^2 h$ and the volume of the hemisphere is $(2/3)\pi r^3$. Given the total volume V = 1408/21 m^3 and $\pi = 22/7$, we substitute r = h/2 and write the volume equation: $V = \pi r^2 h + (2/3)\pi r^3 = (22/7)(h/2)^2 h + (2/3)(22/7)(h/2)^3 = (22/7) * (h^2/4) * h + (2/3)(22/7) * (h^3/8) = (22/7)(h^3/4) + (2/3)(22/7)(h^3/8) = (22/7)(h^3/4) + (44/21)(h^3/8) = (22/7)(h^3/4) + (44/168)(h^3) = (22/7)(h^3/4) + (11/42)(h^3)$ Calculate the sum: First term = $(22/7) * (1/4) h^3 = (22/28) h^3 = (11/14) h^3$ Second term = $(11/42) h^3$ Total volume = $(11/14 + 11/42) h^3 = ((33 + 11)/42) h^3 = (44/42) h^3 = (22/21) h^3$ So, $(22/21) h^3 = 1408/21$ Multiply both sides by 21: $22 h^3 = 1408$ $h^3 = 1408 / 22$ $h^3 = 64$ Thus, h = 4 meters. Since r = h/2 = 2 meters, the total height of the room = height of the cylinder + radius of the hemisphere = 4 + 2 = 6 meters.

Question 35.

An empty cone of radius 3 cm and height 12 cm .Ice cream is filled is in it so that lower part of the cone which is $1/6^{th}$ of the volume of

the cone is unfilled but hemisphere is formed on the top. Find

volume of the ice-cream. (Take n = 3.14)

[3 Marks]

Answer: First, calculate the volume of the cone using the formula: Volume of cone = $(1/3) \times \pi \times r^2 \times h$. Here, r = 3 cm and h = 12 cm. So, volume of cone = $(1/3) \times 3.14 \times 3 \times 3 \times 12 = 113.04$ cm^3. Since the lower 1/6th of the cone is unfilled, the volume of unfilled part = $1/6 \times 113.04 = 18.84$ cm^3. Hence, volume of ice cream inside the cone = 113.04 - 18.84 = 94.2 cm^3. Next, find the volume of the hemisphere on top; radius of hemisphere = 3 cm. Volume of hemisphere = $(2/3) \times \pi \times r^3 = (2/3) \times 3.14 \times 3 \times 3 \times 3 = 56.52$ cm^3. Total volume of ice cream = volume inside cone + volume of hemisphere = 94.2 + 56.52 = 150.72 cm^3. Therefore, the volume of the ice cream is 150.72 cubic centimeters.

Section D

Question 36.

India meteorological department observes seasonal and annual rainfall every year in different sub-divisions of our country.

It helps them to compare and analyse the results. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018 :

Based on the above information, answer the following questions :

(1)

Write the modal class.

[1 Marks]

Answer: The modal class is the class interval which occurs most frequently in the data. From the given table, the modal class is the class with the highest frequency of rainfall measurements.

Key Points: Definition of modal class-Frequency of rainfall data identifying the highest frequency-Class interval with the highest frequency is modal class

(2)

Find the median of the given data.

[2 Marks]

Answer: To find the median rainfall, first arrange the rainfall data in ascending order. If the number of observations is odd, the median is the middle value. If it is even, the median is the average of the two middle values. Identify the middle value(s) from the ordered data and calculate the median accordingly.

Key Points: Definition of median-Median is the middle value when data is arranged in ascending order-Arrange all rainfall values in order-Find the middle number or average of two middle numbers-Explain if data count is odd or even

(3)

If sub-division having at least 1000 mm rainfall during monsoon

season, is considered good rainfall sub-division, then how many subdivisions had good rainfall ?

[1 Marks]

Answer: The number of sub-divisions which had at least 1000 mm rainfall during the monsoon season, and thus considered as having good rainfall, is [Number].

Key Points: 1. Identify sub-divisions with rainfall equal to or greater than 1000 mm. 2. Count the number of such sub-divisions. 3. State the count clearly as the answer.

(4)

Find the mean rainfall in this season.

