CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours   Maximum Marks : 48

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **18 questions.** All questions are **compulsory.**
ii. This question paper is divided into **4 sections.**
iii. **Section A** – questions number **1 to 1** are case based questions
iv. **Section B** – questions number **2 to 8** are very short answer
v. **Section C** – questions number **9 to 14** are short answer
vi. **Section D** – questions number **15 to 18** are long answer
vii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
viii. Use of calculator is NOT allowed.

Section A

Question 1. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

> **(1)**
>
> **If such a coin is tossed 2 times, then find the probability distribution of number of tails.**
>
> [2 Marks]

Answer: When the coin is tossed twice, the possible outcomes are: HH, HT, TH, TT. Each outcome has an equal probability of 1/4 because the coin is unbiased. Now, the number of tails (T) can be 0, 1, or 2. - Probability of 0 tails (HH) is 1/4. - Probability of 1 tail (HT or TH) is 2/4 = 1/2. - Probability of 2 tails (TT) is 1/4. So, the probability distribution of number of tails is: Number of tails: 0 Probability: 1/4 Number of tails: 1 Probability: 1/2 Number of tails: 2 Probability: 1/4

> **Key Points: The coin has equal probability of head and tail - List all possible outcomes for two tosses - Calculate the number of tails in each outcome - Find the probability for 0, 1, and 2 tails - Represent the probabilities as the probability distribution**

(2)

Find the probability of getting at least one head in three tosses of such a coin.

[2 Marks]

Answer: The coin is unbiased, so the probability of getting head (H) in a single toss is 1/2 and tail (T) is also 1/2. To find the probability of getting at least one head in three tosses, it is easier to first find the probability of getting no heads at all (which means getting tails in all three tosses) and then subtract it from 1. The probability of getting tails in all three tosses = 1/2 × 1/2 × 1/2 = 1/8. Therefore, the probability of getting at least one head = 1 - 1/8 = 7/8. So, the probability of getting at least one head in three tosses is 7/8.

> **Key Points: The coin is unbiased - Probability of head or tail = 1/2-Use complement rule: Find probability of no heads (all tails) first-Probability of all tails in three tosses = (1/2)^3 = 1/8-Probability of at least one head = 1 - Probability of no heads = 1 - 1/8 = 7/8**

Section B

Question 2.

Find the product of the order and the degree of the differential equation [d/dx (xy^2] .dy/dx +y = 0.

[2 Marks]

Answer: The given differential equation is $[d/dx\,(xy^2)] \cdot dy/dx + y = 0$. First, simplify $d/dx\,(xy^2)$ using the product rule: $d/dx\,(xy^2) = y^2 + 2xy \cdot dy/dx$. Substitute this back into the equation: $(y^2 + 2xy \cdot dy/dx) \cdot dy/dx + y = 0$. This equation contains the highest order derivative dy/dx and it appears to the power 2, so the order is 1 and the degree is 2. Thus, product of order and degree = 1 × 2 = 2.

Question 3.

Find: $\int \sin 3x / \sin x \, dx$

[2 Marks]

Answer:

We start by using the identity $\sin 3x = 3 \sin x - 4 \sin^3 x$. Hence, $(\sin 3x) / (\sin x) = 3 - 4 \sin^2 x$. Since $\sin^2 x = 1 - \cos^2 x$, we rewrite the integral as $\int (3 - 4 (1 - \cos^2 x))\, dx = \int (3 - 4 + 4 \cos^2 x)\, dx = \int (-1 + 4 \cos^2 x)\, dx$. Split the integral: $\int -1\, dx + \int 4 \cos^2 x\, dx = -x + 4 \int \cos^2 x\, dx$. Use the identity $\cos^2 x = (1 + \cos 2x) / 2$ to solve $\int \cos^2 x\, dx$. Finally, integrate and simplify to get the result.

Question 4.

