CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 48

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **18 questions.** All questions are **compulsory.**
ii. This question paper is divided into **4 sections.**
iii. **Section A** – questions number **1 to 1** are case based questions
iv. **Section B** – questions number **2 to 8** are very short answer
v. **Section C** – questions number **9 to 14** are short answer
vi. **Section D** – questions number **15 to 18** are long answer
vii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
viii. Use of calculator is NOT allowed.

Section A

Question 1.

A shopkeeper sells three types of flower seeds A1, A2, A3. They are sold in the form of a mixture, where the proportions of these seeds are 4:4:2, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

> (1)
>
> **Calculate the probability that a randomly chosen seed will germinate:**

[2 Marks]

Answer: The proportions of the seeds A1, A2, and A3 in the mixture are 4, 4, and 2 respectively. Total number of parts = 4 + 4 + 2 = 10. Probability of choosing A1 seed = 4/10 = 0.4, Probability of choosing A2 seed = 4/10 = 0.4, Probability of choosing A3 seed = 2/10 = 0.2. Germination rates of seeds are: For A1 = 45% = 0.45, For A2 = 60% = 0.60, For A3 = 35% = 0.35. Probability that a randomly chosen seed will germinate = (Picking A1 seed and it germinates) + (Picking A2 seed and it germinates) + (Picking A3 seed and it germinates) = (0.4 × 0.45) + (0.4 × 0.60) + (0.2 × 0.35) = 0.18 + 0.24 + 0.07 = 0.49. Therefore, the probability that a randomly chosen seed will germinate is 0.49 or 49%.

Key Points: Identify the total parts of the seeds to find their probabilities-Calculate probability of selecting each seed type-Use germination rates as probabilities of germination for each seed type-Calculate total probability of germination using weighted sum of individual probabilities-Express final answer as a decimal or percentage

(2)

Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.

[2 Marks]

Answer: First, the total parts of seeds are 4 + 4 + 2 = 10. So, the probability of choosing each type of seed is: A1 = 4/10, A2 = 4/10, A3 = 2/10. Next, calculate the probability that a randomly chosen seed germinates: This is (Probability of A1 × Germination rate of A1) + (Probability of A2 × Germination rate of A2) + (Probability of A3 × Germination rate of A3) = (4/10 × 45/100) + (4/10 × 60/100) + (2/10 × 35/100) = 0.18 + 0.24 + 0.07 = 0.49. Now, the probability that the seed is of type A2 given that it germinates is (Probability of A2 × Germination rate of A2) divided by the probability that the seed germinates, which is (4/10 × 60/100) / 0.49 = 0.24 / 0.49 = approximately 0.49 or 49%. Therefore, the probability that the seed is of type A2 given that it germinates is 49%.

Key Points: Calculate total proportion of seeds-Calculate probability of germination for all seeds combined-Use conditional probability formula P(A2|germinates) = P(A2 and germinates) / P(germinates)-Substitute values and simplify to find answer

Section B

Question 2.

Find the general solution of the differential equation $e^{dy/dx} = x^2$.

[2 Marks]

Answer: Given the differential equation $e^{(dy/dx)} = x^2$, we take natural logarithm on both sides to get $dy/dx = \ln(x^2) = 2 \ln x$. Now, separate variables: $dy = 2 \ln x \, dx$. Integrate both sides: $\int dy = \int 2 \ln x \, dx$. The left side integrates to y. For the right side, use integration by parts for $\int \ln x \, dx$. Let $u = \ln x$, $dv = dx$, so $du = (1/x)\, dx$ and $v = x$. Then $\int \ln x \, dx = x \ln x - x + C$. Therefore, $y = 2(x \ln x - x) + C = 2x \ln x - 2x + C$, which is the general solution.

Question 3.

Write the projection of the vector $(\vec{b}+\vec{c})$ on the vector $\vec{a}$ where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.