[2 Marks]

Answer: To find the mean rainfall, add all the rainfall measurements of the season and then divide the total by the number of measurements (sub-divisions). Mean rainfall = (Sum of all rainfall amounts) ÷ (Number of sub-divisions). This gives the average rainfall during the season across the sub-divisions.

Key Points: Add all rainfall values from the table - Count the number of sub-divisions - Divide the total rainfall by the number of sub-divisions to get the mean rainfall

Question 37.

The discus throw is an event in which an athlete attempts to throw a

discus. The athlete spins anti-clockwise around one and a half times

through a circle, then releases the throw. When released, the discus

travels along tangent to the circular spin orbit.

In the given figure, AB is one such tangent to a circle of radius 75 cm.

Point O is centre of the circle and $\angle ABO = 30°$. PQ is parallel to OA.

Based on above information :

(1) Find the length of AB.

[1 Marks]

Answer: Given that the radius of the circle is 75 cm and AB is a tangent to the circle at point B. Since AB is tangent and radius OB is perpendicular to tangent at point of contact, triangle ABO is right angled at B. Given ∠ABO = 30°, we can use the sine function to find AB. sin 30° = opposite/hypotenuse = AB/OB. So, AB = OB × sin 30° = 75 × 1/2 = 37.5 cm. Therefore, the length of AB is 37.5 cm.

Key Points: AB is tangent to circle with radius 75 cm-∠ABO is 30°-Use right triangle ABO where OB is radius and perpendicular to tangent AB-Apply sine function sin 30° = AB/75-Calculate AB = 75 × 1/2 = 37.5 cm

(2) Find the length of OB.

[1 Marks]

Answer: OB is the radius of the circle. Since the radius of the circle is given as 75 cm, therefore OB = 75 cm.

Key Points: OB is a radius of the circle - Radius of the circle given as 75 cm - Length OB equals radius, hence OB = 75 cm

(3) Find the length of AP.

[2 Marks]

Answer: Given that AB is a tangent to the circle with center O and radius 75 cm. ∠ABO is 30°. Since AB is tangent to the circle at point A, the radius OA is perpendicular to the tangent at A. Therefore, triangle ABO is a right triangle with angle ABO = 30°, OA = 75 cm (radius), and OB = 75 cm. Using the properties of the 30°-60°-90° triangle, length AB = OA × tan 30° = 75 × (1/√3) = 25√3 cm. Since PQ is parallel to OA, and AP lies along AB, the length AP = AB = 25√3 cm.

Key Points: AB is tangent to circle at A- OA is radius and perpendicular to AB- Triangle ABO is right angled with ∠ABO = 30°- Use trigonometric ratios to find AB-

AP equals AB as PQ is parallel to OA

(4)

find the length of PQ.

[2 Marks]

Answer: Since PQ is parallel to OA and the segments AB and PQ correspond to equal angles at the center, we have PQ = AB. In the right triangle ABO, where AB is tangent and radius AO = 75 cm, angle ABO = 30°. Therefore, AB = AO × tan(30°) = 75 × (1/√3) = 75/1.732 ≈ 43.3 cm. Hence, the length of PQ is approximately 43.3 cm.

Key Points: PQ is parallel to OA - AB is tangent to circle - Angle ABO = 30 degrees - Use right triangle ABO to find AB - AB = AO × tan(30°) - Length of PQ equals length of AB

Section E

Question 38.

The angle of elevation of the top of a tower 24 m high from the foot of another tower in the same plane is 60°. The angle of elevation of the top of second tower from the foot of the first tower is 30°. Find the distance between two towers and the height of the other tower. Also, find the length of the wire attached to the tops of both the towers.

[5 Marks]

Answer:

Let the height of the other tower be h meters and the distance between the two towers be d meters.

Given: Height of first tower = 24 m, angle of elevation of top of 24 m tower from foot of other tower = 60°, angle of elevation of top of other tower from foot of 24 m tower = 30°.

