CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours Maximum Marks : 90

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**
ii. This question paper is divided into **5 sections.**
iii. **Section A** – questions number **1 to 3** are case based questions
iv. **Section B** – questions number **4 to 22** are multiple choice questions
v. **Section C** – questions number **23 to 29** are very short answer
vi. **Section D** – questions number **30 to 38** are short answer
vii. **Section E** – questions number **39 to 44** are long answer
viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.
ix. Use of calculator is NOT allowed.

Section A

Question 1. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark. A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

(1) Express θ in terms of height of the camera installed on the pole and x.

[1 Marks]

(2) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

[2 Marks]

(3) Find dx/dθ.

[1 Marks]

(4)

If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is 3/101 rad/s, then find the speed of the car.

[3 Marks]

Question 2. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights. Assume that an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.

(1) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

[2 Marks]

(2) Find the probability that an airplane reached its destination late.

[2 Marks]

Question 3. If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f. The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function. Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

(1) If $\sin^{-1}(x)$ is defined from $[-1,1]$ to its principal value branch, find the value of $\sin^{-1}(1/2) - \sin^{-1}(1)$.

[1 Marks]

(2) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch.

[2 Marks]

(3) If A is the interval other than principal value branch, give an example of one such interval.

[1 Marks]

(4)

Find the domain and range of $f(x) = 2 \sin^{-1} (1 - x)$.

[2 Marks]

Section B

Question 4.

A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

[1 Marks]

(A) neither one-one nor onto

(B) onto but not one-one

(C) both one-one and onto

(D) one-one but not onto

Question 5.

If a matrix has 36 elements, the number of possible orders it can have is:

[1 Marks]

(A) 13

(B) 9

(C) 3

(D) 5

Question 6.

Which of the following statements is true for the function

[1 Marks]

(A) f(x) is continuous and differentiable $\forall x \in R$

(B) f(x) is continuous $\forall x \in R$

(C) f(x) is discontinuous at infinitely many points

(D) f(x) is continuous and differentiable $\forall x \in R-\{0\}$

Question 7.

Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if

[1 Marks]

(A) $f(x) > 0, \forall x \in (a, b)$

(B) $f'(x) = 0, \forall x \in (a, b)$

(C) $f'(x) > 0, \forall x \in (a, b)$

(D) $f'(x) < 0, \forall x \in (a, b)$

Question 8.

[1 Marks]

(A) 7

(B) 6

(C) 8

(D) 18

Question 9.

[1 Marks]

(A)

(B)

(C)

(D)

Question 10.

Let □ be the angle between two-unit vectors □□and □□such that sin□=3/5. Then, □□□is equal to:

[1 Marks]

(A) ± 3/5

(B) ± 3/4

(C) ± 4/5

(D) ± 4/3

Question 11.

The integrating factor of the differential equation $(1 - x^2)\, dy/dx + xy = ax, -1 < x < 1$, is:

[1 Marks]

(A) $1/x^2 - 1$

(B) $1/1-x^2$

(C) $1/\sqrt{x^2-1}$

(D) $1/\sqrt{1-x^2}$

Question 12.

If the direction cosines of a line are $\sqrt{3}k$, $\sqrt{3}k$, $\sqrt{3}k$ then the value of k is:

[1 Marks]

(A) ±1/3

(B) ±1

(C) $\pm\sqrt{3}$

(D) ±3

Question 13. A linear programming problem deals with the optimization of a/an:

[1 Marks]

(A) logarithmic function

(B) quadratic function

(C) linear function

(D) exponential function

Question 14. If P(A|B) = P(A'|B), then which of the following statements is true?

[1 Marks]

(A) P(A) = P(A')

(B) P(A) = 2 P(B)

(C) P(A ∩ B) = 2 P(B)

(D) P(A ∩ B) = ½ P(B)

Question 15.

[1 Marks]

(A) $2x^3$

(B) 0

(C) 2

(D) $2x^3 - 2$

Question 16.

The derivative of $\sin(x^2)$ with respect to x at $x = \sqrt{\pi}$ is:

[1 Marks]

(A) 1

(B) 2π

(C) $-2\sqrt{\pi}$

(D) -1

Question 17.

