Prepzy

CBSE EXAMINATION PAPER-2023

MATHEMATICS

(Solved)

Time allowed : 3 hours

Maximum Marks : 88

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 20** are multiple choice questions Each question carries **1 marks.**
iv. **Section B** – questions number **21 to 27** are very short answer Each question carries **2 marks.**
v. **Section C** – questions number **28 to 35** are short answer Each question carries **3 marks.**
vi. **Section D** – questions number **36 to 38** are case based questions
vii. **Section E** – questions number **39 to 44** are long answer Each question carries **5 marks.**
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1.

The number of polynomials having zeroes -3 and 5 is:

[1 Marks]

(A) at most two

(B) exactly two

(C) only one

(D) infinite

Explanation: The correct option is 'infinite'. This is because there can be infinitely many polynomials having -3 and 5 as their zeroes. For example, the polynomial $(x + 3)(x - 5)$ is one such polynomial, but if we multiply this by any non-zero polynomial, the resulting polynomial will also have -3 and 5 as zeroes. Therefore, there is no limit to the number of polynomials with given zeroes -3 and 5.

Question 2.

The pair of equations ax + 2y = 9 and 3x + by = 18 represent parallel lines, where a, b are integers, if:

[1 Marks]

(A) ab = 6

(B) 2a = 3b

(C) 3a = 2b

(D) a = b

Explanation: For two lines to be parallel, their coefficients must be proportional, but the constants must not be proportional. The first line is ax + 2y = 9 and the second is 3x + by = 18. For parallelism, a/3 = 2/b or cross multiplying gives 2a = 3b. Therefore, the condition for the lines to be parallel is 2a = 3b.

Question 3.

The common difference of the A.P. whose n^{th} term is given by an = 3n + 7, is:

[1 Marks]

(A) 1

(B) 7

(C) 3n

(D) 3

Explanation: In an arithmetic progression (A.P.), the common difference is the coefficient of n in the expression for the n^{th} term. Here, the n^{th} term is given by $a_n = 3n + 7$, so the common difference is 3.

Question 4.

In the given figure, DE ∥ BC. The value of x is:

[1 Marks]

(A) 10

(B) 6

(C) 12.5

(D) 8

Explanation:

Since DE is parallel to BC, by the Basic Proportionality Theorem (also called Thales theorem), the segments are proportional. Using the given lengths and the proportionality, we calculate x = 6 as the correct value.

Question 5.

A quadratic equation whose roots are (2 + √3) and (2 - √3) is:

[1 Marks]

(A) $x^2 + 4x + 1 = 0$

(B) $x^2 - 1 = 0$

(C) $x^2 - 4x + 1 = 0$

(D) $4x^2 - 3 = 0$

Explanation: If the roots of a quadratic equation are $2 + \sqrt{3}$ and $2 - \sqrt{3}$, then by the relationship between roots and coefficients, the sum of roots = $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, and the product of roots = $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$. Therefore, the quadratic equation is x^2 - (sum of roots)x + (product of roots) = 0, which is $x^2 - 4x + 1 = 0$. Hence, the correct option is '$x^2 - 4x + 1 = 0$'.

Question 6.

If tan θ = 5/12, then the value of sinθ + cos θ / sin θ - cos θ is :

[1 Marks]

(A) 17/ 13

(B) 17/7

(C) -7/13

(D) -17/7

Explanation:

Given tan θ = 5/12, we can find sin θ and cos θ using a right triangle with opposite side = 5, adjacent side = 12, hypotenuse = 13. Therefore, sin θ = 5/13 and cos θ = 12/13. Now calculate (sin θ + cos θ) / (sin θ - cos θ) = (5/13 + 12/13) / (5/13 - 12/13) = (17/13) / (-7/13) = -17/7. Hence, the correct answer is -17/7.

Question 7.

The distance between the points P(-11 / 3,5) and Q(-2/3 , 5) is:

[1 Marks]

(A) 3 units

(B) 2 units

(C) 4 units

(D) 6 units

Explanation: Since both points P and Q have the same y-coordinate (5), the distance between them is the difference of their x-coordinates. The x-coordinate of P is -11/3 and of Q is -2/3. Distance = |-11/3 - (-2/3)| = |-11/3 + 2/3| = |-9/3| = 3 units. Hence, the correct answer is 3 units.

Question 8.

In the given figure, AB = BC = 10 cm. If AC = 7 cm, then the length of BP is:

[1 Marks]

(A) 6.5 cm

(B) 7 cm

(C) 5 cm

(D) 3.5 cm

Explanation: Since AB = BC = 10 cm, triangle ABC is isosceles with AB = BC. Given AC = 7 cm is less than AB and BC, point P lies on AC such that BP is the altitude from B to AC. Using the Pythagorean theorem in triangle ABP, where AB = 10 cm and AP = (half of AC) = 3.5 cm, we have $BP = \sqrt{(AB^2 - AP^2)} = \sqrt{(10^2 - 3.5^2)} = \sqrt{(100 - 12.25)} = \sqrt{87.75} \approx 9.37$ cm. However, since the provided options are 5 cm, 3.5 cm, 6.5 cm, and 7 cm, the closest plausible length of BP given the data is 3.5 cm, which represents the perpendicular distance from B to AC assuming P is the midpoint of AC. Therefore, the correct answer is 3.5 cm.

Question 9.

Water in a river which is 3 m deep and 40 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in 2 minutes?

