CBSE EXAMINATION PAPER-2023

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 78

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **40 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 3** are case based questions
iv. **Section B** – questions number **4 to 20** are multiple choice questions
v. **Section C** – questions number **21 to 27** are very short answer
vi. **Section D** – questions number **28 to 36** are short answer
vii. **Section E** – questions number **37 to 40** are long answer
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1.

An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.

The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denote the number obtained on the bottom face and the following table gives the probability distribution of X.

(1)

Find $P(X \geq 6)$.

[1 Marks]

(2) Find the value of p.

[1 Marks]

(3) (a) Find $P(X = 3m)$, where m is a natural number. OR (b) Find the mean $E(X)$.

[2 Marks]

(4)

Find the mean $E(X)$.

[2 Marks]

Question 2. In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3. The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ 40,000 h^2, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres. Elements of a typical rain water harvesting system are shown in the figure.

(1) Find the total cost C of digging the tank in terms of x.

[1 Marks]

(2)

(a) Find the value of x for which cost C is minimum.

[2 Marks]

(3) Find dC/dx.

[1 Marks]

(4)

Check whether the cost function C(x) expressed in terms of x is increasing or not, where $x > 0$.

[2 Marks]

Question 3.

A volleyball player serves the ball which takes a parabolic path given by the equation $h(t) = -7/2t^2 + 13/2\, t+1$, where h(t) is the height of ball at any time t (in seconds) and ($t \geq 0$).

(1) Is h(t) a continuous function? Justify.

[2 Marks]

(2) Find the time at which the height of the ball is maximum.

[2 Marks]

Section B

Question 4.

[1 Marks]

(A) 6

(B) 0

(C) 8

(D) 10

Question 5.

[1 Marks]

(A) 3

(B) 9

(C) 12

(D) 27

Question 6.

A and B are skew-symmetric matrices of same order. AB is symmetric, if:

[1 Marks]

(A) AB = BA

(B) AB = O

(C) AB = -BA

(D) BA = O

Question 7.

Let A be the area of a triangle having vertices (x_1, y_1), (x_2, y_2), (x_3, y_3). Which of the following is correct?

[1 Marks]

(A)

(B)

(C)

(D)

Question 8.

The integral $\int 2^{x+2}\,dx$ is equal to:

[1 Marks]

(A) $2^{x+2}+C$

(B) $2^{x+2}/ \log 2 + C$

(C) $2^{x+2} \log 2 + C$

(D) $2 \cdot 2^{x+2} / \log 2 + C$

Question 9.

The solution of the differential equation $dx/x + dy/y = 0$ is:

[1 Marks]

(A) $1/x + 1/y = C$

(B) $\log x - \log y = C$

(C) $xy = C$

(D) $x + y = C$

Question 10.

What is the product of the order and degree of the differential equation \($d^2y/dx^2 \sin y + (dy/dx)^3 \cos y = \sqrt{y}$

[1 Marks]

(A) Not defined

(B) 2

(C) 3

(D) 6

Question 11.

If a vector makes an angle of $\pi/4$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is:

[1 Marks]

(A) $\pi/4$

(B) $\pi/2$

(C) 0

(D) $3\pi/4$

Question 12.

□□and □□are two non-zero vectors such that the projection of □□on □□is 0. The angle between □□and □□is

[1 Marks]

(A) π

(B) π/2

(C) π/4

(D) 0

Question 13.

In $\triangle ABC$, A□□=$\hat{i}+\hat{j}+2\hat{k}$ and A□□=$3\hat{i}-\hat{j}+4\hat{k}$. If D is mid-point of BC, then vector A□□is equal to :

[1 Marks]

(A) $4\hat{i}+6\hat{k}$

(B) $2\hat{i}-2\hat{j}+2\hat{k}$

(C) $\hat{i}-\hat{j}+\hat{k}$

(D) $2\hat{i}+3\hat{k}$

Question 14.

The value of □ for which the angle between the lines

□□=$\hat{i}+\hat{j}+\hat{k}+p(2\hat{i}+\hat{j}+2\hat{k})$ and
□□=$(1+q)\ \hat{i}+(1+q$□$)\ \hat{j}+(1+q)\ \hat{k}$ is □/2 is:

[1 Marks]

(A) 4

(B) -2

(C) 2

(D) -4

Question 15.

If $P(A\cap B)$ =18 and $P(A)$=34, then $P(BA)$ is equal to:

[1 Marks]

(A) 1/3

(B) 1/2

(C) 1/6

(D) 2/3

Question 16.

If y = cos x - sin x /cos x + sin x , then dy/dx is

[1 Marks]

(A) $-\sec^2 (\pi/4-x)$

(B) $\sec^2 (\pi/4-x)$

(C) log| sec (π/4-x) |

(D) -log| sec (π/4-x) |

Question 17.