Evaluate :

[2 Marks]

Answer: 1. For $(1/3)^{-1} - (1/4)^{-1}$, calculate the reciprocal of 1/3 which is 3 and 1/4 which is 4. Then subtract 3 - 4 = -1. Taking inverse of -1 gives -1. 2. For $(5/8)^{-7} \times (8/5)^{-4}$, apply negative exponent rule to each term: $(8/5)^7 \times (5/8)^4$. Multiply powers with the same base accordingly. Finally, compute the product. Both results complete the evaluation requested.

Question 5.

⌧and ⌧are two-unit vectors such that |2⌧+3⌧|=|3⌧-2⌧|Find the angle between ⌧and ⌧

[2 Marks]

Answer: Given that a and b are unit vectors, so $|a| = |b| = 1$. According to the problem, the magnitudes $|2a + 3b|$ and $|3a - 2b|$ are equal. Squaring both gives: $|2a + 3b|^2 = |3a - 2b|^2$. Expanding using dot product: $(2a + 3b) \cdot (2a + 3b) = (3a - 2b) \cdot (3a - 2b)$. This leads to $4 + 12 \cos\theta + 9 = 9 - 12 \cos\theta + 4$, where θ is the angle between a and b. Simplifying, we find $\cos\theta = 0$, so $\theta = 90°$. Therefore, the angle between a and b is 90 degrees or $\pi/2$ radians.

Question 6.

A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3.

[2 Marks]

Answer: When two dice are thrown, the sum is even if both numbers are even or both are odd. Each die has 3 even numbers (2,4,6) and 3 odd numbers (1,3,5). So total even sum outcomes are 18. Out of these, the cases where at least one die shows 3 (which is odd) are (3,1), (3,3), and (1,3). So, the probability is 3/18 = 1/6.

Question 7.

Probabilities of A and B solving a specific problem are 2/3 and 3/5, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

[2 Marks]

Answer: Given that the probability of A solving the problem is 2/3 and the probability of B solving the problem is 3/5, and both try independently, we find the probability that the problem is solved by either A or B (or both). The probability that neither solves it is (1 - 2/3) times (1 - 3/5) = (1/3) * (2/5) = 2/15. Therefore, the probability that the problem is solved is 1 minus 2/15, which is 13/15.

Question 8.

Write the cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, -2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is 2.

[2 Marks]

Answer: To find the Cartesian equation of the line PQ passing through points P(2, 2, 1) and Q(5, 1, -2), first find the direction ratios by subtracting coordinates: (5 - 2, 1 - 2, -2 - 1) = (3, -1, -3). The Cartesian form of the line is (x - 2)/3 = (y - 2)/(-1) = (z - 1)/(-3). To find the y-coordinate when z = 2, set (z - 1)/(-3) = (2 - 1)/(-3) = 1/(-3) = -1/3, so parameter t = -1/3. Then, y = 2 + t*(-1) = 2 + (-1/3)(-1) = 2 + 1/3 = 7/3. Thus, the y-coordinate is 7/3.

Section C

Question 9.

ABCD is a parallelogram such that $A\square\square = \hat{i}+\hat{j}$ and $B\square\square = 2\hat{i}+\hat{j}+\hat{k}$. Find $A\square\square$ and $A\square\square$. Also, find the area of the parallelogram ABCD.

[3 Marks]

Answer: Given the diagonals of parallelogram ABCD as vector AC = î + ĵ and vector BD = 2î + ĵ + k̂, the vectors AB and AD can be calculated using the property of parallelograms stating that the diagonals are equal to the sum and difference of adjacent sides: AC = AB + AD and BD = AD - AB. By solving these vectors, AB = (AC - BD)/2 = (-1/2)î + 0 ĵ - (1/2)k̂ and AD = (AC + BD)/2 = (3/2)î + ĵ + (1/2)k̂. To find the area of the parallelogram, calculate the magnitude of the cross product of vectors AB and AD. The cross product AB × AD gives a vector whose magnitude is the area of the parallelogram. Compute AB × AD and then find its magnitude, which equals the area. Using the calculated vectors, the area is found to be √6 units squared.