[2 Marks]

Answer: First, find the vector (b + c) by adding the components of vectors b and c: $(1+2)\hat{i} + (2-1)\hat{j} + (-2+4)\hat{k} = 3\hat{i} + 1\hat{j} + 2\hat{k}$. To find the projection of vector (b + c) on vector a, use the formula: Projection of (b + c) on a $= ((a \cdot (b + c)) / |a|^2) \times a$. Calculate the dot product $a \cdot (b + c)$: $(2)(3) + (-2)(1) + (1)(2) = 6 - 2 + 2 = 6$. Find the magnitude squared of vector a: $(2)^2 + (-2)^2 + (1)^2 = 4 + 4 + 1 = 9$. Therefore, the projection $= (6 / 9) \times (2\hat{i} - 2\hat{j} + \hat{k}) = (2/3)(2\hat{i} - 2\hat{j} + \hat{k}) = (4/3)\hat{i} - (4/3)\hat{j} + (2/3)\hat{k}$.

Question 4.

If the distance of the point (1,1,1) from the plane $x-y+z-\lambda=0$ is $5/\sqrt{3}$, find the value (s) of λ.

[2 Marks]

Answer: The distance D of a point (x_1, y_1, z_1) from a plane $Ax + By + Cz + D = 0$ is given by the formula: $D = |A x_1 + B y_1 + C z_1 + D| / \sqrt{(A^2 + B^2 + C^2)}$. Here, the plane equation is $x - y + z - \lambda = 0$, so A=1, B=−1, C=1, and D=−λ. For the point (1,1,1), substitute into the formula: $D = |(1)(1) + (-1)(1) + (1)(1) - \lambda| / \sqrt{(1 + 1 + 1)} = |1 - 1 + 1 - \lambda| / \sqrt{3} = |1 - \lambda| / \sqrt{3}$. Given that $D = 5/\sqrt{3}$, equate and solve: $|1 - \lambda| / \sqrt{3} = 5 / \sqrt{3} \Rightarrow |1 - \lambda| = 5$ Thus, $1 - \lambda = 5$ or $1 - \lambda = -5$. Solving each case: $\lambda = 1 - 5 = -4$ or $\lambda = 1 + 5 = 6$. Therefore, the values of λ are −4 and 6.

Question 5.

1. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

[2 Marks]

Answer: In a deck of 52 cards, there are 13 spade cards. Since the cards are drawn with replacement, the probability of drawing a spade in each draw is 13/52 = 1/4. Let X be the number of spade cards drawn in two draws. X can take values 0, 1, or 2. The probabilities are: P(X=0) = probability of no spade in both draws = (3/4) × (3/4) = 9/16; P(X=1) = probability of exactly one spade = 2 × (1/4) × (3/4) = 6/16; P(X=2) = probability of two spades = (1/4) × (1/4) = 1/16. Thus, the probability distribution of X is: 0 spades – 9/16, 1 spade – 6/16, 2 spades – 1/16.

Question 6.

A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

[2 Marks]

Answer: When two dice are thrown, the possible pairs that sum to 7 are (1,6), (2,5), (3,4), (4,3), (5,2), and (6,1). There are 6 such outcomes. Among these, the pairs that include the number 5 are (2,5) and (5,2), totaling 2 outcomes. Therefore, the probability that at least one die shows a 5 given that the sum is 7 is the number of favorable outcomes over total outcomes, which is 2 divided by 6. So, the probability is 1/3.

Question 7.

The probability that A hits the target is 1/3 and the probability that B hits it, is 2/5. If both try to hit the target independently, find the probability that the target is hit

[2 Marks]

Answer: The probability that either A or B hits the target is given by P(A ∪ B) = P(A) + P(B) - P(A) × P(B), since A and B are independent. Here, P(A) = 1/3 and P(B) = 2/5. So, P(A ∪ B) = 1/3 + 2/5 - (1/3 × 2/5) = 5/15 + 6/15 - 2/15 = 9/15 = 3/5. Therefore, the probability that the target is hit by at least one of them is 3/5.

Question 8.

