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Introduction to Circles

Circles are fundamental geometric shapes observed in nature and everyday life. A circle is defined as the set of all points on a plane that are equidistant from a fixed point called the centre. The constant distance from the centre to any point on the circle is called the radius.

Definition and Basic Properties

A **circle** is the locus of points equidistant from a given point (the centre) on a plane. The distance from the centre to any point on the circle is the **radius**.

A **chord** is a line segment joining two points on the circle. A chord passing through the centre is called the **diameter**, which is the longest chord of the circle.

Worked Illustration

Consider a circle with centre A and radius AB. Points B and C lie on the circle, and BC is a chord. The angle subtended by chord BC at the centre is angle BAC.

Practice Set

- Level 1: Identify the radius, diameter, and chords in given circles.
- Level 2: Draw circles with given radius and mark chords and diameters.
- Level 3: Prove that the diameter is the longest chord in a circle.

Answer Key

- Level 1: Radius is the segment from centre to any point on the circle; diameter passes through the centre; chords connect two points on the circle.
- Level 2: Construction steps with compass and ruler.
- Level 3: Use triangle inequality and properties of chords.

Quick Reference

- Circle: Locus of points equidistant from centre.
- Radius: Distance from centre to circle.
- Chord: Segment joining two points on circle.
- Diameter: Chord passing through centre.

Glossary

- **Centre:** Fixed point from which all points on the circle are equidistant.
- **Radius:** Distance from centre to any point on the circle.
- **Chord:** Line segment joining two points on the circle.
- **Diameter:** Chord passing through the centre.

Symmetry of Circles

Circles exhibit perfect symmetry. They have infinite lines of reflection symmetry passing through the centre, each line being a diameter. Circles also have rotational symmetry about the centre for any angle of rotation.

Concept Explanation

Rotating a circle about its centre by any angle results in the same circle. Folding a circle along any diameter results in two overlapping halves, showing reflection symmetry.

Practice Set

- Level 1: Identify lines of symmetry in circles and other polygons.
- Level 2: Compare rotational symmetries of circles and regular polygons.
- Level 3: Prove that all diameters are lines of symmetry for a circle.

Answer Key

- Level 1: Circle has infinite lines of symmetry; square has 4; pentagon has 5.
- Level 2: Circle has infinite rotational symmetry; polygons have finite.
- Level 3: Use folding and congruence arguments.

Quick Reference

- Reflection symmetry: Line divides figure into mirror images.
- Rotational symmetry: Figure looks the same after rotation.

Glossary

- **Reflection Symmetry:** Symmetry about a line (axis).
- **Rotational Symmetry:** Symmetry about a point for rotations.

Circles Through Given Points

Given points on a plane, circles can be drawn passing through them under certain conditions.

Circles Through Two Points

Infinite circles can pass through two distinct points A and B. The centres of these circles lie on the perpendicular bisector of segment AB. The smallest such circle has the midpoint of AB as centre and radius half of AB (diameter AB).

Circles Through Three Points

For three non-collinear points A, B, and C, there exists a unique circle passing through all three. The centre of this circle, called the **circumcentre**, is the intersection of the perpendicular bisectors of the sides of triangle ABC. The circle is called the **circumcircle**.

Worked Illustration

Construct triangle ABC. Find midpoints of sides and draw perpendicular bisectors. Their intersection O is the circumcentre. Draw circle with centre O and radius OA passing through A, B, and C.

Practice Set

- Level 1: Draw circles through two given points and identify centres.
- Level 2: Construct circumcircle of a given triangle.
- Level 3: Prove uniqueness of circumcircle for non-collinear points.

Answer Key

- Level 1: Centres lie on perpendicular bisector; radius varies.
- Level 2: Use compass and ruler to construct perpendicular bisectors and circumcircle.
- Level 3: Use intersection properties of lines and congruence.

Quick Reference

- Perpendicular bisector: Locus of points equidistant from segment endpoints.
- Circumcentre: Intersection of perpendicular bisectors of triangle sides.
- Circumcircle: Circle passing through all vertices of a triangle.

Glossary

- **Perpendicular Bisector:** Line dividing a segment into two equal parts at right angles.
- **Circumcentre:** Centre of circumcircle of a triangle.
- **Circumcircle:** Circle passing through all vertices of a triangle.

Properties of Chords

Chords are line segments joining two points on a circle. They have several important properties related to their lengths, angles subtended, and distances from the centre.

Theorem 1: Equal Chords Subtend Equal Angles at the Centre

If two chords of a circle are equal in length, then the angles they subtend at the centre are equal.

Proof

Let AB and DE be equal chords. Join the centre C to points A, B, D, and E. Triangles CAB and CDE are congruent by SSS (radii and chord lengths). Hence, angles subtended at C by AB and DE are equal.

Theorem 2: Chords Subtending Equal Angles at the Centre are Equal

If two chords subtend equal angles at the centre, then the chords are equal in length.