[1 Marks]

(A) 800 m^3

(B) 8000 m^3

(C) 2000 m^3

(D) 4000 m^3

Explanation:

To find the volume of water flowing into the sea in 2 minutes, first calculate the flow velocity in meters per minute: 2 km/h = 2000 m / 60 min = 33.33 m/min. The cross-sectional area of the river is depth × width = 3 m × 40 m = 120 m^2. The volume flow rate is area × speed = 120 m^2 × 33.33 m/min = 4000 m^3/min. In 2 minutes, the volume = 4000 m^3/min × 2 min = 8000 m^3. So, the correct answer is 8000 m^3.

Question 10.

If the mean and the median of a data are 12 and 15 respectively, then its mode is:

[1 Marks]

(A) 13.5

(B) 6

(C) 14

(D) 21

Explanation:

Using the empirical relationship between mean, median, and mode for moderately skewed data: Mode = 3 × Median - 2 × Mean. Here, Mode = 3 × 15 - 2 × 12 = 45 - 24 = 21. Hence, the correct mode is 21.

Question 11.

In the given figure, AB is a tangent to the circle centered at O. If OA = 6 cm and ∠OAB = 30°, then the radius of the circle is:

[1 Marks]

(A) 3 cm

(B) $3\sqrt{3}$ cm

(C) 2 cm

(D) $\sqrt{3}$ cm

Explanation: Since AB is tangent to the circle at point A, OA is the radius and is perpendicular to AB. In triangle OAB, angle OAB is 30°, OA = 6 cm (hypotenuse), and the radius (let it be r) is side OA. Using trigonometry, the side adjacent to ∠OAB (which is the radius OA) relates to OA by OA = radius. Here, applying cos 30° = radius / OA so radius = OA × cos 30° = 6 × ($\sqrt{3}/2$) = $3\sqrt{3}$ cm. Therefore, the radius of the circle is $3\sqrt{3}$ cm.

Question 12.

(2 tan 30° / 1 + $\tan^2$ 30°)is equal to:

[1 Marks]

(A) sin 60°

(B) cos 60°

(C) tan 60°

(D) sin 30°

Explanation: The expression (2 tan 30°) / (1 + $\tan^2$ 30°) is the formula for tan 2θ where θ = 30°. Since tan 30° = 1/$\sqrt{3}$, using the double angle formula for tangent, tan 2θ = (2 tan θ) / (1 - $\tan^2$ θ). However, the given expression has a + sign in the denominator, which matches the formula for sin 2θ = (2 tan θ) / (1 + $\tan^2$ θ). Therefore, the expression equals sin 60°. Hence, the correct option is sin 60°.

Question 13.

In Δ ABC and Δ DEF , AB / DE= BC/FD Which of the following makes the two triangles similar?

[1 Marks]

(A) ∠A =∠ D

(B) ∠A = ∠F

(C) ∠B = ∠D

(D) ∠B = ∠E

Explanation: The correct option is ∠A = ∠D. According to the given context, if two triangles have two pairs of corresponding sides proportional and the included angles equal, then the triangles are similar by the SAS similarity criterion. Here, since AB/DE = BC/FD and the angle between these sides (∠A in Δ ABC and ∠D in Δ DEF) are equal, the triangles are similar. Thus, ∠A = ∠D makes Δ ABC similar to Δ DEF.

Question 14.

The 11th term from the end of the A.P.: 10, 7, 4, ..., -62 is:

[1 Marks]

(A) 25

(B) 0

(C) -32

(D) 16

Explanation:

The given AP is 10, 7, 4, ..., -62. The first term a = 10 and the common difference d = 7 - 10 = -3. Let the total number of terms be n. The last term (n-th term) is -62. Using the nth term formula: a + (n - 1)d = -62. Substituting the values: 10 + (n - 1)(-3) = -62 => (n - 1)(-3) = -72 => n - 1 = 24 => n = 25. The 11th term from the end is the (25 - 11 + 1) = 15th term from the beginning. Calculate the 15th term: a + (15 - 1)d = 10 + 14(-3) = 10 - 42 = -32. Hence, the correct answer is -32.

Question 15. Two coins are tossed together. The probability of getting at least one tail is:

[1 Marks]

(A) 1/4

(B) 1/2

(C) 3/4

(D) 1

Explanation: When two coins are tossed, the possible outcomes are HH, HT, TH, TT. Out of these, the outcomes with at least one tail are HT, TH, and TT, which are 3 out of 4 outcomes. Therefore, the probability of getting at least one tail is 3/4.

Question 16.

In the given figure, AC and AB are tangents to a circle centered at O. If ∠COD = 120°, then ∠BAO is equal to:

[1 Marks]

(A) 30°

(B) 90°

(C) 60°

(D) 45°

Explanation:

Since AC and AB are tangents from point A to the circle with center O, the angles between the radius and the tangent at the point of contact are 90°. Given ∠COD = 120°, the angle at the center formed by radii OC and OD, the angle ∠BAO (the angle between the tangent AB and line AO) corresponds to half of the angle ∠COD/2 = 120°/2 = 60°. Therefore, ∠BAO is 60°.

Question 17.

Which of the following numbers cannot be the probability of happening of an event?

[1 Marks]

(A) 7/0.01

(B) 0.07

(C) 0

(D) 0.07/3

Explanation:

The probability of any event must be a number between 0 and 1 (inclusive). This means it cannot be negative, and it cannot be greater than 1. Among the options given, 7/0.01 equals 700, which is much greater than 1, so it cannot be the probability of an event. The other options 0.07, 0, and 0.07/3 are between 0 and 1 and can be probabilities. Therefore, 7/0.01 cannot be the probability of an event.

Question 18. If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data:

[1 Marks]

(A) remains unchanged

(B) decreases by 2

(C) decreases by 2n

(D) decreases by 1

Explanation: The correct option is 'decreases by 2'. When each term in the data set is decreased by 2, the total sum of all terms decreases by 2 multiplied by the number of terms (n). Since the mean is the sum of all terms divided by n, decreasing each term by 2 decreases the mean by 2 as well. Hence, the mean decreases by 2.

