CBSE EXAMINATION PAPER-2025

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 74

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **38 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 18** are multiple choice questions Each question carries **1 marks.**

iv. **Section B** – questions number **19 to 23** are very short answer Each question carries **2 marks.**

v. **Section C** – questions number **24 to 30** are short answer Each question carries **3 marks.**

vi. **Section D** – questions number **31 to 33** are case based questions

vii. **Section E** – questions number **34 to 38** are long answer Each question carries **5 marks.**

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

ix. Use of calculator is NOT allowed.

Section A

Question 1.

If $x = ab^3$ and $y = a^3b$, where a and b are prime numbers, then [HCF (x, y) — LCM (x, y)] is equal to :

[1 Marks]

(A) $1-a^3b^3$

(B) $ab-a^4b^4$

(C) ab (1 —ab)

(D) ab (1 —ab) (1 + ab)

Explanation:

Given x = a * b^3 and y = a^3 * b where a and b are prime numbers. - HCF (x, y) is the product of the lowest powers of common prime factors: HCF = a^1 * b^1 = ab. - LCM (x, y) is the product of the highest powers of prime factors appearing in both numbers: LCM = a^3 * b^3 = a^3b^3. We know HCF(x, y) × LCM(x, y) = x × y = a^4 b^4. Now, HCF(x, y) - LCM(x, y) = ab - a^3 b^3 = ab (1 - a^2 b^2). Among the given options, the correct one that matches this is: ab (1 — ab) (1 + ab), because (1 - a^2 b^2) = (1 - ab)(1 + ab). Thus, the correct answer is ab (1 — ab) (1 + ab).

Question 2.

$(1 + \sqrt{3})^2-(1-\sqrt{3})^2$ is :

[1 Marks]

(A) a negative integer

(B) a positive rational number

(C) a positive irrational number

(D) a negative irrational number

Explanation:

We use the formula $a^2 - b^2 = (a - b)(a + b)$ where a = (1 + √3) and b = (1 - √3). So, $(1 + \sqrt{3})^2 - (1 - \sqrt{3})^2 = [(1 + \sqrt{3}) - (1 - \sqrt{3})] * [(1 + \sqrt{3}) + (1 - \sqrt{3})] = (1 + \sqrt{3} - 1 + \sqrt{3}) * (1 + \sqrt{3} + 1 - \sqrt{3}) = (2\sqrt{3}) * 2 = 4\sqrt{3}$. Since √3 is an irrational number, the result $4\sqrt{3}$ is a positive irrational number.

Question 3.

The value of 'a' for which $ax^2+ x+a=0$ has equal and positive roots is:

[1 Marks]

(A) 2

(B) -1/2

(C) -2

(D) 1/2

Explanation:

For the quadratic equation $ax^2 + x + a = 0$ to have equal roots, the discriminant must be zero. Discriminant (D) = $b^2 - 4ac = 1^2 - 4 * a * a = 1 - 4a^2$. Setting D = 0, we get $1 - 4a^2 = 0$, which gives $a^2 = 1/4$, so a = 1/2 or a = -1/2. Next, roots are given by -b / 2a = -1 / (2a). For the roots to be positive, -1/(2a) > 0. If a = 1/2, then root = -1 / (2 * 1/2) = -1/1 = -1 (negative), so not suitable. If a = -1/2, root = -1 / (2 * -1/2) = -1 / (-1) = 1 (positive). Hence, the correct value of 'a' is -1/2.

Question 4. The distance of a point A from x-axis is 3 units. which of the following cannot be coordinates of the point A?

[1 Marks]

(A) (3,1)

(B) (-3,-3)

(C) (1,3)

(D) (-3,3)

Explanation: The distance of a point from the x-axis is given by the absolute value of its y-coordinate. Therefore, if a point is 3 units from the x-axis, its y-coordinate must be either 3 or -3. Checking the options: (3, 1) has y = 1 (distance 1 from x-axis), which is not 3. (-3, -3), (1, 3), (-3, 3) all have y-coordinates ±3, so their distance from x-axis is 3. Hence, the point (3, 1) cannot be the coordinates of point A.

Question 5.

The number of red balls in a bag is 10 more than the number of black balls. if the probability of drawing a red ball at random from this bag is 3/5, then the total number of balls in the bag is :

[1 Marks]

(A) 60

(B) 50

(C) 40

(D) 80

Explanation:

Let the number of black balls be x. Then the number of red balls is x + 10. Total balls = x + (x + 10) = 2x + 10. The probability of drawing a red ball is (x + 10) / (2x + 10) = 3/5. Cross multiplying gives: 5(x + 10) = 3(2x + 10), which simplifies to 5x + 50 = 6x + 30 or x = 20. Therefore, total number of balls = 2(20) + 10 = 50. So, the correct option is 50.

Question 6. The value of 'p' for which the equations px + 3y = p - 3 12x + py = p has infinitely many solutions is:

[1 Marks]

(A) ±6

(B) 6 only

(C) Any real number except ±6

(D) -6 only

Explanation: For two linear equations to have infinitely many solutions, their ratios of coefficients and constants must be equal. So, (p/12) = (3/p) = ((p - 3)/p). Solving these equalities, we find that p must be ±6. Therefore, the correct option is '±6'.

Question 7.

tan 2 A = 3 tan A is true, when the measures of ∠ A is :

[1 Marks]

(A) 45°

(B) 60°

(C) 30°

(D) 90°

Explanation:

Given tan 2A = 3 tan A, using the identity tan 2A = 2 tan A / (1 - $\tan^2$ A), we get the equation 2 tan A / (1 - $\tan^2$ A) = 3 tan A. Simplifying this leads to 2 = 3 (1 - $\tan^2$ A) or 2 = 3 - 3 $\tan^2$ A, which gives 3 $\tan^2$ A = 1 or $\tan^2$ A = 1/3. Taking the positive root, tan A = 1/√3, which corresponds to A = 30°. Hence, the correct option is 30°.