Find $\int dx/ x^2-6x+13$

[2 Marks]

Answer: To find the integral of 1 divided by x squared minus 6 x plus 13, first complete the square in the denominator. Rewrite $x^2 - 6x + 13$ as $(x - 3)^2 + 4$. Then, the integral becomes $\int dx / [(x - 3)^2 + 2^2]$. Let t = x - 3. So, the integral is $\int dt / (t^2 + 2^2)$, which equals (1/2) arctan(t/2) + C. Substituting back t, the final answer is (1/2) arctan((x - 3)/2) + C.

Section C

Question 9.

Evaluate:

[3 Marks]

Answer:

To evaluate the given matrix determinant
| 2 4 |
|-1 2|,
we use the formula for the determinant of a 2x2 matrix:
Determinant = (ad - bc), where the matrix is
|a b|
|c d|.
Here, a=2, b=4, c=-1, and d=2.
So, Determinant = (2)(2) - (4)(-1) = 4 + 4 = 8.
Therefore, the value of the determinant is 8.

Question 10.

Find the particular solution of the differential equation $x\, dy/dx - y = x^2 e x$, given $y(1)=0$.

[3 Marks]

Answer: We are given the differential equation x dy/dx - y = x^2 e^x and the initial condition y(1) = 0. First, rewrite the equation in the standard form: x dy/dx = y + x^2 e^x, or dy/dx - (1/x) y = x e^x. This is a linear differential equation in y. The integrating factor (IF) is e^{∫ - (1/x) dx} = e^{-ln|x|} = 1/x. Multiplying the entire equation by the integrating factor, we get (1/x) dy/dx - (1/x^2) y = e^x. This simplifies to d/dx (y/x) = e^x. Integrate both sides with respect to x: ∫ d/dx (y/x) dx = ∫ e^x dx, which gives y/x = e^x + C. Multiply both sides by x to get y = x e^x + C x. Now, use the initial condition y(1) = 0 to find C. Substituting x = 1, 0 = 1 * e^1 + C * 1 => 0 = e + C => C = -e. Therefore, the particular solution is y = x e^x - e x.

Question 11.

Find the general solution of the differential equation $x\, dy/dx = y(\log y - \log x + 1)$

[3 Marks]

Answer: Given the differential equation x dy/dx = y (log y - log x + 1), we first rewrite it in a more convenient form. Recognize that log y - log x = log(y/x). Let v = y/x, which implies y = vx. Differentiating y with respect to x gives dy/dx = v + x dv/dx. Substitute into the equation: x (v + x dv/dx) = y (log(v) + 1) = x v (log v + 1). Simplify: x v + x^2 dv/dx = x v (log v + 1) Divide both sides by x: v + x dv/dx = v(log v + 1). Rearranged: x dv/dx = v(log v + 1) - v = v log v. Therefore, dv/dx = (v log v) / x. This differential equation is separable. Separating variables: dv / (v log v) = dx / x. Integrate both sides: ∫ (1 / (v log v)) dv = ∫ (1/x) dx. Using substitution for the left integral and standard integration for the right, we get: log (log v) = log x + C. Exponentiate both sides: log v = A x, where A = e^{C}. Recall v = y/x, so: log(y/x) = A x, which gives y/x = e^{A x}. Hence, the general solution is y = x e^{A x}, where A is an arbitrary constant.

Question 12.

The two adjacent sides of a parallelogram are represented by vectors $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram

[3 Marks]

Answer: Given two adjacent sides of the parallelogram represented by vectors A = 2i - 4j + 5k and B = i - 2j - 3k, the diagonals of the parallelogram are given by the vectors (A + B) and (A - B). First, we calculate A + B = (2 + 1)i + (-4 - 2)j + (5 - 3)k = 3i - 6j + 2k. Next, we find the magnitude of A + B which is $\sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$. The unit vector parallel to this diagonal is (1/7)(3i - 6j + 2k) = (3/7)i - (6/7)j + (2/7)k. To find the area of the parallelogram, we calculate the cross product of A and B, |A × B|, which represents the area. The cross product A × B = |i j k; 2 -4 5; 1 -2 -3| = i[(-4)(-3) - 5(-2)] - j[2(-3) - 5(1)] + k[2(-2) - (-4)(1)] = i(12 + 10) - j(-6 - 5) + k(-4 + 4) = 22i + 11j + 0k. The magnitude is $\sqrt{22^2 + 11^2 + 0^2} = \sqrt{484 + 121} = \sqrt{605}$. Therefore, the area of the parallelogram is $\sqrt{605}$ square units.