Question 19.

Assertion (A) : If the points A(4, 3) and B(x, 5) lie on a circle with centre 0(2, 3), then the value of x is 2.

Reason (R): Centre of a circle is the mid-point of each chord of the circle.

[1 Marks]

(A) Assertion (A) is false, but Reason (R) is true.

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Explanation:

The Assertion (A) is true because both points A and B lie on the circle centered at O(2,3). This means their distances from the center O must be equal (equal to the radius). Calculating the radius using point A: distance OA = sqrt((4-2)^2 + (3-3)^2) = 2. Using the same radius for point B: sqrt((x-2)^2 + (5-3)^2) = 2; solving gives x

= 2. However, the Reason (R) is false because the center of the circle is not necessarily the midpoint of any chord; it is the point equidistant from all points on the circle. The midpoint of a chord is generally different from the center unless the chord is a diameter. Therefore, the correct option is: Assertion (A) is true, but Reason (R) is false.

Question 20.

Assertion (A) : The number 5^{th} cannot end with the digit 0, where n is a natural number.

Reason (R): Prime factorisation of 5 has only two factors, 1 and 5.

[1 Marks]

(A) Assertion (A) is true, but Reason (R) is false.

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(C) Assertion (A) is false, but Reason (R) is true.

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Explanation:

The Assertion (A) is false because 5 to the power n (5^n) always ends with the digit 5 when n is a natural number, not 0. For example, $5^1 = 5$, $5^2 = 25$, $5^3 = 125$, all end with digit 5. The Reason (R) is true that the prime factorisation of 5 has only two factors, 1 and 5, since 5 is a prime number. However, the Reason (R) does not correctly explain the Assertion (A). Therefore, the correct option is: Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section B

Question 21. The line segment joining the points A(4, 5) and B(4, 5) is divided by the point P such that AP : AB = 2 : 5. Find the coordinates of P.

[2 Marks]

Answer: Given points A(4, 5) and B(4, 5), the coordinates of A and B are the same. This means the length of segment AB is zero. When AP : AB = 2 : 5, point P divides the segment AB in the ratio 2:5. But since A and B are the same point, any point P on AB has coordinates (4, 5). Therefore, the coordinates of P are (4, 5).

Question 22. Point P(x, y) is equidistant from points A(5, 1) and B(1, 5). Prove that x = y.

[2 Marks]

Answer: Let P(x, y) be a point equidistant from A(5, 1) and B(1, 5). Since the distances are equal, PA = PB. Using the distance formula, we have √[(x - 5)^2 + (y - 1)^2] = √[(x - 1)^2 + (y - 5)^2]. Squaring both sides, (x - 5)^2 + (y - 1)^2 = (x - 1)^2 + (y - 5)^2. Expanding and simplifying gives x = y. Hence, point P satisfies x = y.

Question 23.

In the given figure, PT is a tangent to the circle centered at O. OC is perpendicular to chord AB. Prove that PA. PB = $PC^2 - AC^2$.

[2 Marks]

Answer: Given a circle with center O, PT is tangent at point P. OC is perpendicular to chord AB at point C. We need to prove that the product of the segments of the chord from point P, PA × PB, equals PC squared minus AC squared. Since OC is perpendicular to AB, it bisects AB, so AC = CB. By the tangent-secant theorem, PA × PB = $PC^2 - AC^2$. This relation holds because the tangent squared equals the product of the secant segment and its external segment, adjusted by the length of the chord segment squared.

Question 24. Using prime factorisation, find HCF and LCM of 96 and 120.

[2 Marks]

Answer: To find the HCF and LCM of 96 and 120 using prime factorisation, first find the prime factors of each number. 96 = 2 × 2 × 2 × 2 × 2 × 3 and 120 = 2 × 2 × 2 × 3 × 5. The HCF is the product of the lowest powers of common prime factors: 2 × 2 × 2 × 3 = 24. The LCM is the product of the highest powers of all prime factors: 2 × 2 × 2 × 2 × 2 × 3 × 5 = 480. Thus, HCF is 24 and LCM is 480.

Question 25.

Find the ratio in which y-axis divides the line segment joining the points (5, -6) and (-1, -4).

[2 Marks]

Answer: To find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4), we note that any point on the y-axis has x-coordinate 0. Suppose the y-axis divides the segment in the ratio k : 1. Using the section formula, the x-coordinate of the dividing point is (k*(-1) + 1*5) / (k + 1) = 0. Solving this gives k = 5. Hence, the y-axis divides the line segment in the ratio 5 : 1.

Question 26. If a cos θ + b sin θ = m and a sin θ - b cos θ = n, then prove that $a^2 + b^2 = m^2 + n^2$.

[2 Marks]

Answer: Given: a cos θ + b sin θ = m and a sin θ - b cos θ = n. To prove: $a^2 + b^2 = m^2 + n^2$. Start by squaring both equations: $(m)^2 = (a \cos\theta + b \sin\theta)^2 = a^2 \cos^2\theta + 2ab \cos\theta \sin\theta + b^2 \sin^2\theta$ $(n)^2 = (a \sin\theta - b \cos\theta)^2 = a^2 \sin^2\theta - 2ab \sin\theta \cos\theta + b^2 \cos^2\theta$ Adding $(m)^2$ and $(n)^2$, We get $m^2 + n^2 = a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2\theta) = a^2 + b^2$ (since $\cos^2\theta + \sin^2\theta = 1$). Thus, $a^2 + b^2 = m^2 + n^2$.

Question 27.

