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Introduction to Coordinate Systems

A system of coordinates is a structured framework that enables us to use numbers to describe the exact physical locations of points or objects. The concept of grid-based thinking has deep historical roots in Bhārat, where ancient civilizations like the Sindhu-Sarasvatī used grids for urban planning. Early mathematicians such as Baudhāyana developed geometric constructions that laid the foundation for coordinate geometry.

Coordinate systems became crucial for navigation and astronomy, with scholars like Āryabhaṭa and Brahmagupta formalizing concepts such as zero and negative numbers, which are essential for the Cartesian plane. The coordinate system evolved through contributions from Arabic scholars and European mathematicians like René Descartes, who formalized the representation of points in a two-dimensional plane using two perpendicular axes.

This chapter introduces the coordinate system, enabling precise location of points and visualization of algebraic equations as geometric shapes.

Worked Illustration

Consider a room floor plan divided into a bedroom and a bathroom. Using a scale of 1 cm : 1 foot, pins mark key points, and threads connect corners of objects, allowing tactile understanding of positions.

Practice Set

- Describe the historical significance of coordinate systems in ancient civilizations.
- Explain the role of Brahmagupta in the development of the Cartesian plane.
- Identify the contributions of René Descartes to coordinate geometry.

Answer Key

- Ancient civilizations like Sindhu-Sarasvatī used grid systems for city planning, enabling precise location of places.
- Brahmagupta formalized zero and negative numbers, essential for defining the origin and negative axes in the Cartesian plane.
- René Descartes formalized the representation of points in a two-dimensional plane using two perpendicular axes.

Quick Reference

- Coordinate system: Framework to locate points using numbers.
- Origin: Point (0,0) where axes intersect.
- Axes: x-axis (horizontal), y-axis (vertical).
- Coordinates: Ordered pair (x, y) representing point location.

Glossary

- **Coordinate System:** A method to specify locations using numbers.
- **Origin:** The point where x-axis and y-axis intersect, denoted (0,0).
- **Axes:** The two perpendicular lines used to define the coordinate plane.
- **Coordinates:** The pair of numbers (x, y) that locate a point in the plane.

The Two-Dimensional Cartesian Coordinate System

The two-dimensional Cartesian coordinate system uses two perpendicular lines, the x-axis (horizontal) and the y-axis (vertical), intersecting at the origin O (0,0). This system allows us to locate any point in 2-D space using coordinates (x, y), where x is the distance from the y-axis and y is the distance from the x-axis.

Distances to the right of the origin or upwards are positive, while distances to the left or downwards are negative. Points on the axes have coordinates of the form (x, 0) or (0, y).

Formula Derivation

The coordinate plane is divided into four quadrants:

- First quadrant: (+, +)
- Second quadrant: (–, +)
- Third quadrant: (–, –)
- Fourth quadrant: (+, –)

Worked Illustration

Points on the axes:

- B = (4.5, 0) lies 4.5 units to the right on the x-axis.
- G = (0, –4.5) lies 4.5 units down on the y-axis.
- H = (0, 4) lies 4 units up on the y-axis.

Practice Set

- Plot points (3, 4), (–2, 5), (–3, –4), and (5, –2) on the Cartesian plane.

- Identify the quadrant for each point.
- Explain the sign of coordinates in each quadrant.

Answer Key

- (3, 4) lies in the first quadrant (+, +).
- (–2, 5) lies in the second quadrant (–, +).
- (–3, –4) lies in the third quadrant (–, –).
- (5, –2) lies in the fourth quadrant (+, –).

Quick Reference

- Origin: (0, 0)
- x-axis: Horizontal axis
- y-axis: Vertical axis
- Quadrants: Four parts of the plane divided by axes

Glossary

- **Quadrant:** One of the four regions in the coordinate plane.
- **Origin:** The point (0,0) where axes intersect.
- **x-axis:** Horizontal axis.
- **y-axis:** Vertical axis.

Distance Between Two Points in the Plane

To find the distance between two points in the coordinate plane, we use the Baudhāyana–Pythagoras theorem. If the points are not aligned parallel to the axes, we form a right triangle and calculate the hypotenuse.

Formula Derivation

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$. The horizontal distance is $|x_2 - x_1|$ and the vertical distance is $|y_2 - y_1|$.

By the Pythagorean theorem, the distance d between the points is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Worked Illustration

Find the distance between points A(3, 4) and D(7, 1):

- Horizontal distance: $7 - 3 = 4$
- Vertical distance: $4 - 1 = 3$
- Distance: $\sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$ units

Practice Set

- Calculate the distance between points (2, 3) and (5, 7).
- Find the distance between points (–1, –2) and (3, 4).
- Determine the distance between points (–3, 5) and (4, –1).

Answer Key

- Distance between (2, 3) and (5, 7): $\sqrt{(5-2)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = 5$
- Distance between (–1, –2) and (3, 4): $\sqrt{(3+1)^2 + (4+2)^2} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52}$
- Distance between (–3, 5) and (4, –1): $\sqrt{(4+3)^2 + (-1-5)^2} = \sqrt{7^2 + (-6)^2} = \sqrt{49 + 36} = \sqrt{85}$

Quick Reference

- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Horizontal distance: $|x_2 - x_1|$
- Vertical distance: $|y_2 - y_1|$

Glossary

- **Distance:** The length between two points in the plane.
- **Baudhāyana–Pythagoras theorem:** A theorem used to calculate the hypotenuse of a right triangle.
- **Coordinates:** Ordered pairs representing points.