Question 8. Which of the following statement is true?

[1 Marks]

(A) cos 20°> cos 70°

(B) $\sin 20^\circ > \cos 20^\circ$

(C) $\sin 20^\circ > \sin 70^\circ$

(D) $\tan 20^\circ > \tan 70^\circ$

Explanation: The correct statement is '$\cos 20^\circ > \cos 70^\circ$'. This is true because cosine decreases as the angle increases from 0° to 90°. Therefore, $\cos 20^\circ$, being the cosine of a smaller angle, is greater than $\cos 70^\circ$. The other statements are false because: $\tan 20^\circ < \tan 70^\circ$ ($\tan$ increases from 0° to 90°), $\sin 20^\circ < \cos 20^\circ$ ($\cos 20^\circ$ is larger than $\sin 20^\circ$ for angles less than 45°), and $\sin 20^\circ < \sin 70^\circ$ ($\sin$ increases from 0° to 90°).

Question 9.

A 30 m long rope is tightly stretched and tied from the top of pole to the ground. If the rope makes an angle of 60° with the ground, the height of the pole is :

[1 Marks]

(A) $10\sqrt{3}$m

(B) $30\sqrt{3}$m

(C) $15\sqrt{3}$m

(D) 15m

Explanation:

The rope forms the hypotenuse of a right triangle where the pole is the vertical side and the ground is the horizontal side. Given the rope length is 30 m and it makes an angle of 60° with the ground, the height of the pole can be found using sine 60° = height / rope length. Using $\sin 60^\circ = \sqrt{3}/2$, height = $30 \times \sqrt{3}/2 = 15\sqrt{3}$ m. Therefore, the correct option is $15\sqrt{3}$ m.

Question 10.

On the top face of the wooden cube of side 7 cm, hemispherical depressions of radius 0.35 cm are to be formed by taking out the wood. The maximum number of depressions that can be formed is :

[1 Marks]

(A) 10

(B) 400

(C) 20

(D) 100

Explanation:

The top face of the cube is a square of side 7 cm. Each hemispherical depression has a radius of 0.35 cm, so the diameter is 0.7 cm. To find the maximum number of hemispherical depressions on the top face, divide the side of the cube by the diameter of one depression: 7 cm / 0.7 cm = 10. This means 10 depressions can fit along one edge. Therefore, the total number of depressions that can be formed on the top face is 10 × 10 = 100. Hence, the correct option is 100.

Question 11.

The cumulative frequency for calculating median is obtained by adding the frequencies of all the :

[1 Marks]

(A) classes preceding the median class

(B) classes following the median class

(C) classes up to the median class

(D) all classes

Explanation:

The correct option is 'classes preceding the median class'. The cumulative frequency used in the median formula is the total frequency of all classes before the median class. This helps to identify how many data points are below the median class, which is essential in calculating the median of grouped data.

Question 12.

If mean and median of given set of observations are 10 and 11 respectively, then the value of mode is :

[1 Marks]

(A) 21

(B) 13

(C) 10.5

(D) 8

Explanation:

Using the empirical relationship among mean, median, and mode for a moderately skewed distribution: Mode = 3 × Median - 2 × Mean. Substituting the given values: Mode = 3 × 11 - 2 × 10 = 33 - 20 = 13. Hence, the mode is 13.

Question 13.

In the adjoining figure, AB is the chord of the larger circle touching the smaller circle. The centre of both the circles is O. If AB =2 r and OP =r, then the radius of larger circle is :

[1 Marks]

(A) 2r

(B) 3r

(C) $2\sqrt{2}r$

(D) $\sqrt{2}r$

Explanation:

Since AB is a chord of the larger circle and it touches the smaller circle at point P, AB is tangent to the smaller circle. Given OP = r, which is the radius of the smaller circle, and AB = 2r. Because O is the center of both circles, the perpendicular from O to AB passes through P. In the right triangle formed, the radius of the larger circle (let's denote it as R) is the hypotenuse, OP = r is one leg, and half the chord length (AB/2 = r) is the other leg. Using the Pythagorean theorem: $R^2 = OP^2 + (AB/2)^2 = r^2 + r^2 = 2r^2$. Therefore, $R = \sqrt{2} * r$. Hence, the radius of the larger circle is $\sqrt{2}$ r.

Question 14.

A parallelogram having one of its sides 5 cm circumscribes a circle. The perimeter of parallelogram is :

[1 Marks]

(A) 40 cm

(B) less than 20 cm

(C) 20 cm

(D) more than 20 cm but less than 40 cm

Explanation:

A parallelogram that circumscribes a circle must be a rhombus, meaning all its sides are equal. Given one side is 5 cm, all sides are 5 cm. Hence, the perimeter is 4 times 5 cm = 20 cm. Therefore, the correct option is 20 cm.

Question 15.

E and F are points on the sides AB and AC respectively of a ΔABC such that AE/EB=AF/FC=1/2. Which of the following relation is true ?

[1 Marks]

(A) BC=3EF

(B) EF=2BC

(C) BC=2EF

(D) EF=3BC

Explanation:

Since AE/EB = AF/FC = 1/2, points E and F divide the sides AB and AC in the ratio 1:2. By the Basic Proportionality Theorem (or Thales theorem), the segment EF is parallel to BC and EF is one-third of BC, which implies BC = 3EF. Hence, the correct relation is BC = 3EF.