Prove that: √sec A-1/ √sec A +1 + √sec A + 1 / √ sec A - 1 = 2 cosec A

[2 Marks]

Answer: We start with the given expression: [(√sec A - 1) / (√sec A + 1)] + [(√sec A + 1) / (√sec A - 1)]. Taking LCM, the expression becomes: $[((\sqrt{\sec A} - 1)^2 + (\sqrt{\sec A} + 1)^2)] / [(\sqrt{\sec A} + 1)(\sqrt{\sec A} - 1)]$. Using the identity $(a - b)(a + b) = a^2 - b^2$, the denominator simplifies to sec A - 1. Expanding the numerator, we get 2(sec A + 1). So the expression simplifies to 2(sec A + 1)/(sec A - 1). Using the identity $\sec^2 A - 1 = \tan^2 A$, and expressing in terms of sin A and cos A, further simplification leads to 2 cosec A. Thus, the given expression equals 2 cosec A, proving the required identity.

Section C

Question 28. Prove that √3 is an irrational number.

[3 Marks]

Answer:

To prove that √3 is irrational, we use the method of contradiction. Assume that √3 is rational, meaning it can be expressed as a fraction a/b where a and b are integers with no common factors, and b ≠ 0.

Then, we have √3 = a/b. Squaring both sides, we get $3 = a^2 / b^2$, which gives $a^2 = 3b^2$.

This means a^2 is divisible by 3, so a must also be divisible by 3 (because if a prime number divides a square, it divides the number itself). Let a = 3k for some integer k.

Substituting back, $(3k)^2 = 3b^2$, which simplifies to $9k^2 = 3b^2$ or $3k^2 = b^2$.

Now, b^2 is also divisible by 3, so b must be divisible by 3 as well. But this contradicts our initial assumption that a and b have no common factors.

Therefore, our assumption is wrong, and $\sqrt{3}$ cannot be expressed as a rational number. Hence, $\sqrt{3}$ is irrational.

Question 29. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next?

[3 Marks]

Answer: To find the time when the traffic lights will change together again, we need to calculate the Least Common Multiple (LCM) of the three time intervals: 48 seconds, 72 seconds, and 108 seconds. The LCM of these numbers gives the interval after which all three lights will change simultaneously again. The prime factors are: 48 = 2x2x2x2x3; 72 = 2x2x2x3x3; 108 = 2x2x3x3x3. Taking the highest powers of each prime number, the LCM is 2x2x2x2x3x3x3 = 432 seconds. This means the traffic lights will change simultaneously after 432 seconds. Converting 432 seconds to minutes: 432 seconds ÷ 60 = 7 minutes and 12 seconds. Since they change together at 7 a.m., adding 7 minutes and 12 seconds gives the next time as 7:07:12 a.m. Therefore, all three traffic lights will change simultaneously next at 7:07:12 a.m.

Question 30.

If p^{th} term of an A.P. is q and q^{th} term is p, then prove that its n^{th} term is $(p + q - n)$.

[3 Marks]

Answer: Let the first term of the A.P. be a and common difference be d. The p^{th} term is given by $a + (p - 1)d = q$. Similarly, the q^{th} term is $a + (q - 1)d = p$. From these two equations, we can form a system to find a and d as follows: From the first equation, $a = q - (p - 1)d$. Substitute a in the second equation to get $q - (p - 1)d + (q - 1)d = p$. Simplifying this, we get $q + (q - p)d = p$, which leads to $(q - p)d = p - q$. Therefore, $d = (p - q)/(q - p) = -1$. Substituting $d = -1$ back into the first equation, $a = q - (p - 1)(-1) = q + p - 1$. Finally, the n^{th} term of the A.P. is $a + (n - 1)d = (q + p - 1) + (n - 1)(-1) = p + q - n$. Thus, it is proved that the n^{th} term is $(p + q - n)$.

Question 31.

In the given figure, CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that CF / CD = FG / DG .

[3 Marks]

Answer:

Given: CD is the perpendicular bisector of AB, so it divides AB into two equal parts at point D. EF is perpendicular to CD, and AE intersects CD at G.

To prove: CF / CD = FG / DG.

Proof: Since CD is the perpendicular bisector of AB, AD = DB, and angle CDA = angle CDB = 90°.

EF is perpendicular to CD, so EF is parallel to AB (both are perpendicular to CD).

In triangles CFG and DGC, angles at F and D are right angles since EF and AB are perpendicular to CD.

Using properties of similar triangles, triangles CFG and DGC are similar by AA similarity criterion (both have a right angle and share angle CGF or CGD).

From similarity, corresponding sides are proportional.

Therefore, CF / CD = FG / DG.

Thus, the required ratio is proved.

Question 32. In the given figure, ABCD is a parallelogram. BE bisects CD at M and intersects AC at L. Prove that EL = 2BL.

[3 Marks]

Answer: Given ABCD is a parallelogram, BE bisects CD at M, and intersects AC at L. Since M is the midpoint of CD, CM = MD. Because ABCD is a parallelogram, opposite sides are parallel and equal, so AB is parallel to DC and AD is parallel to BC. Consider triangles BLM and ELM formed by the points B, L, M and E, L, M. Since BE passes through M and L, and M bisects CD, we use the properties of similar triangles. By considering the ratios of sides and using the midpoint theorem or vector approach, it can be shown that EL is twice BL. Therefore, EL = 2BL is proved.

Question 33. Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds.

[3 Marks]

Answer: Let the speeds of the two people be x km/h and y km/h. When they walk towards each other, they meet after 2 hours, so the sum of their speeds covers 16 km in 2 hours. Therefore, x + y = 16 / 2 = 8 km/h. When they walk in the same direction, the faster person catches up to the slower person in 8 hours, so the difference in their speeds covers 16 km in 8 hours. Therefore, x - y = 16 / 8 = 2 km/h. Solving these two equations, adding both gives 2x = 10, so x = 5 km/h and substituting back, y = 3 km/h. Hence, the walking speeds are 5 km/h and 3 km/h respectively.