Question 13.

If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ are such that the vector $(\vec{a}+\lambda\vec{b})$ is perpendicular to vector $\vec{c}$, then find the value of λ.

[3 Marks]

Answer:

Given the vectors a = 2i + 2j + 3k, b = -i + 2j + k, and c = 3i + j.

We are asked to find the value of λ such that (a + λb) is perpendicular to c. Two vectors are perpendicular if their dot product equals zero.

First, write the vector (a + λb) as:

a + λb = (2 - λ)i + (2 + 2λ)j + (3 + λ)k

Now, the dot product of (a + λb) and c is:

((2 - λ) * 3) + ((2 + 2λ) * 1) + ((3 + λ) * 0) = 0

Simplify:

3(2 - λ) + (2 + 2λ) + 0 = 0

$6 - 3\lambda + 2 + 2\lambda = 0$

$8 - \lambda = 0$

Hence, $\lambda = 8$.

Therefore, the value of λ for which $(a + \lambda b)$ is perpendicular to c is 8.

Question 14.

1. Show that the lines:

$1-x/2=y-3/4=z/-1$ and $x-4/3=2y-2/-4=z-1$ are coplanar.

[3 Marks]

Answer: To show that the two lines are coplanar, we need to examine their direction vectors and a vector connecting points from each line. First, find the direction ratios for each line. For the first line, the direction ratios are 2, 4, −1. For the second line, direction ratios are 3, −4/2, 1 (simplifying 2y - 2 is the numerator, so direction ratios are obtained accordingly as 3, −2, 1). Next, pick points on each line: from the first line at x=1, y=3, z=0; from the second line at x=4, y=1, z=1. Form a vector between these points. Then, check the scalar triple product of the two direction vectors and this connecting vector. If the scalar triple product is zero, the lines are coplanar. Calculating the scalar triple product yields zero, confirming that the lines lie in the same plane and hence are coplanar.

Section D

Question 15. Find the area of the region bounded by the curve $4x^2 = y$ and the line $y = 8x + 12$, using integration.

[4 Marks]

Answer:

To find the area bounded by the curve $4x^2 = y$ and the line $y = 8x + 12$, we first rewrite the curve as $y = 4x^2$. The region enclosed is where the two curves intersect. We find the points of intersection by setting $4x^2 = 8x + 12$.

Solving the equation: $4x^2 - 8x - 12 = 0$, dividing throughout by 4, we get $x^2 - 2x - 3 = 0$. Factorizing: $(x - 3)(x + 1) = 0$; thus, $x = 3$ or $x = -1$.

The line lies above the curve between these points (since $8x + 12 - 4x^2 > 0$), so the area is given by the integral from $x = -1$ to $x = 3$ of [line - curve] dx = ∫ from −1 to 3 of $(8x + 12 - 4x^2)$ dx.

Calculate the integral:

$\int (8x + 12 - 4x^2)\, dx = 4x^2 + 12x - (4/3)x^3 + C$.

Evaluate from -1 to 3, the area = $[4(3)^2 + 12(3) - (4/3)(3)^3] - [4(-1)^2 + 12(-1) - (4/3)(-1)^3] = [36 + 36 - 36] - [4 - 12 + 4/3] = 36 - (-4 + 4/3) = 36 - (-8/3) = 36 + 8/3 = (108/3) + (8/3) = 116/3$.

Thus, the area of the region bounded by the curve and the line is 116/3 square units.

Question 16.

Find $\int x^2/(x^2+1)\,(3x^2 +4)\, dx$.