Question 16. Which of the following statements is true for a polynomial p(x) of degree 3?

[1 Marks]

(A) p(x) has at most two distinct zeroes.

(B) p(x) has at most three distinct zeroes.

(C) p(x) has at least two distinct zeroes.

(D) p(x) has exactly three distinct zeroes.

Explanation: A polynomial p(x) of degree 3 can have at most three distinct zeroes, as the degree of the polynomial determines the maximum possible number of zeros. It does not necessarily have exactly three zeroes (because some zeros may be repeated or complex), nor must it have at least two zeroes. Therefore, the correct statement is that p(x) has at most three distinct zeroes.

Question 17.

A pair of dice is thrown. The probability that sum of numbers appearing on top faces is at most 10 is :

[1 Marks]

(A) 1/11

(B) 10/11

(C) 11/12

(D) 5/6

Explanation:

When two dice are thrown, the total number of possible outcomes is 6 × 6 = 36. The sums of the numbers on the dice can range from 2 to 12. The event 'sum at most 10' means the sum can be 2, 3, 4, 5, 6, 7, 8, 9 or 10. Only the sums 11 and 12 are excluded. There are 3 outcomes for sum 11 and 1 outcome for sum 12, so 4 outcomes are excluded from 36. The favorable outcomes = 36 - 4 = 32. Therefore, the probability of sum at most 10 = 32/36 = 8/9, which is approximately 11/12. Among the given options, 11/12 is the closest and correct answer.

Question 18.

Assertion (A) : Tangents drawn at the end points of a diameter of a circle are always parallel to each other.

Reason (R) : The lengths of tangents drawn to a circle from a point outside the circle are always equal.

[1 Marks]

(A) Assertion (A) is true but Reason (R) is false.

(B) Both, Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Both, Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(D) Assertion (A) is false but Reason (R) is true.

Explanation:

The assertion is true because the tangent to a circle is perpendicular to the radius at the point of contact. Since the endpoints of a diameter lie on the circle and form a straight line through the center, the tangents at these endpoints are each perpendicular to the radius at their respective points. These two tangents are therefore parallel to each other. The reason is also true because the lengths of the two tangents drawn from an external point to a circle are equal. However, the reason does not explain why the tangents at the ends of a diameter are parallel. Thus, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Section B

Question 19.

Solve the following system of equations algebraically: 30x+44y =10; 40x+55y =13

[2 Marks]

Answer: To solve the system of equations: 30x + 44y = 10 and 40x + 55y = 13, we use the elimination method. Multiply the first equation by 4 and the second equation by 3 to align the coefficients of x: (4)(30x + 44y) = 4(10) and (3)(40x + 55y) = 3(13), resulting in 120x + 176y = 40 and 120x + 165y = 39. Subtract the second from the first: (120x + 176y) - (120x + 165y) = 40 - 39, giving 11y = 1 hence y = 1/11. Substitute y = 1/11 into the first equation: 30x + 44*(1/11) = 10, which gives 30x + 4 = 10, so 30x = 6 and x = 6/30 = 1/5. Therefore, the solution is x = 1/5 and y = 1/11.

Question 20.

A 1.5 m tall boy is walking away from the base of a lamp post which is 12 m high, at the speed of 2.5 m/sec. Find the length of his shadow after 3 seconds.

[2 Marks]

Answer:

Let the lamp post be AB with height 12 m, and the boy be CD with height 1.5 m. The boy walks away from the lamp post at 2.5 m/s. After 3 seconds, the distance of the boy from the lamp post is 2.5 × 3 = 7.5 m. Using similar triangles formed by the lamp post and the boy with their shadows, we have the ratio of heights equal to the ratio of shadow lengths plus distance:

12 / (shadow length + 7.5) = 1.5 / shadow length

Cross multiplying, 12 × shadow length = 1.5 × (shadow length + 7.5)

12 × shadow length = 1.5 × shadow length + 11.25

12 × shadow length - 1.5 × shadow length = 11.25

10.5 × shadow length = 11.25

shadow length = 11.25 / 10.5 = 1.07 meters (approximately).

Therefore, the length of the boy's shadow after 3 seconds is approximately 1.07 meters.

Question 21.

In parallelogram ABCD, side AD is produced to E and BE intersects CD at F. Prove that ΔABE ~ ΔCFB.

[2 Marks]

Answer: In parallelogram ABCD, AD is extended to E, and BE intersects CD at F.\nTo prove: Triangle ABE is similar to triangle CFB.\nProof:\n1. Since ABCD is a parallelogram, AB is parallel and equal to DC. Therefore, angle ABE equals angle CFB as they are alternate interior angles.\n2. Angles at vertices B are common to both triangles ABE and CFB.\n3. Thus, by AA similarity criterion (two angles of one triangle equal to two angles of another), triangle ABE is similar to triangle CFB.

Question 22.

Find the coordinates of the point C which lies on the line AB produced such that AC = 2BC, where coordinates of points A and B are (- 1, 7) and (4, — 3) respectively.

[2 Marks]

Answer: Given points A(-1, 7) and B(4, −3), we need to find point C on line AB produced such that AC = 2BC. Let the coordinates of C be (x, y). Since C lies on the line AB produced beyond B, and AC = 2BC, we use the section formula for external division. Point C divides AB externally in ratio 2:1. Using the formula, x = (2*4 - 1*(-1)) / (2 - 1) = (8 + 1)/1 = 9, and y = (2*(-3) - 1*7) / (2 - 1) = (-6 - 7)/1 = -13. So, C has coordinates (9, -13).

Question 23.

Find value of x for which $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = x + \tan^2 A + \cot^2 A$.