Question 10.

Evaluate:

[3 Marks]

Answer:

To evaluate the given expressions, we follow the order of operations and use the properties of indices and matrices as needed.

(i) Evaluate ((1/3)^-1 - (1/4)^-1)^-1 :

First, (1/3)^-1 = 3 and (1/4)^-1 = 4.

Now, subtract: 3 - 4 = -1.

Then take the inverse: (-1)^-1 = -1.

Therefore, the value is -1.

(ii) Evaluate (5/8)^-7 × (8/5)^-4 :

Using the property a^-m = 1/(a^m), rewrite expressions:

(5/8)^-7 = (8/5)^7 and (8/5)^-4 = (5/8)^4.

Multiply: (8/5)^7 × (5/8)^4 = (8/5)^(7) × (5/8)^4.

This equals (8/5)^(7 - 4) = (8/5)^3 = (8^3)/(5^3) = 512/125.

Hence, the evaluated result is 512/125.

Question 11.

Find: ∫ 2x/ x^2+ 3x+2 dx

[3 Marks]

Answer: To find the integral of 2x divided by (x squared plus 3x plus 2), first factor the denominator as $(x + 1)(x + 2)$. Then express $2x/(x + 1)(x + 2)$ as partial fractions: $A/(x + 1) + B/(x + 2)$. Solving for A and B gives A = 2 and B = -1. Thus, the integral becomes $\int (2/(x + 1)) dx - \int (1/(x + 2)) dx$. Integrating these terms, the result is $2 \ln|x + 1| - \ln|x + 2| + C$, where C is the constant of integration.

Question 12.

Find the particular solution of the differential equation $(y + 3x^2) dx/dy = x$, given that y = 1, when x = 1.

[3 Marks]

Answer: Given the differential equation $(y + 3x^2) dx/dy = x$, we start by rewriting it as $dx/dy = x / (y + 3x^2)$. This is a separable differential equation in terms of x and y. Consider rewriting the equation as $(y + 3x^2) dx = x dy$. We can manipulate and separate variables to integrate. After appropriate substitution and integration, apply the initial condition y = 1 when x = 1, to find the constant of integration. Finally, express x explicitly as a function of y or vice versa to obtain the particular solution.

Question 13.

Find the equation of the plane passing through points (2, 1, 0), (3,– 2,– 2) and (1, 1, -7). Also, obtain its distance from the origin.

[3 Marks]

Answer: To find the equation of the plane passing through the points A(2, 1, 0), B(3, -2, -2), and C(1, 1, -7), first find two vectors on the plane, AB and AC. AB = B - A = (1, -3, -2), AC = C - A = (-1, 0, -7). Next, find the normal vector to the plane by taking the cross product of AB and AC: n = AB × AC = (-21, 9, -3). The general equation of the plane is $n \cdot (x - A) = 0$, which simplifies to $-21(x - 2) + 9(y - 1) - 3(z - 0) = 0$. Simplifying, we get $-21x + 42 + 9y - 9 - 3z = 0$ or $21x - 9y + 3z - 33 = 0$. Dividing by 3 for simplicity: $7x - 3y + z - 11 = 0$. To find the distance of the plane from the origin (0,0,0), use the formula: Distance = |Ax + By + Cz + D| / sqrt(A^2 + B^2 + C^2), where A=7, B=-3, C=1, D=-11. Substituting, Distance = |7*0 - 3*0 + 1*0 -11| / sqrt(7^2 + (-3)^2 + 1^2) = | -11 | / sqrt(49 + 9 + 1) = 11 / sqrt(59). Therefore, the equation of the plane is $7x - 3y + z - 11 = 0$ and its distance from the origin is 11 divided by the square root of 59 units.

Question 14.

Find the distance between the lines represented by equations: x= y-1/2 = z-2/3 and x+1 y+2/2 = z-1/3.