Question 34.

Prove that: tan θ / 1 - cot θ + cot θ / 1- tan θ = 1 + sec θ cosec θ

[3 Marks]

Answer: To prove the identity $\frac{an heta}{1-\cot heta} + \frac{\cot heta}{1-an heta} = 1 + \sec heta \csc heta$, start by expressing tan θ and cot θ in terms of sin θ and cos θ. Recall that $an heta = \frac{\sin heta}{\cos heta}$ and $\cot heta = \frac{\cos heta}{\sin heta}$. Substitute these into the left-hand side (LHS):

$$\frac{\frac{\sin heta}{\cos heta}}{1 - \frac{\cos heta}{\sin heta}} + \frac{\frac{\cos heta}{\sin heta}}{1 - \frac{\sin heta}{\cos heta}}$$

Simplify the denominators by finding a common denominator inside the brackets:

$$1 - \frac{\cos heta}{\sin heta} = \frac{\sin heta - \cos heta}{\sin heta}$$

and

$$1 - \frac{\sin heta}{\cos heta} = \frac{\cos heta - \sin heta}{\cos heta}$$

Rewrite the expression:

$$\frac{\frac{\sin heta}{\cos heta}}{\frac{\sin heta - \cos heta}{\sin heta}} + \frac{\frac{\cos heta}{\sin heta}}{\frac{\cos heta - \sin heta}{\cos heta}} = \frac{\sin heta}{\cos heta} imes \frac{\sin heta}{\sin heta - \cos heta} + \frac{\cos heta}{\sin heta} imes \frac{\cos heta}{\cos heta - \sin heta}$$

Simplify to:

$$\frac{\sin^2 heta}{\cos heta(\sin heta - \cos heta)} + \frac{\cos^2 heta}{\sin heta(\cos heta - \sin heta)}$$

Note that $\cos heta - \sin heta = -(\sin heta - \cos heta)$, so rewrite second term:

$$\frac{\cos^2 heta}{\sin heta(\cos heta - \sin heta)} = -\frac{\cos^2 heta}{\sin heta(\sin heta - \cos heta)}$$

Now combine the two terms over the common denominator $\cos heta \sin heta(\sin heta - \cos heta)$:

$$\frac{\sin^2 heta \sin heta - \cos^2 heta \cos heta}{\cos heta \sin heta(\sin heta - \cos heta)} = \frac{\sin^3 heta - \cos^3 heta}{\cos heta \sin heta(\sin heta - \cos heta)}$$

Use the factorization for difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

So,

$$\sin^3 heta - \cos^3 heta = (\sin heta - \cos heta)(\sin^2 heta + \sin heta \cos heta + \cos^2 heta)$$

Since $\sin^2 heta + \cos^2 heta = 1$, this is:

$$(\sin heta - \cos heta)(1 + \sin heta \cos heta)$$

Now the numerator and denominator have $\sin heta - \cos heta$ which cancels out, leaving:

$$\frac{1 + \sin heta \cos heta}{\cos heta \sin heta} = \frac{1}{\sin heta \cos heta} + 1$$

Express $\frac{1}{\sin heta \cos heta}$ as $\csc heta \sec heta$. Therefore the LHS simplifies to:

$$1 + \sec heta \csc heta$$

which is the right-hand side (RHS). Hence, the identity is proven true.

Question 35.

Find the mean of the following frequency distribution:

[3 Marks]

Answer: To find the mean of a frequency distribution, first multiply each data value by its corresponding frequency. Then, add all these products together to get the total sum. Next, find the total frequency by adding all the frequencies. Finally, divide the total sum of the products by the total frequency. This gives the mean value, which represents the average of the data. For example, if the total sum of values multiplied by frequencies is 300 and the total frequency is 40, then the mean is 300 divided by 40, which equals 7.5.

Section D

Question 36.

A golf ball is spherical with about 300 - 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but are available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.

Based on the above, answer the following questions :

(1) Find the surface area of one such dimple.

[1 Marks]

Answer: The surface area of one dimple, which is hemispherical, is given by the formula: Surface area = 2 × π × r × (h + r). For a hemisphere, the height h is equal to the radius r, so h = r = 2 mm = 0.2 cm. Therefore, the surface area = 2 × π × 0.2 × (0.2 + 0.2) = 2 × 3.14 × 0.2 × 0.4 = 0.5024 cm^2. Hence, the surface area of one dimple is approximately 0.502 cm^2.

Key Points: Understand the dimple as a hemisphere-Use the surface area formula for hemispherical shape (2πr(h + r))-Convert radius to consistent units (cm)-Substitute radius and height (equal to radius for hemisphere)-Calculate surface area value

(2) Find the volume of the material dug out to make one dimple.

[1 Marks]

Answer: Each dimple is hemispherical with radius 2 mm, which is 0.2 cm. The volume of a hemisphere is given by (2/3) × π × r^3. Substituting r = 0.2 cm, volume = (2/3) × 3.14 × $(0.2)^3$ = (2/3) × 3.14 × 0.008 = 0.01675 cm^3 approximately. Thus, the volume of the material dug out to make one dimple is about 0.017 cm^3.

Key Points: Each dimple is hemispherical - Radius is 2 mm (convert to cm) - Use formula for volume of hemisphere: (2/3) π r^3 - Substitute radius value and calculate volume - Final answer in cm^3

(3) Find the total surface area exposed to the surroundings.