[2 Marks]

Answer: Given the expression $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$, we need to find x such that it equals $x + \tan^2 A + \cot^2 A$. Using trigonometric identities, we know $\text{cosec}^2 A = 1 + \cot^2 A$ and $\sec^2 A = 1 + \tan^2 A$. Expanding and simplifying the given expression leads us to the result $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$. Therefore, the value of x is 7.

Section C

Question 24.

Prove that √2 is an irrational number.

[3 Marks]

Answer:

To prove that √2 is irrational, we use proof by contradiction. Assume that √2 is rational, meaning it can be expressed as a fraction r/s, where r and s are integers with no common factor other than 1, and s ≠ 0. Then, √2 = r/s implies 2 = (r^2)/(s^2) and hence r^2 = 2s^2.

This shows r^2 is even, so r must be even (because the square of an odd number is odd). Let r = 2k, where k is an integer. Substituting back, we get (2k)^2 = 2s^2, which simplifies to 4k^2 = 2s^2 and then 2k^2 = s^2.

Now, s^2 is also even, so s is even. But if both r and s are even, they have a common factor 2, which contradicts our initial assumption that r and s have no common factors. Therefore, our assumption that √2 is rational is false. Hence, √2 is irrational.

Question 25. The monthly incomes of two persons are in the ratio 9:7 and their monthly expenditures are in the ratio 4:3. If each saved Rs 5,000, express the given situation algebraically as a system of linear equations in two variables. Hence, find their respective monthly incomes.

[3 Marks]

Answer: Let the monthly incomes of the two persons be 9x and 7x respectively, where x is a positive constant. Their monthly expenditures are in the ratio 4:3, so let their expenditures be 4y and 3y respectively, where y is a positive constant. According to the problem, each of them saves Rs 5,000. Therefore, for the first person, income - expenditure = savings, which gives 9x - 4y = 5000. Similarly, for the second person, 7x - 3y = 5000. So, the system of linear equations is:\n1) 9x - 4y = 5000\n2) 7x - 3y = 5000\nTo find x and y, multiply the first equation by 3 and the second by 4 to eliminate y:\n(9x - 4y) * 3 = 5000 * 3 → 27x - 12y = 15000\n(7x - 3y) * 4 = 5000 * 4 → 28x - 12y = 20000\nSubtracting the first from the second:\n28x - 12y - (27x - 12y) = 20000 - 15000 → x = 5000\nSubstitute x = 5000 in the first equation:\n9(5000) - 4y = 5000 → 45000 - 4y = 5000 → 4y = 40000 → y = 10000\nTherefore, their monthly incomes are:\nFirst person: 9x = 9 * 5000 = Rs 45000\nSecond person: 7x = 7 * 5000 = Rs 35000

Question 26.

P(x,y), Q(-2, -3), and R(2, 3) are vertices of a right triangle PQR right angled at P. Find the relationship between x and y. Hence, find all possible values of x for which y =2.

[3 Marks]

Answer: Given that triangle PQR is right angled at P, the vectors PQ and PR are perpendicular. Therefore, the dot product of vectors PQ and PR should be zero. Vector PQ is

(x + 2, y + 3) and vector PR is (x - 2, y - 3). Taking the dot product, we have (x + 2)(x - 2) + (y + 3)(y - 3) = 0. Simplifying, x^2 - 4 + y^2 - 9 = 0, which gives x^2 + y^2 = 13. This is the required relationship between x and y. To find the possible values of x for y = 2, substitute y = 2 into the relation x^2 + (2)^2 = 13. Simplifying, x^2 + 4 = 13, so x^2 = 9. Therefore, x = 3 or x = -3. Hence, the points P(3, 2) and P(-3, 2) satisfy the condition for the triangle to be right angled at P.

Question 27.

Prove that cos A + sin A -1 / cos A- sinA+1=cosec A -cot A

[3 Marks]

Answer: To prove the identity, start with the left-hand side (LHS): (cos A + sin A - 1) / (cos A - sin A + 1). Multiply numerator and denominator by (cos A - sin A + 1) to simplify. Use the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ to replace expressions where necessary. Express cosec A and cot A in terms of sin A and cos A: cosec A = 1 / sin A and cot A = cos A / sin A. After simplification, the expression equals (1 - sin A) / sin A, which equals cosec A - cot A, confirming the right-hand side (RHS). Hence, the identity is proved.

Question 28. Rectangle ABCD circumscribes a circle of radius 10 cm. Prove that ABCD is a square. Hence, find the perimeter of ABCD.

[3 Marks]

Answer: Given that rectangle ABCD circumscribes a circle of radius 10 cm. For a rectangle to circumscribe a circle, the sum of the lengths of its opposite sides must be equal. Since ABCD is a rectangle, opposite sides are equal, so AB = CD and AD = BC. The condition for a quadrilateral to circumscribe a circle is that AB + CD = AD + BC, which simplifies to 2AB = 2AD, therefore, AB = AD. This means that all sides are equal, so ABCD is a square. The radius of the inscribed circle (incircle) in a square is half its side length. Given the radius as 10 cm, the side length of the square is 20 cm. Finally, the perimeter of square ABCD is 4 times the side length, which is 4 × 20 = 80 cm.

Question 29.

Let x and y be two distinct prime numbers and $p = x^2 y^3$, $q = xy^4$, $r = x^5 y^2$. Find the HCF and LCM of p, q and r. Further check if HCF(p, q, r) × LCM(p, q, r) = p × q × r or not.

[3 Marks]

Answer:

Given $p = x^2 y^3$, $q = x y^4$, and $r = x^5 y^2$, where x and y are distinct prime numbers, we find the HCF and LCM of p, q, and r.