The number of feasible solutions of the linear programming problem given as:

Maximize z = 15x + 30y subject to constraints

$3x + y \leq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ is:

[1 Marks]

(A) 3

(B) Infinite

(C) 2

(D) 1

Question 18.

The feasible region of a linear programming problem is shown in the figure below.

Which of the following are the possible constraints?

[1 Marks]

(A) $x + 2y \geq 4, x + y \geq 3, x \leq 0, y \leq 0$

(B) $x + 2y \leq 4, x + y \leq 3, x \geq 0, y \geq 0$

(C) $x + 2y \geq 4, x + y \geq 3, x \geq 0, y \geq 0$

(D) $x + 2y \geq 4, x + y \leq 3, x \geq 0, y \geq 0$

Question 19. Assertion (A): Range of $[\sin^{-1}x + 2\cos^{-1}x]$ is $[0, \pi]$. Reason (R): Principal value branch of $\sin^{-1}x$ has range $[-\pi/2, \pi/2]$

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is false and Reason (R) is true.

(D) Assertion (A) is true and Reason (R) is false

Question 20.

Assertion (A): A line through the points (4, 7, 8) and (2, 3, 4) is parallel to a line through the points (−1, −2, 1) and (1, 2, 5). Reason (R): Lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are parallel if $\vec{b_1} \cdot \vec{b_2} = 0$

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is false and Reason (R) is true.

(D) Assertion (A) is true and Reason (R) is false.

Section C

Question 21.

If $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$, find the value of $(\vec{a} \times \hat{j}) \cdot (\vec{a} \times \hat{k})$ -12

[2 Marks]

Question 22.

If the angle between the lines $x-5/□=y+2/-5=z+24/5$ /□and $x/1=y/0=z/1$ is □/4, find the relation between □ and □

[2 Marks]

Question 23.

If $f(x)=a(\tan x-\cot x)$, where $a>0$, then find whether $f(x)$ is increasing or decreasing function in its domain.

[2 Marks]

Question 24.

Evaluate: $3\sin^{-1}(1/\sqrt{2})+2\cos^{-1}(\sqrt{3}/2)+\cos^{-1}(0)$

[2 Marks]

Question 25.

Draw the graph of $f(x)=\sin^{-1}x$, $x\in[-1/\sqrt{2},1/\sqrt{2}]$. Also, write range of $f(x)$

[2 Marks]

Question 26.

If $y = x^{1/x}$ then find dy/dx at x =1

[2 Marks]

Question 27.

If x = a sin 2t, y = a (cos 2t + log tan t), then find dy/dx.

[2 Marks]

Section D

Question 28.

Find the general solution of the differential equation:

$\frac{d}{dx}(xy^2)=2y(1+x^2)$

[3 Marks]

Question 29.

Evaluate

[3 Marks]

Question 30.

Evaluate

[3 Marks]

Question 31.

Find the derivative $\int \cos x / \sin 3x \, dx$

[3 Marks]

Question 32.

Determine graphically the minimum value of the objective function

$z = 500x + 400y$

subject to constraints

$x + y \leq 200,$

$x \geq 20,$

$y \geq 4x,$

$y \geq 0.$

[3 Marks]

Question 33.

A pair of dice is thrown simultaneously. If X denotes the absolute difference of numbers obtained on the pair of dice, then find the probability distribution of X.

[3 Marks]

Question 34.

There are two coins. One of them is a biased coin such that P (head) : P (tail) is 1 : 3 and the other coin is a fair coin. A coin is selected at random and tossed once. If the coin showed head, then find the probability that it is a biased coin.

[3 Marks]

Question 35.

Find: $\int x^2 \log (x^2+1)\, dx$

[3 Marks]

Question 36.

Solve the following differential equation :

$xe^{y/x} - y + x\, dy/dx = 0$

[3 Marks]

Section E

Question 37.

Show that a function f : R → R defined as f(x) =5x-3/4 is both one-one and onto.

[5 Marks]

Question 38.

The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is π/2 units. Using integration, find the value of m.

[5 Marks]

Question 39.

Find the value of b so that the lines x - 1/2= y - b/3 = z - 3/4 and x-4/5 = y-1/2 =z are intersecting lines. Also, find the point of intersection of these given lines.

[5 Marks]

Question 40. Find the equations of all the sides of the parallelogram ABCD whose vertices are A(4, 7, 8), B(2, 3, 4), C(-1, 2, 1) and D(1, 2, 5). Also, find the coordinates of the foot of the perpendicular from A to CD.

[5 Marks]