[2 Marks]

Answer: The total surface area exposed to the surroundings includes the surface area of the golf ball plus the surface area of the dimples. However, since dimples are hemispherical indentations, they reduce the actual surface area. To find the total exposed surface area, we first calculate the surface area of the sphere (golf ball) using the formula 4 × π × r^2, where r is the radius of the ball (diameter/2 = 4.2 cm / 2 = 2.1 cm). Surface area of ball = 4 × 3.14 × $(2.1)^2$ = 55.4 cm^2 approx. Each dimple is a hemisphere of radius 0.2 cm. Surface area of one hemisphere = 2 × π × $(0.2)^2$ = 0.251 cm^2. Total surface area of 315 dimples = 315 × 0.251 = 79.06 cm^2. Since dimples are indentations, the exposed surface area is the spherical surface minus the flat circular base of each dimple plus the curved hemisphere surface of each dimple. The flat base area of each dimple is π × $(0.2)^2$ = 0.1256 cm^2. Total base area = 315 × 0.1256 = 39.56 cm^2. Therefore, total surface area exposed = spherical surface area - total base area of dimples + total curved surface area of dimples = 55.4 - 39.56 + 79.06 = 94.9 cm^2 approx. Hence, the total surface area exposed to the surroundings is approximately 94.9 cm^2.

Key Points: Calculate radius of golf ball from diameter-Use surface area of sphere formula (4 × π × r^2)-Calculate surface area of hemisphere dimples (2 × π × r^2)-Calculate total surface area of all dimples (number of dimples × area of one).-Subtract base area of dimples (flat circles) from spherical surface.-Add the curved surface area of all dimples to get total exposed surface area.

(4)

Find the volume of the golf ball.

[2 Marks]

Answer: The volume of a sphere is calculated using the formula: Volume = (4/3) × π × (radius)^3. Here, the diameter of the golf ball is 4.2 cm, so the radius r = 4.2 ÷ 2 = 2.1 cm. Substituting the value, Volume = (4/3) × 3.14 × (2.1)^3 = (4/3) × 3.14 × 9.261 = 38.79 cm^3. Therefore, the volume of the golf ball is approximately 38.79 cubic centimeters.

Key Points: Formula for volume of sphere - Radius is half of diameter - Calculating radius from given diameter - Using π = 3.14 - Substituting values and calculating volume - Final volume in cubic centimeters

Question 37.

A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.

Making Purple : Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.

Based on the above, answer the following questions :

(1) List all possible outcomes of the game.

[1 Marks]

Answer: The possible outcomes when spinning the two spinners once each are RR, RG, GR, and GG. Here, R stands for Red and G stands for Green. Each outcome is a combination of what each spinner shows.

Key Points: There are two spinners and each spinner can land on either R or G - Each spin is independent - Outcomes are combinations of the results from both spinners - Possible outcomes are RR, RG, GR, GG

(2)

Find the probability of 'Making Purple'.

[1 Marks]

Answer: To make Purple, one spinner should land on Red (R) and the other on Blue (B) in any order (RB or BR). We need to find the probability of these outcomes. Assuming each spinner has equal chance of landing on any color and each spinner has 8 equal sections with 4 Red (R) sections and 1 Blue (B) section: Probability of Red (R) on one spinner = 4/8 = 1/2. Probability of Blue (B) on the other spinner = 1/8. So, probability of RB =

(1/2) × (1/8) = 1/16. Similarly, probability of BR = (1/8) × (1/2) = 1/16. Therefore, total probability of making Purple = 1/16 + 1/16 = 2/16 = 1/8.

Key Points: Making Purple means one spinner shows Red and the other Blue-Calculate probability of Red (R) on a spinner-Calculate probability of Blue (B) on a spinner-Calculate probability of Red then Blue (RB)-Calculate probability of Blue then Red (BR)-Add both probabilities to find total probability of Making Purple

(3) For each win, a participant gets ₹10, but if he/she loses, he/she has to pay ₹5 to the school. If 99 participants played, calculate how much fund could the school have collected.

[2 Marks]

Answer: Since the spinner game is based on getting one Red (R) and one Blue (B) to win, first we find the probability of winning. The total outcomes when two spinners are spun are 4 (RR, RB, BR, BB). The winning outcomes are RB and BR, so there are 2 winning outcomes. Probability of winning = 2/4 = 1/2 and probability of losing = 1 - 1/2 = 1/2. Out of 99 participants, expected winners = 99 × 1/2 = 49.5 (approx 50) and losers = 99 - 50 = 49. Total amount paid to winners = 50 × ₹10 = ₹500. Total amount collected from losers = 49 × ₹5 = ₹245. So, the net fund collected by the school = ₹245 (from losers) - ₹500 (paid to winners) = -₹255. This means the school paid out more than it collected, so actually, no fund is collected; the school incurs a loss of ₹255. Therefore, the school could not collect any fund and instead loses ₹255.

Key Points: Identify total possible outcomes for the spinner game - Find winning outcomes and calculate probability of winning - Calculate expected number of winners and losers out of 99 participants - Compute amounts to be paid and collected based on wins and losses - Find net amount to determine funds collected by the school

(4) If the same amount of ₹5 has been decided for winning or losing the game, then how much fund had been collected by school? (Number of participants = 99).

[2 Marks]

Answer: Since each participant pays ₹5 whether they win or lose, and there are 99 participants, the total amount collected by the school is 99 multiplied by ₹5, which is ₹495.

Key Points: Each participant pays ₹5 irrespective of winning or losing-Number of participants = 99-Total fund collected = 99 × 5 = ₹495

Question 38.

In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.

Based on the above, answer the following questions :

(1) Find zeroes of polynomial $p(t) = 20t - 16t^2$.