The order and degree of the differential equation $[1+(dy/dx)^2]^3=d^2y/dx^2$ respectively are:

[1 Marks]

(A) 2, 3

(B) 1, 2

(C) 2, 6

(D) 2, 1

Question 18.

The distance of point P(a, b, c) from y-axis is:

[1 Marks]

(A) b

(B) b^2

(C) $a^2 + c^2$

(D) $\sqrt{(a^2+c^2}$

Question 19. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is:

[1 Marks]

(A) 0

(B) 1

(C) 2

(D) 3

Question 20.

If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then:

[1 Marks]

(A) $AB = 0$

(B) $AB = BA$

(C) $AB = -BA$

(D) $BA = 0$

Question 21.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(C) Assertion (A) is false, but Reason (R) is true.

(D) Assertion (A) is true, but Reason (R) is false.

Question 22. Assertion (A) : A line in space cannot be drawn perpendicular to x, y and z axes simultaneously. Reason (R) : For any line making angles, α, β, γ, with the positive directions of x, y and z axes respectively, $\cos^2 a + \cos^2 b + \cos^2 g = 1$.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Assertion (A) is true, but Reason (R) is false

(C) Assertion (A) is false, but Reason (R) is true.

(D) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Section C

Question 23.

Check whether the function $f(x) = x^2 |x|$ is differentiable at $x = 0$ or not.

[2 Marks]

Question 24.

If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\, dy/dx = 1+y^4/4y$

[2 Marks]

Question 25.

Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

[2 Marks]

Question 26.

Find: $\int x\sqrt{1+2x}\, dx$

[2 Marks]

Question 27.

Evaluate:

[2 Marks]

Question 28.

If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b}) \perp \vec{a}$ and $(2\vec{a}+\vec{b}) \perp \vec{b}$, then prove that $|\vec{b}|=2\surd|\vec{a}|$

[2 Marks]

Question 29.

In the given figure, ABCD is a parallelogram. If $\vec{AB}=2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{DB}=3\hat{i}-6\hat{j}+2\hat{k}$, then find $\vec{AD}$ and hence find the area of parallelogram ABCD.

[2 Marks]

Section D

Question 30. A relation R on set A = {1, 2, 3, 4, 5} is defined as R = {(x, y) : $|x^2 - y^2| < 8$}. Check whether the relation R is reflexive, symmetric and transitive.

[3 Marks]

Question 31. A function f is defined from R → R as f(x) = ax + b, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

[3 Marks]

Question 32.

If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, prove that $dy/dx = \sqrt{1-y^2}/1-x^2$.

[3 Marks]

Question 33.

If $y = (\tan x)^{x}$, then find dy/dx.

[3 Marks]

Question 34.

Find: $\int x^2/\ x^2+4(x^2+9)\ dx$

[3 Marks]

Question 35.

Evaluate:

[3 Marks]

Question 36.

Find the particular solution of the differential equation given by $x^2 dy/dx - xy = x^2 \cos^2(y/2x)$, given that when $x = 1$, $y = \pi/2$

[3 Marks]

Question 37.

Solve the following linear programming problem graphically:

Maximise $z = 500x + 300y$,

subject to constraints

$x + 2y \leq 12$,

$2x + y \leq 12$,

$4x + 5y \geq 20$,

$x \geq 0, y \geq 0$.

[3 Marks]

Question 38.

[3 Marks]

Section E

Question 39.

system of equations : $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$

[5 Marks]

Question 40.

find the value of $(a + x) - (b + y)$.

[5 Marks]

Question 41.

Evaluate

[5 Marks]

Question 42.

Evaluate

[5 Marks]

Question 43.

Using integration, find the area of the ellipse $x^2/16+y^2/4 = 1$ included between the lines $x = -2$ and $x = 2$.

[5 Marks]

Question 44.

The image of point P(x, y, z) with respect to line $x/1 = y-1/2 = z-2/3$ is P(1, 0, 7). Find the coordinates of point P.

[5 Marks]