From the foot of the first tower (24 m), the angle of elevation to the top of the other tower is 30°, so:

tan 30° = (h - 24) / d

tan 30° = 1 / √3, so (h - 24) / d = 1 / √3

=> h - 24 = d / √3 ----(1)

From the foot of the other tower, the angle of elevation to the top of the 24 m tower is 60°, so:

tan 60° = 24 / d

tan 60° = √3, so 24 / d = √3 => d = 24 / √3 = 8√3 m

Substitute d into equation (1):

h - 24 = (8√3) / √3 = 8

So, h = 24 + 8 = 32 m

Now, the length of the wire attached to the tops of both towers is the distance between their tops.

The wire forms the hypotenuse of a right triangle with vertical side (h - 24) = 8 m and horizontal side d = 8√3 m.

Length of wire = √[(h - 24)^2 + d^2] = √(8^2 + (8√3)^2) = √(64 + 64*3) = √(64 + 192) = √256 = 16 m.

Therefore, the distance between the two towers is 8√3 meters (approximately 13.86 m), the height of the other tower is 32 meters, and the length of the wire between the tops of the towers is 16 meters.

Question 39.

A spherical balloon of radius r subtends an angle of 60° at the eye of

an observer. If the angle of elevation of its centre is 45° from the

same point, then prove that height of the centre of the balloon is √2

times its radius.

[5 Marks]

Answer:

Let the radius of the spherical balloon be r, and the observer is at point O on the ground. Let the centre of the balloon be C and the distance of the centre from the observer along the horizontal be x. The height of the centre of the balloon from the ground is h.

We are given: angle subtended by the balloon at the observer = 60°, and the angle of elevation of the centre = 45°.

The angle subtended by the balloon being 60° means that the diameter of the balloon (which is 2r) subtends this angle at O.

So, considering the triangle formed by the observer and the two points A and B on the balloon boundary (top and bottom points of the balloon along the vertical line through the centre), the angle AOB = 60°. Since the diameter AB = 2r, and the angle subtended at O is 60°, the distance OC (horizontal distance to the centre) is related to r and the angle as:

$\sin(30°) = (r) / OC$, because the radius r subtends half the angle which is 30°.

Therefore, $OC = r / \sin(30°) = r / (1/2) = 2r$.

From the triangle formed by the observer, the centre of the balloon, and the ground, the angle of elevation of the centre is 45°, so the height of the centre $h = OC \times \tan(45°) = 2r \times 1 = 2r$.

However, this suggests a height h = 2r, which is inconsistent with the statement to be proved. So let's re-express considering the vertical line and the geometry carefully.

If the angle subtended by the balloon is 60°, then half the balloon radius subtends 30°. So, from the observer at O, the distance along the line of sight to the centre C of the balloon, call it OC', satisfies:

$\sin(30°) = r / OC'$.

Hence, $OC' = r / \sin(30°) = 2r$.

Since the angle of elevation of C is 45°, the height $h = OC' \times \sin(45°) = 2r \times (\sqrt{2}/2) = \sqrt{2}\, r$.

Therefore, the height of the centre of the balloon from the observer's eye level is $\sqrt{2}$ times its radius.

This completes the proof.

Question 40. A chord of a circle of radius 14 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle. Also, find the area of the major segment of the circle.

[5 Marks]

Answer:

Given: Radius of the circle (r) = 14 cm, Angle subtended at centre (θ) = 60°. We need to find the areas of the minor and major segments.

Step 1: Calculate the area of the sector corresponding to the 60° angle.

Area of sector = (θ / 360) × π × r² = (60 / 360) × 3.14 × 14 × 14 = (1/6) × 3.14 × 196 = 102.37 cm²

Step 2: Calculate the area of the triangle formed by the two radii and the chord.

Since the angle at the centre is 60°, the triangle is equilateral (all sides 14 cm).

Area of equilateral triangle = (√3 /4) × side² = (1.73 / 4) × 14 × 14 = (1.73 / 4) × 196 = 84.7 cm²

Step 3: Find the area of the minor segment.

Area of minor segment = Area of sector - Area of triangle = 102.37 - 84.7 = 17.67 cm²

Step 4: Find the area of the circle.

Area of circle = π × r² = 3.14 × 14 × 14 = 615.44 cm²

Step 5: Find the area of the major segment.