Proof

Given equal angles subtended by chords AB and DE at centre C, triangles CAB and CDE are congruent by SAS. Hence, AB = DE.

Theorem 3: Line from Centre to Midpoint of Chord is Perpendicular to Chord

The line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

Proof

Let M be the midpoint of chord AB. Triangles CMA and CMB are congruent by SAS, so angles at M are equal and sum to 180 degrees, implying CM is perpendicular to AB.

Theorem 4: Perpendicular from Centre to Chord Bisects the Chord

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Theorem 5: Chords of Equal Length are Equidistant from the Centre

Chords of equal length lie at the same perpendicular distance from the centre.

Theorem 6: Longer Chord is Closer to the Centre

Among two unequal chords, the longer chord is nearer to the centre of the circle.

Practice Set

- Level 1: Identify equal chords and their angles subtended at the centre.
- Level 2: Prove perpendicularity of line from centre to chord midpoint.
- Level 3: Compare distances of chords from centre and prove related theorems.

Answer Key

- Level 1: Use congruence of triangles and properties of radii.
- Level 2: Use triangle congruence and angle sum properties.
- Level 3: Apply Pythagoras theorem and congruence criteria.

Quick Reference

- Equal chords subtend equal angles at centre.
- Line from centre to chord midpoint is perpendicular.
- Perpendicular from centre bisects chord.
- Equal chords are equidistant from centre.
- Longer chord is closer to centre.

Glossary

- **Chord:** Line segment joining two points on a circle.
- **Radius:** Segment from centre to circle.
- **Perpendicular Bisector:** Line perpendicular to a segment at its midpoint.
- **Congruent Triangles:** Triangles with equal corresponding sides and angles.

Arcs and Angles in Circles

An arc is a connected portion of a circle defined by two points on the circle. Arcs can be major or minor depending on their length.

Angle Subtended by an Arc at the Centre

The angle subtended by an arc at the centre is the angle formed by the two radii joining the centre to the endpoints of the arc.

Angle Subtended by an Arc at a Point on the Circle

The angle subtended by an arc at any point on the circle outside the arc is half the angle subtended at the centre by the same arc.

Theorem: Angle at Centre is Double the Angle at Circumference

For an arc AB, the angle subtended at the centre is twice the angle subtended at any point on the circle outside the arc.

Corollary: Angle Subtended by Diameter is 90 degrees

The angle subtended by a diameter at any point on the circle is a right angle.

Practice Set

- Level 1: Identify major and minor arcs and measure angles subtended.
- Level 2: Prove the angle at centre is double the angle at circumference.
- Level 3: Apply the corollary to solve problems involving diameters and right angles.

Answer Key

- Level 1: Use protractor to measure angles and identify arcs.
- Level 2: Use isosceles triangle properties and exterior angle theorem.
- Level 3: Use theorem and corollary to find unknown angles.

Quick Reference

- Arc: Part of circle between two points.
- Minor arc: Smaller arc between two points.
- Major arc: Larger arc between two points.
- Angle at centre: Angle between radii to arc endpoints.
- Angle at circumference: Half the angle at centre.

Glossary

- **Arc:** Connected part of a circle between two points.
- **Minor Arc:** Smaller arc between two points.
- **Major Arc:** Larger arc between two points.
- **Central Angle:** Angle subtended at centre by an arc.
- **Inscribed Angle:** Angle subtended at a point on the circle by an arc.

Cyclic Quadrilaterals

A quadrilateral is cyclic if all its vertices lie on a single circle.

Theorem 1: Equal Angles Subtended by a Chord at Two Points Implies Concyclicity

If a chord AB subtends equal angles at two points C and D on the same side of AB, then points A, B, C, D lie on the same circle.

Theorem 2: Opposite Angles of a Cyclic Quadrilateral Sum to 180 degrees

In a cyclic quadrilateral ABCD, the sum of opposite angles is 180 degrees, i.e., angle A plus angle C equals 180 degrees and angle B plus angle D equals 180 degrees.

Theorem 3: Converse of Opposite Angles Theorem

If in a quadrilateral, the sum of a pair of opposite angles is 180 degrees, then the quadrilateral is cyclic.

Practice Set

- Level 1: Identify cyclic quadrilaterals from diagrams.
- Level 2: Prove that a given quadrilateral is cyclic using angle properties.
- Level 3: Solve problems involving cyclic quadrilaterals and angle sums.

Answer Key

- Level 1: Use angle measurements and circle properties.
- Level 2: Use theorems on angles subtended by chords and cyclicity.
- Level 3: Apply angle sum properties and cyclic quadrilateral criteria.

Quick Reference

- Cyclic Quadrilateral: Quadrilateral inscribed in a circle.
- Opposite angles sum to 180 degrees.
- Equal angles subtended by chord imply concyclicity.

Glossary

- **Cyclic Quadrilateral:** Quadrilateral with vertices on a circle.
- **Concyclic Points:** Points lying on the same circle.
- **Opposite Angles:** Angles across from each other in a quadrilateral.