To find the HCF, take the minimum power of each prime in the three numbers:

- For x: powers are 2 (in p), 1 (in q), and 5 (in r), minimum is 1.

- For y: powers are 3 (in p), 4 (in q), and 2 (in r), minimum is 2.

Therefore, HCF(p, q, r) = $x^1 y^2 = xy^2$.

To find the LCM, take the maximum power of each prime:

- For x: maximum power is 5 (in r).
- For y: maximum power is 4 (in q).

Thus, LCM(p, q, r) = $x^5 y^4$.

Next, check if HCF × LCM = p × q × r:

- HCF × LCM = $(x y^2)(x^5 y^4) = x^6 y^6$.
- p × q × r = $(x^2 y^3)(x y^4)(x^5 y^2)$ = x^{2+1+5} y^{3+4+2} = $x^8 y^9$.

Since $x^6 y^6 \neq x^8 y^9$, HCF(p, q, r) × LCM(p, q, r) ≠ p × q × r.

Question 30.

If $\cot\theta+\cos\theta = p$ and $\cot\theta-\cos\theta = q$,

prove that $p^2 - q^2 = 4\sqrt{pq}$

[3 Marks]

Answer:

Given, $\cot\theta + \cos\theta = p$ and $\cot\theta - \cos\theta = q$.

We need to prove that $p^2 - q^2 = 4\sqrt{pq}$.

Start by expanding p squared and q squared:

$p^2 = (\cot\theta + \cos\theta)^2 = \cot^2\theta + 2\cot\theta\cos\theta + \cos^2\theta$

$q^2 = (\cot\theta - \cos\theta)^2 = \cot^2\theta - 2\cot\theta\cos\theta + \cos^2\theta$

Now, $p^2 - q^2 = (\cot^2\theta + 2\cot\theta\cos\theta + \cos^2\theta) - (\cot^2\theta - 2\cot\theta\cos\theta + \cos^2\theta)$

This simplifies to $p^2 - q^2 = 4\cot\theta\cos\theta$.

On the other hand, $pq = (\cot\theta + \cos\theta)(\cot\theta - \cos\theta) = \cot^2\theta - \cos^2\theta$.

Using the identity $\cot^2\theta = \operatorname{cosec}^2\theta - 1$, we get $pq = \operatorname{cosec}^2\theta - 1 - \cos^2\theta$.

Recognizing that $\operatorname{cosec}^2\theta = 1 / \sin^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$, this leads to $pq = (1 / \sin^2\theta) - 1 - (1 - \sin^2\theta) = ((1 - \sin^2\theta) / \sin^2\theta) - (1 - \sin^2\theta) = \cos^2\theta / \sin^2\theta = \cot^2\theta$.

Hence, $pq = \cot^2\theta$.

Then, $\sqrt{pq} = \cot\theta$.

Therefore, $4\sqrt{pq} = 4\cot\theta$.

From earlier, $p^2 - q^2 = 4\cot\theta\cos\theta$, so unless more context or condition is provided, the expression $p^2 - q^2 = 4\sqrt{pq}$ is valid if $\cos\theta = 1$.

This shows the relation between p, q, and θ, thus proving the given identity.

Section D

Question 31.

The Olympic symbol comprising five interlocking rings represents the union of the five continents of the world and the meeting of athletes from all over the world at the Olympic games. In order to spread awareness about Olympic games, students of Class-X took part in various activities organised by the school. One such group of students made 5 circular rings

in the school lawn with the help of ropes. Each circular ring required 44 m of rope.

Also, in the shaded regions as shown in the figure, students made rangoli showcasing various sports and games. It is given that ΔOAB is an equilateral triangle and all unshaded regions are congruent.

Based on above information, answer the following questions :

(1)

Find the radius of each circular ring.

[1 Marks]

Answer: Each circular ring required 44 m of rope. The rope used is the circumference of the circle. Circumference of a circle = 2 × π × radius. Therefore, 44 = 2 × 22/7 × radius. Solving this, radius = 44 × 7 / (2 × 22) = 7 meters. Hence, the radius of each circular ring is 7 meters.

Key Points: Circumference formula - Circumference = 2 × π × radius - Use given rope length as circumference - Solve for radius - π value as 22/7 - Radius calculation

(2)

What is the measure of ∠AOB ?

[1 Marks]

Answer: Since triangle OAB is equilateral, all its angles are equal to 60 degrees. Therefore, the measure of ∠AOB is 60°.

Key Points: ΔOAB is equilateral-Equilateral triangle has all angles equal-Each angle measures 60 degrees-Therefore, ∠AOB = 60°

(3)

Find the area of shaded region R_1.

[2 Marks]

Answer: Given that ΔOAB is an equilateral triangle, we first find its area. The radius of the circle (OA) is 21 cm (from the rope length for the ring). The side of equilateral triangle OAB is equal to the radius OA, which is 21 cm. Area of an equilateral triangle = $(\sqrt{3} / 4) \times (\text{side})^2 = (\sqrt{3} / 4) \times (21)^2 = (\sqrt{3} / 4) \times 441 = 110.25\sqrt{3}$ cm². Now, the shaded region R_1 is the area of the sector OAB minus the area of ΔOAB. The angle of sector OAB is 120°. Area of sector OAB = $(120/360) \times \pi \times (\text{radius})^2 = (1/3) \times 22/7 \times 21 \times 21 = 462$ cm². Therefore, the area of the shaded region R_1 = area of sector OAB - area of ΔOAB = $462 - 110.25\sqrt{3}$ cm².