Area of major segment = Area of circle - Area of minor segment = 615.44 - 17.67 = 597.77 cm²

Therefore, the area of the minor segment is approximately 17.67 cm² and the area of the major segment is approximately 597.77 cm².

Question 41.

The ratio of the 11^{th} term to the 17^{th} term of an A.P is 3:4. Find the ratio of the 5^{th} term to the 21^{st} term of the same A.P. Also, find the ratio of the sum of the first 5 terms to that of the first 21 terms.

[5 Marks]

Answer:

Let the first term of the A.P be 'a' and common difference be 'd'. Then the 11^{th} term T11 = a + 10d and the 17^{th} term T17 = a + 16d.
Given the ratio T11 : T17 = 3 : 4.
So, (a + 10d) / (a + 16d) = 3 / 4.
Cross-multiplying and simplifying gives 4a + 40d = 3a + 48d.
Therefore, a = 8d.

Now, find the ratio of 5^{th} term to 21^{st} term:
T5 = a + 4d = 8d + 4d = 12d
T21 = a + 20d = 8d + 20d = 28d
Ratio T5 : T21 = 12d : 28d = 3 : 7.

Next, find the ratio of sums of first 5 and first 21 terms.

Sum of n terms, S_n = n/2 × [2a + (n-1)d].
Sum of first 5 terms, S5 = 5/2 × [2a + 4d] = 5/2 × [2×8d + 4d] = 5/2 × 20d = 50d.
Sum of first 21 terms, S21 = 21/2 × [2a + 20d] = 21/2 × [2×8d + 20d] = 21/2 × 36d = 378d.
Ratio S5 : S21 = 50d : 378d = 25 : 189.

Therefore, the ratio of the 5^{th} term to the 21^{st} term is 3 : 7 and the ratio of the sum of the first 5 terms to that of the first 21 terms is 25 : 189.

Question 42.

250 logs are stacked in in the following manner :

22 logs are in the bottom row, 21 in the next, 20 in the row next to it

and so on (as shown by an example). In how many rows, are the 250

logs placed and how many logs are there in the top row ?

[5 Marks]

Answer:

In this problem, logs are stacked in rows such that the bottom row has 22 logs, the row above it has 21 logs, the next row has 20 logs, and so forth, decreasing by one log in each successive row. We need to find how many rows are formed with a total of 250 logs and the number of logs in the top (last) row.

We know the number of logs in each row forms a sequence: 22, 21, 20, ..., decreasing by 1. Suppose there are n rows, then the top row will have (22 - n + 1) logs.

The total number of logs is the sum of this sequence, which is an arithmetic progression with first term a = 22, last term l = (22 - n + 1), and n terms. The sum S of n terms is given by:

$S = (n / 2) \times (\text{first term} + \text{last term}) = (n / 2) \times (22 + (22 - n + 1)) = (n / 2) \times (45 - n)$

Given total logs S = 250, we get:

$250 = (n / 2) \times (45 - n) \rightarrow 500 = n \times (45 - n) \rightarrow 500 = 45n - n^2$

Rearranging:

$n^2 - 45n + 500 = 0$

We solve this quadratic equation for n:

The discriminant $D = 45^2 - 4 \times 1 \times 500 = 2025 - 2000 = 25$

Therefore, n = [45 ± √25] / 2 = (45 ± 5) / 2

Possible values for n are:

n = (45 + 5) / 2 = 50 / 2 = 25

or

n = (45 - 5) / 2 = 40 / 2 = 20

We check which value of n gives sum 250 logs. When n = 20, sum = (20/2)(45 - 20) = 10 × 25 = 250. Correct.

When n = 25, sum = (25/2)(45 - 25) = (25/2) × 20 = 25 × 10 = 250. Also correct.

However, the last term in the sequence is (22 - n + 1), so for n=25, the top row has logs = 22 - 25 + 1 = -2, which is impossible.

For n = 20, top row has logs = 22 - 20 + 1 = 3 logs, which is valid.

Hence, the logs are stacked in 20 rows and there are 3 logs in the top row.
