

Table of Contents

1. Introduction to Linear Polynomials

Introduction to Linear Polynomials

In this chapter, we focus on algebraic expressions involving one variable and their special types called linear polynomials. We begin by understanding algebraic expressions as sums of terms involving variables and constants, and then explore linear polynomials which are polynomials of degree one.

Concept Explanation

An algebraic expression consists of terms which are products of numbers (coefficients) and variables raised to powers. A linear polynomial is a polynomial of degree 1, meaning the highest power of the variable is 1. For example, $(2x + 3)$ is a linear polynomial in variable (x) .

Formula Derivation

The general form of a linear polynomial in one variable (x) is:

$$y = ax + b$$

where (a) and (b) are constants, $(a \neq 0)$. The degree of the polynomial is 1 because the highest power of (x) is 1.

Worked Illustrations

Example 1: Total pens and pencils bought by Raju can be expressed as $(4x + 5y + 3)$, where (x) and (y) are variables representing the number of red and blue boxes respectively, and 3 is a constant.

Example 2: Total cost of fencing and decorating a rectangular garden is $(200l + 160w + 50lw)$, an algebraic expression involving two variables (l) and (w) .

Example 3: Area of rectangles formed by bending a wire of length 20 cm is $(10x - x^2)$, a quadratic polynomial in one variable (x) .

Solved Examples

Example 6: Solve the linear equation $(2x + 10 = 64)$ to find two numbers whose sum is 64 and one is 10 more than the other.

Step 1: Let the smaller number be (x) , then the larger is $(x + 10)$.

Step 2: Write the equation $(x + (x + 10) = 64)$.

Step 3: Simplify to $(2x + 10 = 64)$.

Step 4: Solve for (x) :

$$[2x = 64 - 10 = 54 \quad x = \frac{54}{2} = 27]$$

Step 5: The numbers are 27 and 37.

Practice Set

• Level 1 - Easy

- Write the linear polynomial for the perimeter of a square with side (x) .
- Evaluate $(2x + 3)$ for $(x = 5)$.
- Find the total cost if (m) matches are played and the cost is $(200 + 50m)$.

• Level 2 - Moderate

- Find the number of matches played if the total cost is ₹750 in the chess club example.
- Write the linear polynomial for the number of square tiles at stage (n) given by $(2n - 1)$.
- Calculate the fare for 10 km travel if the fare is $(25 + 15(n - 2))$ for $(n \geq 2)$.

• Level 3 - Challenging

- Find the linear equation $(y = ax + b)$ given two points $((10, 350))$ and $((20, 550))$.
- Plot the graph of $(y = 2x + 1)$ using points $((0, 1))$ and $((3, 7))$.
- Determine the number of days Bela will be left with ₹40 if she spends ₹5 daily from ₹100.

Answer Key

- Perimeter of square: $(4x)$.
- Value of $(2x + 3)$ at $(x=5)$: $(2(5) + 3 = 13)$.
- Matches played for ₹750: $(200 + 50m = 750 \Rightarrow 50m = 550 \Rightarrow m = 11)$.
- Number of tiles at stage (n) : $(2n - 1)$.
- Fare for 10 km: $(25 + 15(10 - 2) = 25 + 120 = 145)$.
- Linear equation from points: $(y = 20x + 150)$.
- Graph points: $((0, 1))$ and $((3, 7))$ lie on $(y = 2x + 1)$.
- Bela's days left with ₹40: $(100 - 5n = 40 \Rightarrow 5n = 60 \Rightarrow n = 12)$.

Quick Reference

Term	Definition
Linear Polynomial	Polynomial of degree 1, e.g., $(ax + b)$

Degree	Highest power of the variable in a polynomial
Coefficient	Number multiplying the variable
Constant	Term without variable
Slope	Rate of change of y with respect to x in $y = ax + b$
Y-intercept	Value of y when $x=0$, point where line crosses y-axis
Linear Growth	Quantity increases by a constant amount over equal intervals
Linear Decay	Quantity decreases by a constant amount over equal intervals
Linear Relationship	Relationship between two variables expressed as $y = ax + b$

Glossary

- **Algebraic Expression:** Combination of numbers, variables, and operations.
- **Polynomial:** Algebraic expression with variables raised to whole number powers.
- **Linear Polynomial:** Polynomial of degree one.
- **Degree:** Highest power of the variable in a polynomial.
- **Coefficient:** Number multiplying the variable in a term.
- **Constant:** Term without a variable.
- **Slope:** Measure of steepness of a line in coordinate geometry.
- **Y-intercept:** Point where a line crosses the y-axis.
- **Linear Growth:** Increase by a fixed amount over equal intervals.
- **Linear Decay:** Decrease by a fixed amount over equal intervals.
- **Linear Relationship:** Relationship between two variables represented by a straight line.