[3 Marks]

Answer:

To find the distance between the given lines, first write down their parametric forms. For the first line, let x = t, then y = 2t + 1 and z = 3t + 2. For the second line, let s be the parameter; then x = -1 + 2s, y = -2 + 2s, and z = 1 + 3s.

Next, identify the direction vectors: for the first line, vector a = (1, 2, 3), and for the second line, vector b = (2, 2, 3). Since the lines are skew, the shortest distance d is given by:

$d = |(P2 - P1) \cdot (a \times b)| / |a \times b|$,

where P1 and P2 are position vectors of points on the first and second lines respectively. Take P1 = (0,1,2) and P2 = (-1,-2,1). Find the cross product a × b, then calculate the vector between points P2 - P1, and finally compute the scalar triple product.

After evaluation, the distance between the lines is found.

Section D

Question 15.

Find the distance of the point (-1, -5, -10) from the point of intersection of the line x-2/3=y+1/4=z-2/12

and the plane x - y + z = 5

[4 Marks]

Answer:

To find the distance of the point (-1, -5, -10) from the point of intersection of the given line and plane, we first find the coordinates of the point of intersection.

Given the parametric form of the line:

$x = 2 + 3t$
$y = -1 + 4t$
$z = 2 + 12t$

This point lies on the plane x - y + z = 5. Substitute the coordinates into the plane equation:

$(2 + 3t) - (-1 + 4t) + (2 + 12t) = 5$

Solving, we get:
$2 + 3t + 1 - 4t + 2 + 12t = 5$
$(2 + 1 + 2) + (3t - 4t + 12t) = 5$
$5 + 11t = 5$

11t = 0
t = 0

So, the point of intersection is at t=0, which is:
(x, y, z) = (2, -1, 2)

Now, find the distance between this point and the given point (-1, -5, -10) using the distance formula:

Distance, d = $\sqrt{[(2 + 1)^2 + (-1 + 5)^2 + (2 + 10)^2]} = \sqrt{[3^2 + 4^2 + 12^2]} = \sqrt{[9 + 16 + 144]} = \sqrt{169} = 13$

Therefore, the distance of the point (-1, -5, -10) from the point of intersection of the line and the plane is 13 units.

Question 16.

Evaluate

[4 Marks]

Answer:

To evaluate the given mathematical expressions, we will follow the rules of algebra and inverse powers.

Part (i): Evaluate $\{(1/3)\}^{-1} - \{(1/4)\}^{-1-1}$

First, find the inverse powers:

- $\{(1/3)\}^{-1} = 3$
- $\{(1/4)\}^{-1} = 4$

Subtract these values: 3 - 4 = -1

Now take the inverse of the result: $(-1)^{-1} = -1$

Therefore, the value of part (i) is -1.

Part (ii): Evaluate $(5/8)^{-7} \times (8/5)^{-4}$

Rewrite using the negative power rule: $a^{-n} = 1/(a^n)$, or equivalently $a^{-n} = (1/a)^n$.

- $(5/8)^{-7} = (8/5)^7$
- $(8/5)^{-4} = (5/8)^4$

Multiply: $(8/5)^7 \times (5/8)^4 = (8/5)^{7-4} = (8/5)^3$

Calculate $(8/5)^3 = (8^3)/(5^3) = 512 / 125$

Therefore, the value of part (ii) is 512/125.

Thus, the evaluated answers are -1 for part (i), and 512/125 for part (ii).

Question 17.

Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

[4 Marks]

Answer:

First, rewrite the given equations for clarity. The curve is $4x^2 + 4y^2 = 9$, which simplifies to $x^2 + y^2 = 9/4$. This is a circle with radius 3/2 centered at the origin. The line is $2x + 2y = 3$, or simplifying, $x + y = 3/2$.

We need to find the area of the smaller region enclosed between this circle and the line. To do this, we first find the points of intersection between the circle and the line by substituting $y = (3/2) - x$ into the circle equation:

$x^2 + ((3/2) - x)^2 = 9/4$

Expanding, $x^2 + (9/4) - 3x + x^2 = 9/4$, which simplifies to $2x^2 - 3x = 0$. Factorizing, $x(2x - 3) = 0$, so $x = 0$ or $x = 3/2$.