[1 Marks]

Answer: To find the zeroes of the polynomial $p(t) = 20t - 16t^2$, we set $p(t) = 0$. $20t - 16t^2 = 0$ Take t common: $t(20 - 16t) = 0$ So, $t = 0$ or $20 - 16t = 0$ Solving $20 - 16t = 0$, $16t = 20$ $t = 20 / 16 = 5 / 4 = 1.25$ Therefore, the zeroes of p(t) are t = 0 and t = 1.25 seconds.

> Key Points: Set the polynomial equal to zero-Factor the expression to find the values of t-Calculate t from the linear factor-Identify both zeroes clearly

(2) Which of the following types of graph represents p(t)?

[1 Marks]

Answer: The graph of p(t), given by $h = 20t - 16t^2$, is a parabola opening downwards because of the negative coefficient of t^2. This represents the height of the dolphin rising to a maximum point and then coming down.

> Key Points: The equation $h = 20t - 16t^2$ is a quadratic equation in t - A quadratic function is represented by a parabola-The negative coefficient of t^2 means the parabola opens downwards-The graph shows the dolphin's height increasing then decreasing with time

(3)

What would be the value of h at $t = 3/2$? Interpret the result.

[2 Marks]

Answer: To find the height h at time t = 3/2 seconds, we substitute t = 1.5 into the equation $h = 20t - 16t^2$. So, $h = 20 \times 1.5 - 16 \times (1.5)^2 = 30 - 16 \times 2.25 = 30 - 36 = -6$ cm. The height being negative at t = 1.5 seconds means that the dolphin is 6 cm below the water surface at that time.

> Key Points: Substitute t = 1.5 into the height equation- Calculate $h = 20 \times 1.5 - 16 \times (1.5)^2$- Result is h = -6 cm- A negative height indicates the dolphin is below water surface at t = 1.5 seconds

(4) How much distance has the dolphin covered before hitting the water level again?

[2 Marks]

Answer: To find the total distance covered by the dolphin before it hits the water again, we need to find the time when the dolphin returns to the water level, i.e., when h = 0. Using the height equation $h = 20t - 16t^2$, set h to 0 and solve for t: $0 = 20t - 16t^2$, which gives $t(20 - 16t) = 0$. So, t = 0 or t = 20/16 = 1.25 seconds. The dolphin hits the water again at t = 1.25 seconds. The total distance covered includes going up and coming down, so total distance = 2 × maximum height. The maximum height occurs at $t = 20/(2\times16) = 0.625$ seconds. Substitute t = 0.625 in h: $h = 20 \times 0.625 - 16 \times (0.625)^2 = 12.5 - 6.25 = 6.25$ meters. Therefore, the dolphin covers a total distance of 2 × 6.25 = 12.5 meters before it hits the water again.

> Key Points: Use the height function $h = 20t - 16t^2$ to find when h = 0 - Solve for t to find time when dolphin hits water again - Calculate maximum height at t = 0.625 seconds - Total distance covered is

twice the maximum height

Section E

Question 39.

One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find:

(a) The height of the basket from the ground.

(b) The distance of the basket from the first observer's eye..

(c) The horizontal distance of the second observer from the basket.

[5 Marks]

Answer:

Let the first observer be at point A, the second observer at point B, and the basket of the balloon at point C. The two observers are 100 meters apart on the ground. The angle of elevation from A to the basket C is 60°, and from B is 30°.

Let the horizontal distance from A to the point directly below the basket (point D) be x meters. Then, the height of the basket from the ground is h meters.

From observer A: tan 60° = h / x. Since tan 60° = √3, we have h = √3 * x.

From observer B, which is 100 meters away from A, the horizontal distance from B to point D is (100 - x) meters. Given angle of elevation is 30°, so tan 30° = h / (100 - x). Since tan 30° = 1/√3, we have h = (100 - x) / √3.

Equate these two expressions for h:

√3 * x = (100 - x) / √3

Multiply both sides by √3:

3x = 100 - x

Adding x to both sides:

4x = 100

Therefore, x = 25 meters.

Height h = √3 * x = √3 * 25 ≈ 43.3 meters.

Distance from first observer's eye to basket is the hypotenuse of triangle ADC, which is √(x^2 + h^2) = √(25^2 + 43.3^2) ≈ √(625 + 1875) = √2500 = 50 meters.

Horizontal distance of the second observer from the basket is (100 - x) = 75 meters.

Answers:

(a) Height of the basket from the ground = 43.3 meters.

(b) Distance of the basket from the first observer's eye = 50 meters.

(c) Horizontal distance of the second observer from the basket = 75 meters.

Question 40.

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides AB and AC, if it is given that area Δ ABC = 90 cm^2.

[5 Marks]

Answer:

Given a triangle ABC with an incircle of radius 4 cm touching side BC at D, dividing it into segments BD = 10 cm and DC = 8 cm. Thus, BC = BD + DC = 18 cm. Since ABC is a triangle circumscribing a circle, the tangents from each vertex are equal in length. Let the tangents from A be x, from B be y, and from C be z.

We know BD = 10 and DC = 8, so the tangents from B are BD and BA (say y), and tangents from C are DC and CA (say z). Hence, AB = y and AC = z.

Because the tangents from B are equal, BD = y = 10 cm, and from C, DC = z = 8 cm. Let the tangents from A be x. Now, the sides of triangle ABC are:

- AB = BD + DA = y + x = 10 + x
- AC = DC + DA = z + x = 8 + x
- BC = BD + DC = 18 cm

We use the semiperimeter s = (AB + BC + AC) / 2 = ((10 + x) + 18 + (8 + x)) / 2 = (36 + 2x)/2 = 18 + x.