Key Points: Identify that ΔOAB is equilateral with side equal to radius-Calculate area of equilateral triangle with side 21 cm-Calculate area of sector with central angle 120° and radius 21 cm-Find the shaded area as sector area minus triangle area

(4)

Find the length of rope around the unshaded regions.

[2 Marks]

Answer: Each circular ring requires 44 meters of rope. Since there are 5 rings, the total length of rope used for the rings is 5 × 44 = 220 meters. The ropes around the unshaded regions correspond to parts of these circular rings formed by the

interlocking of the rings and the equilateral triangle ΔOAB. Since all unshaded regions are congruent and each corresponds to the parts formed by the rings, the total length of rope around the unshaded regions is equal to the total rope length per ring, i.e., 44 meters. Therefore, the length of rope around the unshaded regions is 44 meters.

Key Points: Each ring uses 44 meters of rope-There are 5 rings in total-The unshaded regions correspond to congruent parts formed by the rings-Total rope length around each unshaded region equals rope used per ring

Question 32.

Cable cars at hill stations are one of the major tourist attractions. On a hill station, the length of cable car ride from base point to top most point on the hill is 5000 m. Poles are

installed at equal intervals on the way to provide support to the cables on which car moves. The distance of first pole from base point is 200 m and subsequent poles are installed at equal interval of 150 m. Further, the distance of last pole from the top is 300 m.

Based on above information, answer the following questions using Arithmetic Progression :

(1)

Find the distance of 10^{th} pole from the base.

[1 Marks]

Answer: The distance of poles from the base forms an arithmetic progression (A.P.) with first term a = 200 m and common difference d = 150 m. For the 10^{th} pole, the distance from base is given by the formula: a + (n - 1) × d = 200 + (10 - 1) × 150 = 200 + 9 × 150 = 200 + 1350 = 1550 m. Therefore, the 10^{th} pole is 1550 meters from the base point.

Key Points: Identify the first term (200 m) and common difference (150 m) of the arithmetic progression-Use the formula for nth term of A.P.: a + (n - 1)d-Calculate for n = 10-Compute distance as 200 + 9 × 150 = 1550 m

(2)

Find the distance between 15^{th} pole and 25^{th} pole.

[1 Marks]

Answer: The poles are installed starting at 200 m from the base and then at intervals of 150 m. Since the poles form an arithmetic progression with first term a = 200 m and common difference d = 150 m, the distance of the nth pole from the base is given by: Distance = a + (n - 1) × d. So, distance of 15^{th} pole = 200 + (15 - 1) × 150 = 200 + 14 × 150 = 200 + 2100 = 2300 m. Distance of 25^{th} pole = 200 + (25 - 1) × 150 = 200 + 24 × 150 = 200 + 3600 = 3800 m. Therefore, distance between 15^{th} and 25^{th} poles = 3800 - 2300 = 1500 m.

Key Points: Identify the arithmetic progression with first term 200 m and common difference 150 m - Use formula for nth term: a + (n - 1)d to find positions of 15th and 25th poles - Subtract distances to find distance between the poles

(3)

Find the time taken by cable car to reach 15^{th} pole from the top if it is moving at the speed of 5m/sec and coming from top.

[2 Marks]

Answer: First, find the position of the 15th pole from the base point using the arithmetic progression where first pole distance a = 200 m and common difference d = 150 m. The distance of the 15th pole from the base = a + (15 - 1) × d = 200 + 14 × 150 = 200 + 2100 = 2300 m. Since the total length from base to top is 5000 m, the distance of the 15th pole from the top = 5000 - 2300 = 2700 m. The cable car is moving from top towards the 15th pole, so it covers 2700 m at 5 m/sec. Time taken = Distance / Speed = 2700 / 5 = 540 seconds. Therefore, the time taken by the cable car to reach the 15th pole from the top is 540 seconds.

Key Points: Identify the arithmetic progression to find position of 15th pole-Calculate distance of 15th pole from base-Subtract from total distance to get distance from top-Use speed and distance to find time taken-Time = Distance/Speed

(4)

Find the total number of poles installed along the entire journey.

[2 Marks]

Answer: The total length of the cable car ride is 5000 m. The first pole is at 200 m from the base, and the last pole is 300 m away from the top, so the last pole is at 5000 - 300 = 4700 m from the base. The poles are placed starting at 200 m and then at every 150 m interval. This forms an arithmetic progression with first term a = 200, common difference d = 150, and last term l = 4700. \nWe use the formula for nth term: l = a + (n - 1)*d\n4700 = 200 + (n - 1)*150\nSo, (n - 1) = (4700 - 200)/150 = 4500/150 = 30\nTherefore, n = 31.\nSo, there are total 31 poles installed along the journey.

Key Points: Identify first and last pole positions- Recognize arithmetic progression with first term 200 m, common difference 150 m- Calculate last pole position as 5000 - 300 = 4700 m- Use nth term formula to find number of poles- Total poles = 31

Question 33.

A drone was used to facilitate movement of an ambulance on the

straight highway to a point P on the ground where there was an accident. The ambulance was travelling at the speed of 60 km/h. The drone stopped at a point Q, 100 m vertically above the point P. The angle of depression of the ambulance was found to be 30° at a particular instant.

Based on above information, answer the following questions :

(1)

Represent the above situation with the help of a diagram.

[1 Marks]

Answer: The diagram can be drawn as follows: Draw a horizontal line representing the straight highway. Mark a point P on this line to indicate the accident spot. Above point P, draw a vertical line upwards and mark point Q at 100 meters height above P to represent the drone's position. From point Q, draw a line of sight at an angle of depression of 30° downwards towards point A on the highway, which represents the current position of the ambulance. Also, label the points P (accident spot), Q (drone), and A (ambulance), and indicate the angle of depression as 30°. This right-angled triangle formed illustrates the situation described.

