CBSE EXAMINATION PAPER-2023

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 85

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **42 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 3** are case based questions

iv. **Section B** – questions number **4 to 21** are multiple choice questions

v. **Section C** – questions number **22 to 27** are very short answer

vi. **Section D** – questions number **28 to 36** are short answer

vii. **Section E** – questions number **37 to 42** are long answer

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

ix. Use of calculator is NOT allowed.

Section A

Question 1. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure. Types of Yoga: Hatha Yoga, Bikram Yoga, Vinyasa Yoga, Kundalini Yoga, Anusara Yoga. The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44.

(1) Find the value of x.

[1 Marks]

(2) Find the value of y.

[1 Marks]

(3)

Find P(C/B)

[2 Marks]

(4)

Find the probability that a randomly selected person does Yoga of type A or B but not C.

[2 Marks]

Question 2. A tank, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45 degrees.

(1) Find the volume of water in the tank in terms of its radius r.

[1 Marks]

(2) Find rate of change of radius when $r = 2\sqrt{2}$ cm.

[1 Marks]

(3)

Find the rate at which the wet surface area of the conical tank is decreasing at the instant when radius r = 2√2 cm.

[2 Marks]

(4)

Find the rate of change of height when height 'h'at an instant when slant height is 4 cm.

[2 Marks]

Question 3.

The equation of the path traced by a roller-coaster is given by the polynomial f(x) = a(x + 9)(x + 1)(x - 3). If the roller-coaster crosses y-axis at point (0, -1), answer the following:

(1) Find the value of a.

[2 Marks]

(2)

Find f (x) at x = 1.

[2 Marks]

Section B

Question 4.

If |A| = 2, where A is a 2x2 matrix, then $|4A^{-1}|$ equals:

[1 Marks]

(A) 8

(B) 1/32

(C) 4

(D) 2

Question 5. Let A be a 3x3 matrix such that |adj A| = 64. Then |A| is equal to:

[1 Marks]

(A) 8 only

(B) 8 only

(C) 64

(D) 8 or -8

Question 6.

[1 Marks]

(A)

(B)

(C)

(D)

Question 7.

If $d/dx(f(x)) = \log x$, then $f(x)$ equals:

[1 Marks]

(A) $-1/x + c$

(B) $x(\log x - 1) + C$

(C) $x(\log x + x) + C$

(D) $1/x + c$

Question 8.

[1 Marks]

(A) $1/\sqrt{3}$

(B) $-1/\sqrt{3}$

(C) $\sqrt{3}$

(D) $-\sqrt{3}$

Question 9.

The sum of the order and the degree of the differential equation $y \sin \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^3 = \sin y$ is:

[1 Marks]

(A) 2

(B) 5

(C) 4

(D) 3

Question 10.

The value of p for which the vectors $2\hat{i}+p\hat{j}+\hat{k}$ and $-4\hat{i}-6\hat{j}+26\hat{k}$ are perpendicular to each other, is:

[1 Marks]

(A) -3

(B) 3

(C) -17/3

(D) 17/3

Question 11.

The value of $(\hat{i}\times\hat{j})\cdot\hat{j}+(\hat{j}\times\hat{i})\cdot\hat{k}$ is

[1 Marks]

(A) 2

(B) 1

(C) -1

(D) 0

Question 12.

If $\vec{a}=\hat{i}$ and $\vec{b}=2\hat{i}-2\hat{j}+2\hat{k}$, then $|\vec{a}|$ equals

[1 Marks]

(A) √17

(B) √12

(C) √14

(D) 3

Question 13.

If P(A|B) = 0.3, P(A) = 0.4 and P(B) = 0.8, then P(B|A) is equal to:

[1 Marks]

(A) 0.4

(B) 0.06

(C) 0.3

(D) 0.6

Question 14.

[1 Marks]

(A) 11

(B) -11/4

(C) 11/4

(D) 4/11

Question 15.

[1 Marks]

(A) ±√7

(B) 25

(C) 0

(D) ±5

Question 16.

The general solution of the differential equation $x\,dy-(1+x^2)\,dx=dx$ is:

[1 Marks]

(A) $y = 2\log x + x^2/2 + C$

(B) $y = 2\log x + x^3/3 + C$

(C) $y = x^2/2 + C$

(D) $y = 2x + x^3/3 + C$

Question 17.