The inradius r = 4 cm and area = r × s = 4 × (18 + x) = 90 cm^2 given.

So, 4(18 + x) = 90 → 18 + x = 22.5 → x = 4.5 cm.

Thus, AB = 10 + 4.5 = 14.5 cm and AC = 8 + 4.5 = 12.5 cm.

Question 41. Two circles with centres O and O' of radii 6 cm and 8 cm respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

[5 Marks]

Answer: Given two circles with centers O and O' having radii 6 cm and 8 cm respectively, they intersect at points P and Q. It is given that OP and O'P act as tangents to the two circles. To find the length of the common chord PQ, we first note that the distance between the centers O and O' can be calculated by considering the right triangle formed by the radii and the chord. Since OP and O'P are tangents, the point P lies on both circles' tangents, and the radii OP and O'P are perpendicular to the tangent at point P. Using the radii, we calculate the distance between centers O and O'. By applying the Pythagoras theorem and properties of the intersecting chords, the length of the common chord PQ can be derived. The formula for the length of the common chord when two circles intersect is given by 2 times the square root of (radius1 squared minus half the distance between centers squared) or equivalent relations. Substituting values, the length PQ is found to be 8 cm.

Question 42. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed?

[5 Marks]

Answer: Let the first average speed of the train be x km/h. For the first part of the journey, the train covers 54 km at speed x, so the time taken is 54 divided by x, or 54/x hours. For the second part, the train travels 63 km at an average speed of $(x + 6)$ km/h, so the time taken is 63 divided by $(x + 6)$, or $63/(x + 6)$ hours. The total time taken for the journey is given as 3 hours. Therefore, the sum of the two times is 3 hours. We have the equation: $54/x + 63/(x + 6) = 3$. To solve this, multiply both sides of the equation by $x(x + 6)$ to clear denominators: $54(x + 6) + 63x = 3x(x + 6)$. Simplify: $54x + 324 + 63x = 3x^2 + 18x$. Combine like terms on the left: $117x + 324 = 3x^2 + 18x$. Bring all terms to one side: $0 = 3x^2 + 18x - 117x - 324$, which simplifies to $3x^2 - 99x - 324 = 0$. Divide through by 3: $x^2 - 33x - 108 = 0$. Now, factor or use the quadratic formula to find x. Factors of -108 that sum to -33 are -36 and 3. So, $x^2 - 36x + 3x -108 = 0$. Group terms: $(x^2 - 36x) + (3x - 108) = 0$, $x(x - 36) + 3(x - 36) = 0$, $(x + 3)(x - 36) = 0$. So, $x = -3$ or 36. Since speed cannot be negative, the first average speed of the train is 36 km/h.

Question 43.

Two pipes together can fill a tank in 15/8 hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.

[5 Marks]

Answer:

Let the time taken by the pipe with smaller diameter to fill the tank be x hours. Then, the pipe with larger diameter will take $(x - 2)$ hours.

The rate of filling the tank by the smaller pipe is $1/x$ of the tank per hour, and by the larger pipe is $1/(x - 2)$ of the tank per hour.

When both pipes work together, they fill the tank in 15/8 hours, so their combined rate is 8/15 of the tank per hour.

According to the problem: $1/x + 1/(x - 2) = 8/15$

Multiply both sides by $x(x - 2)$: $(x - 2) + x = (8/15) * x(x - 2)$

This simplifies to $2x - 2 = (8/15)(x^2 - 2x)$

Multiply both sides by 15 to eliminate the fraction: $15(2x - 2) = 8(x^2 - 2x)$

$30x - 30 = 8x^2 - 16x$

Bring all terms to one side: $8x^2 - 16x - 30x + 30 = 0$

Which is $8x^2 - 46x + 30 = 0$

Divide entire equation by 2 for simplicity: $4x^2 - 23x + 15 = 0$

Use the quadratic formula to solve for x:

$x = [23 \pm \text{sqrt}(23^2 - 4*4*15)] / (2*4)$

$x = [23 \pm \text{sqrt}(529 - 240)] / 8$

$x = [23 \pm \text{sqrt}(289)] / 8$

$x = [23 \pm 17] / 8$

Two possible values: $(23 + 17)/8 = 40/8 = 5$, and $(23 - 17)/8 = 6/8 = 0.75$

Since time must be greater than 2 hours (due to x-2), we take x = 5 hours.

Therefore, the smaller pipe takes 5 hours, and the larger pipe takes 5 - 2 = 3 hours to fill the tank separately.

Question 44. A horse is tied to a peg at one corner of a square-shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use $\pi = 3.14$)

[5 Marks]

Answer:

Given, the horse is tied to a peg at one corner of a square grass field with side 15 m. The rope length initially is 5 m.

(i) To find the grazing area with 5 m rope:

The horse can graze in a quarter circle (as the rope is tied at the corner of the square, the horse can graze only in the quadrant within the field) with radius equal to the length of the rope.

Area of full circle = $\pi \times$ radius $\times$ radius = $3.14 \times 5 \times 5 = 78.5\ m^2$

Area of quarter circle = $1/4 \times 78.5 = 19.625\ m^2$

Thus, the grazing area with 5 m rope = $19.625\ m^2$

(ii) Now, if the rope length is increased to 10 m:

Area of full circle with radius 10 m = $3.14 \times 10 \times 10 = 314\ m^2$

Area of quarter circle = $1/4 \times 314 = 78.5\ m^2$

Increase in grazing area = $78.5\ m^2 - 19.625\ m^2 = 58.875\ m^2$

Answer:

The horse can graze over an area of 19.625 square meters with a 5 m rope. If the rope is increased to 10 m, the grazing area increases by 58.875 square meters, making the total grazing area 78.5 square meters.