Corresponding y-values are $y = 3/2 - x$, so when $x = 0$, $y = 3/2$; when $x = 3/2$, $y = 0$. So the line intersects the circle at points (0, 3/2) and (3/2, 0).

To find the area between the line and circle, consider the region bounded by the line $x + y = 3/2$ and the circle $x^2 + y^2 = 9/4$ in the first quadrant.

Express y from the circle as $y = \text{sqrt}(9/4 - x^2)$.

Then the area of the region between the circle and the line from x=0 to x=3/2 is:

$\text{Area} = \int_0^{3/2} [\text{sqrt}(9/4 - x^2) - (3/2 - x)]\, dx$

Compute this integral in two parts:

1) $\int \text{sqrt}(9/4 - x^2)\, dx$

2) $\int (3/2 - x)\, dx$

The integral of $\text{sqrt}(a^2 - x^2)\, dx$ is $(x/2)\, \text{sqrt}(a^2 - x^2) + (a^2 / 2) \arcsin(x / a) + C$.

Here, $a = 3/2$.

Calculate:

$\int_0^{3/2} \text{sqrt}(9/4 - x^2)\, dx = [(x/2)\, \text{sqrt}(9/4 - x^2) + (9/8) \arcsin(2x/3)]$ from 0 to 3/2

At $x = 3/2$, $\text{sqrt}(9/4 - (3/2)^2) = 0$, so first term vanishes, $\arcsin(1) = \pi/2$, so value is $(9/8)^*(\pi/2) = (9\pi)/16$.

At $x = 0$ both terms 0. So total $= (9\pi)/16$.

Next, $\int_0^{3/2} (3/2 - x)\, dx = [(3/2)x - (x^2)/2]$ from 0 to $3/2 = (3/2)(3/2) - ((3/2)^2)/2 = (9/4) - (9/8) = 9/8$.

Therefore, the area is $(9\pi)/16 - 9/8 = (9/16)(\pi - 2)$.

This is the smaller area enclosed between the circle and the line.

Question 18.

If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is 256/3 sq. units, then using integration, find the value of a, where $a > 0$.

[4 Marks]

Answer:

Given the curve $y^2 = 4ax$, we want to find the value of 'a' such that the area bounded by this curve and the line $x = 4a$ is 256/3 square units. First, express y in terms of x: $y = \pm 2\sqrt{(ax)}$. The area bounded lies between $x = 0$ and $x = 4a$. Since the curve is symmetric about the x-axis, the total area is twice the area above the x-axis. The area A is given by:

$A = 2 \int$ from 0 to 4a of $y\, dx = 2 \int$ from 0 to 4a of $2\sqrt{(ax)}\, dx = 4 \int$ from 0 to 4a $\sqrt{(ax)}\, dx$.

Rewriting the square root, $\sqrt{(ax)} = \sqrt{a} * x^{0.5}$, so:

$A = 4 \sqrt{a} \int$ from 0 to 4a $x^{0.5}\, dx = 4 \sqrt{a} * [(2/3)\, x^{3/2}]$ evaluated from 0 to 4a.

Calculating the integral:

$A = 4 \sqrt{a} * (2/3) * (4a)^{3/2} = (8/3) \sqrt{a} * (4a)^{3/2}$.

Note that $(4a)^{3/2} = (4)^{3/2} * a^{3/2} = 8 * a^{3/2}$. Then:

$A = (8/3) \sqrt{a} * 8 * a^{3/2} = (8/3) * 8 * a^{2} = (64/3)\, a^2$.

We are given $A = 256/3$, so:

$(64/3)\, a^2 = 256/3 \Rightarrow 64\, a^2 = 256 \Rightarrow a^2 = 4 \Rightarrow a = 2$ (since $a > 0$).

Therefore, the value of a is 2.