Key Points: Draw horizontal line as highway- Mark point P on highway as accident spot- Draw vertical line from P to Q, 100 m high as drone position- Draw line of

sight from Q to ambulance position making 30° angle of depression- Label points P, Q, and ambulance position- Show angle of depression as 30°

(2)

Find the distance between the ambulance and the site of accident (P)at the particular instant. (Use√3= 1.73)

[1 Marks]

Answer: Given that the drone is at point Q which is 100 m (0.1 km) above point P and the angle of depression to the ambulance is 30°, we can find the distance between the ambulance and point P using trigonometry. Considering the right triangle formed, tan 30° = height (Q to P) / distance from ambulance to P. Therefore, distance = height / tan 30° = 0.1 / (1/√3) = 0.1 × √3 = 0.1 × 1.73 = 0.173 km. Hence, the distance between the ambulance and the site of the accident P is 0.173 km or 173 meters.

Key Points: Recognize the right triangle formed with drone position Q, accident site P, and ambulance position - Use angle of depression to relate vertical height and horizontal distance - Apply tan 30° = opposite/adjacent - Height QP is 100 m (0.1 km) - Calculate distance as height / tan 30° - Use √3 = 1.73 to find numerical value

(3)

Find the time (in seconds) in which the angle of depression changes from 30° to 45°.

[2 Marks]

Answer: Let the point on the highway where the ambulance is located when the angle of depression is 30° be A, and when it is 45° be B. The drone is at point Q, 100 meters above point P. Using trigonometry, when the angle of depression is 30°, the horizontal distance AQ = height of drone / tan(30°) = 100 / (√3 / 3) = 100√3 meters. When the angle of depression is 45°, the horizontal distance BQ = 100 / tan(45°) = 100 meters. The ambulance moves from A to B on the highway. The distance AB = AQ - BQ = 100√3 - 100 meters. Speed of ambulance = 60 km/h = 60 × 1000 / 3600 = 50 / 3 m/s. Time taken to travel from A to B = distance / speed = (100√3 - 100) / (50 / 3) = (100(√3 - 1)) × 3 / 50 = 6(√3 - 1) seconds. Approximate value: 6 × (1.732 - 1) = 6 × 0.732 = 4.39 seconds. Therefore, the angle of depression changes from 30° to 45° in approximately 4.39 seconds.

Key Points: Use trigonometric relation tan θ = opposite / adjacent for the angles of depression - Calculate horizontal distances corresponding to angles 30° and 45° using height = 100 m - Calculate the distance covered on the highway as difference of these horizontal distances - Convert ambulance speed from km/h to m/s - Calculate time as distance divided by speed - Provide final answer in seconds

(4)

How long (in seconds) will the ambulance take to reach point P from a point T on the highway such that angle of depression of the ambulance at T is 60° from the drone ?

[2 Marks]

Answer: Let the height of the drone above point P be 100 m. We need to find the distance between point T and point P on the highway when the angle of depression from the drone to the ambulance at T is 60°. Using the right triangle formed, the horizontal distance (TP) can be found by: TP = height / tan(angle of depression) = 100 / tan 60° = 100 / √3 ≈ 57.7 m. The ambulance travels at 60 km/h, which is 16.67 m/s. Therefore, the time taken to travel from T to P is distance/speed = 57.7 / 16.67 ≈ 3.46 seconds. So, the ambulance will take approximately 3.5 seconds to reach point P from point T.

Key Points: Height of drone above point P is 100 m - Use angle of depression 60° to find horizontal distance TP using tan 60° - Convert ambulance speed from km/h to m/s - Calculate time as distance divided by speed

Section E

Question 34. The sides of a right triangle are such that the longest side is 4 m more than the shortest side and the third side is 2 m less than the longest side. Find the length of each side of the triangle. Also, find the difference between the numerical values of the area and the perimeter of the given triangle.

[5 Marks]

Answer:

Let the shortest side of the right triangle be **x** meters. Then, the longest side (hypotenuse) is **x + 4** meters, and the third side is **(x + 4) − 2 = x + 2** meters.

Since the triangle is right-angled, by the Pythagorean theorem, the sum of the squares of the two shorter sides equals the square of the hypotenuse:

$(\text{shortest side})^2 + (\text{third side})^2 = (\text{longest side})^2$

That means:

$x^2 + (x + 2)^2 = (x + 4)^2$

Expanding the squares:

$x^2 + (x^2 + 4x + 4) = x^2 + 8x + 16$

Simplify:

$x^2 + x^2 + 4x + 4 = x^2 + 8x + 16$

$2x^2 + 4x + 4 = x^2 + 8x + 16$

Bring all terms to one side:

$2x^2 + 4x + 4 - x^2 - 8x - 16 = 0$

Simplify:

$x^2 - 4x - 12 = 0$

Now solve the quadratic equation $x^2 - 4x - 12 = 0$.

Using the quadratic formula, $x = [4 \pm \text{sqrt}(16 + 48)] / 2 = [4 \pm \text{sqrt}(64)] / 2 = [4 \pm 8] / 2$.

Two solutions:

1. $x = (4 + 8)/2 = 12/2 = 6$

2. $x = (4 - 8)/2 = -4/2 = -2$ (not possible as length cannot be negative)

Therefore, the shortest side is 6 meters.

Then the longest side is 6 + 4 = 10 meters and the third side is 10 - 2 = 8 meters.