If $f(x) = a(x - \cos x)$ is strictly decreasing in R then 'a' belongs to

[1 Marks]

(A) $(-\infty, \infty)$

(B) $(-\infty, 0)$

(C) $(0, \infty)$

(D) $\{0\}$

Question 18. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then:

[1 Marks]

(A) z is maximum at (15, 20), minimum at (40, 15)

(B) z is maximum at (2, 72), minimum at (15, 20)

(C) z is maximum at (40, 15), minimum at (15, 20)

(D) z is maximum at (40, 15), minimum at (2, 72)

Question 19. The number of corner points of the feasible region determined by the constraints $x + y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is:

[1 Marks]

(A) 2

(B) 3

(C) 4

(D) 5

Question 20. Assertion (A) : The range of the function $f(x) = 2\sin^{-1} x + 3\pi/2$, where $x \in [-1, 1]$, is $[\pi/2, 5\pi/2]$. Reason (R) : The range of the principal value branch of $\sin^{-1}(x)$ is $[0,\pi]$.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is true and Reason (R) is false

(C) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(D) Assertion (A) is false and Reason (R) is true.

Question 21. Assertion (A) : Equation of a line passing through the points (1, 2, 3) and (3, -1, 3) is $x-3/2 = y+1/3 = z-3/0$. Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $x- x_1/x_2-x_1 = y-y_1/y_2-y_1 = z-z_1/z_2-z_1$

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is false and Reason (R) is true.

(C) Assertion (A) is true and Reason (R) is false.

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section C

Question 22. A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

[2 Marks]

Question 23.

Evaluate:

$\sin^{-1}(\sin 3\pi/4) + \cos^{-1}(\cos 3\pi/4) + \tan^{-1}(1)$.

[2 Marks]

Question 24.

Find all the vectors of magnitude $\sqrt{3}$ which are collinear to vector $\hat{i}+\hat{j}+\hat{k}$.

[2 Marks]

Question 25.

Check whether the lines given by $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.

[2 Marks]

Question 26.

If $y = (x + \sqrt{x^2-1})^2$, then show that $(x^2 - 1)(dy/dx)^2 = 4y^2$.

[2 Marks]

Question 27.

Show that the function $f(x) = 16 \sin x/4 + \cos x - x$ is strictly decreasing in $(\pi/2, \pi)$.

[2 Marks]

Section D

Question 28.

Evaluate :

[3 Marks]

Question 29.

Find:

$\int 1/\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)\,dx$

[3 Marks]

Question 30.

Find the particular solution of the differential equation $dy/dx + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$ given that $y(0) = 0$.

[3 Marks]

Question 31.

Solve the differential equation given by

$x\,dy - y\,dx - \sqrt{x^2 + y^2}\,dx = 0$

[3 Marks]

Question 32.

Solve graphically the linear programming problem:

Maximise $z = 6x + 3y$,

subject to constraints

$4x + y \geq 80$,

$3x + 2y \leq 150$,

$x + 5y \geq 115$,

$x \geq 0, y \geq 0$.

[3 Marks]

Question 33.

The probability distribution of random variable X is given below:

(i) Find the value of k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find $E(X)$, the mean of X.

[3 Marks]

Question 34.

A and B are independent events such that $P(A \cap \bar{B}) = 14$ and $P(\bar{A} \cap B) = 16$. Find $P(A)$ and $P(B)$.

[3 Marks]

Question 35.

Evaluate :

[1 Marks]

Question 36.

Find:

$\int 1/ \cos (x-a) \cos (x-b)\, dx$

[3 Marks]

Section E

Question 37. A relation R is defined on a set of real numbers as R = {(x, y) : x · y is an irrational number}. Check whether R is reflexive, symmetric and transitive or not.

[5 Marks]

Question 38.

If A

[5 Marks]

Question 39.

Solve the following system of equations by matrix method:

$x + 2y + 3z = 6,$

$2x - y + z = 2,$

$3x + 2y - 2z = 3.$

[5 Marks]

Question 40.

Find the vector and the Cartesian equations of a line passing through the point (1, 2, -4) and parallel to the line joining the points A (3, 3, -5) and B (1, 0, -11). Hence, find the distance between the two lines.

[5 Marks]

Question 41. Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.

[5 Marks]

Question 42.

Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and the x-axis, using integration.

[5 Marks]