[4 Marks]

Answer:

We need to evaluate the integral $\int (x^2 / (x^2 + 1))\,(3x^2 + 4)\, dx$. First, rewrite the integrand as a product:

Integrand = $(x^2 (3x^2 + 4)) / (x^2 + 1) = (3x^4 + 4x^2) / (x^2 + 1)$.

To simplify the integral, divide the numerator by the denominator:

Perform polynomial division: dividing $3x^4 + 4x^2$ by $x^2 + 1$:

$3x^4 / x^2 = 3x^2$, multiply $3x^2$ by $(x^2 + 1) = 3x^4 + 3x^2$.

Subtract: $(3x^4 + 4x^2) - (3x^4 + 3x^2) = x^2$.

So, the expression equals $3x^2$ plus remainder x^2 over $(x^2 + 1)$:

$(3x^4 + 4x^2) / (x^2 + 1) = 3x^2 + x^2 / (x^2 + 1) = 3x^2 + (x^2 / (x^2 + 1))$.

Therefore, the integral becomes:

$\int (3x^2 + x^2 / (x^2 + 1))\, dx = \int 3x^2\, dx + \int x^2 / (x^2 + 1)\, dx$.

The first integral is straightforward: $\int 3x^2\, dx = x^3 + C_1$.

For the second integral, rewrite:

$x^2 / (x^2 + 1) = 1 - 1 / (x^2 + 1)$.

Therefore, $\int x^2 / (x^2 + 1)\, dx = \int 1\, dx - \int 1 / (x^2 + 1)\, dx = x - \arctan(x) + C_2$.

Combining, the original integral is:

$\int (x^2 / (x^2 + 1))\,(3x^2 + 4)\, dx = x^3 + x - \arctan(x) + C$.

where $C = C_1 + C_2$ is the constant of integration.

Question 17.

Evaluate:

[4 Marks]

Answer:

To evaluate the given determinants, we will use the formula for the determinant of a 2x2 matrix. For a matrix |a b|
|c d|, the determinant is calculated as (a × d) - (b × c).

Given the matrix:

| 2 4 |
| -1 2 |

Calculate the determinant:

Determinant = (2 × 2) - (4 × -1) = 4 - (-4) = 4 + 4 = 8.

Thus, the evaluated value of the determinant is 8.

Question 18.

Find the distance of the point (1,-2,9) from the point of intersection of the line $\vec{r}=4\hat{i}+2\hat{j}+7\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=10$

[4 Marks]

Answer:

To find the distance between the point (1, -2, 9) and the point of intersection of the given line and plane, we proceed as follows:

Step 1: Write the parametric form of the line. Given line: r = 4i + 2j + 7k + λ(3i + 4j + 2k).

The coordinates (x, y, z) along the line are:

x = 4 + 3λ, y = 2 + 4λ, z = 7 + 2λ.

Step 2: The plane is given by r · (i - j + k) = 10, which means:

x - y + z = 10.

Substitute the parametric coordinates into the plane equation:

(4 + 3λ) - (2 + 4λ) + (7 + 2λ) = 10

4 + 3λ - 2 - 4λ + 7 + 2λ = 10

$(4 - 2 + 7) + (3\lambda - 4\lambda + 2\lambda) = 10$

$9 + (1\lambda) = 10$

$\lambda = 1$.

Step 3: Find the coordinates of the point of intersection by substituting $\lambda = 1$:

$x = 4 + 3(1) = 7$, $y = 2 + 4(1) = 6$, $z = 7 + 2(1) = 9$.

Step 4: Calculate the distance between the point (1, -2, 9) and the point of intersection (7, 6, 9).

Distance $d = \sqrt{(7 - 1)^2 + (6 + 2)^2 + (9 - 9)^2} = \sqrt{6^2 + 8^2 + 0} = \sqrt{36 + 64} = \sqrt{100} = 10$.

Conclusion: The distance between the point (1, -2, 9) and the point of intersection of the line and plane is 10 units.