Now, find the perimeter:

Perimeter = 6 + 8 + 10 = 24 meters

Find the area of the right triangle (area = 1/2 × product of the two legs):

Area = 1/2 × 6 × 8 = 24 square meters

Finally, the difference between the numerical values of the area and the perimeter is:

|Area - Perimeter| = |24 - 24| = 0

Answer: The sides of the triangle are 6 m, 8 m, and 10 m. The difference between the numerical values of the area and perimeter is zero.

Question 35.

The corresponding sides of ΔABC and Δ PQR are in the ratio 3 : 5. AD⊥BC and PS⊥QR as shown in the following figures :

(i) Prove that ΔADC ~ ΔPSR

(ii) If AD =4 cm, find the length of PS.

(iii) Using (ii) find ar (ΔABC) : ar (PQΔR)

[5 Marks]

Answer:

(i) To prove ΔADC ~ ΔPSR:
Given that the sides of ΔABC and ΔPQR are in the ratio 3:5, so corresponding sides are proportional. Since AD⊥BC and PS⊥QR, both AD and PS are altitudes, making right angles with BC and QR respectively. In triangles ADC and PSR, angles ADC and PSR are right angles. Also, ∠DAC = ∠RPS as they correspond to the same angle ratios in similar triangles. Hence, by AA (Angle-Angle) similarity criterion, ΔADC ~ ΔPSR.

(ii) Finding PS if AD = 4 cm:
The sides are in ratio 3:5, so lengths scale similarly. Since AD corresponds to PS, PS = (5/3) × AD = (5/3) × 4 = 20/3 = 6.67 cm.

(iii) Finding the ratio of areas of ΔABC and ΔPQR:
The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Given, ratio of sides = 3:5,
So, ratio of areas = $3^2 : 5^2 = 9 : 25$,
Therefore, ar(ΔABC) : ar(ΔPQR) = 9 : 25.

Question 36.

State basic proportionality theorem. Use it to prove the following: If three parallel lines l, m, n are intersected by transversals p and q as shown in the adjoining figure, then AB/BC = DE/EF.

[5 Marks]

Answer:

Basic Proportionality Theorem:

The Basic Proportionality Theorem (also called Thales Theorem) states that if a line is drawn parallel to one side of a triangle and intersects the other two sides, then it divides those two sides in the same ratio. In other words, if DE is parallel to BC in triangle ABC, then AD/DB = AE/EC.

Proof of the given statement:

Here, we are given three parallel lines l, m, n intersected by two transversals p and q. Points A, B, C lie on transversal p and points D, E, F lie on transversal q such that l, m, n are parallel and intersect the transversals in the order A, B, C and D, E, F respectively.

Since l, m, n are parallel, by the Basic Proportionality Theorem, the segments they form on the transversals are proportional. This means, the ratio of AB to BC on one transversal is equal to the ratio of DE to EF on the other transversal.

Therefore, we have

AB / BC = DE / EF

This is because the three parallel lines divide the two transversals proportionally.

Hence, it is proved that if three parallel lines l, m, n are intersected by transversals p and q, then AB/BC = DE/EF.

Question 37.

wooden cubical die is formed by forming hemispherical depressions on each face of the cube such that face 1 has one depression, face 2 has two depressions and so on. The sum of number of hemispherical depressions on opposite faces is always 7. If the edge of the cubical die measures 5 cm

and each hemispherical depression is of diameter 1.4 cm, find the total surface area of the die so formed.

[5 Marks]

Answer: Given a cube with edge length 5 cm, there are hemispherical depressions on each face of the cube. Faces 1 to 6 have 1 to 6 hemispherical depressions respectively such that the sum on opposite faces is 7. The diameter of each hemisphere is 1.4 cm, so radius r = 0.7 cm. The surface area of the original cube is 6 × (5 × 5) = 150 cm squared. Each hemispherical depression removes some part of the cube's face area but reveals the inner curved surface of the hemisphere. For one hemisphere, curved surface area is 2 × pi × r^2. Since we have 21 depressions in total (1+2+3+4+5+6=21), total curved surface area introduced is 21 × 2 × pi × $(0.7)^2$. The area removed from the cube's faces equals the base area of each hemisphere, which is the area of a circle of radius 0.7 cm, i.e., pi × $(0.7)^2$ per hemisphere. So total area removed from faces is 21 × pi × $(0.7)^2$. Therefore, the total

surface area of the die is: Surface area of cube - area removed from faces + curved surface area of depressions = 150 - (21 × pi × 0.49) + (21 × 2 × pi × 0.49) = 150 + (21 × pi × 0.49) = 150 + 21 × 3.1416 × 0.49 ≈ 150 + 32.3 = 182.3 cm squared. Hence, the total surface area of the die is approximately 182.3 cm squared.

Question 38.

The following table shows the number of patients of different age group who were discharged from the hospital in a particular month :

Find the 'mean' and the 'mode' of the above data.

[5 Marks]

Answer:

To find the mean and mode of the given data on patients discharged in different age groups, we first identify the age groups and the number of patients in each group. The mode is the age group that has the highest frequency of patients discharged. This means it is the most common age group among the discharged patients. For example, if the age group 55 to 65 years has 5 patients, and this is the highest among all groups, then this age group is the mode.

Next, we calculate the mean age. Since the data is grouped, we find the midpoint of each age group and multiply it by the number of patients in that group. Then, sum all these products and divide by the total number of patients. This gives the mean age, which represents the average age group of patients discharged from the hospital.

The mean provides a measure of the central tendency by considering all data points, while the mode only considers the most frequent age group. Both together help us understand the distribution of patient ages effectively. For example, if the mean is close to the mode, it indicates that the data is fairly symmetrical, whereas a large difference may indicate skewness.