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Relay Race and Staggered Starts

A 4 d7 100 meter relay race involves teams of four runners, each running 100 meters in sequence. Athletes start at staggered positions in their lanes to ensure fairness because outer lanes have longer curved paths. The stagger is the distance between starting points of adjacent lanes, calculated to equalize the total running distance for all lanes.

Concept Explanation

Each runner runs 100 meters, passing a baton to the next runner. The stagger compensates for the extra distance on the curved track in outer lanes.

Formula Derivation

The stagger length is based on the difference in arc lengths of semicircular curves of different radii corresponding to each lane.

Worked Illustration

For a 400 m track with two semicircular ends of radius 36.5 m and lane width 1.22 m, the stagger for the second lane is calculated by the difference in circumference of circles with radii 36.5 m and 36.5 + 1.22 m.

Solved Example

Calculate the stagger between the first and second lanes on a 400 m track with lane width 1.22 m.

Step 1: Radius of first lane semicircle = 36.5 m

Step 2: Radius of second lane semicircle = 36.5 + 1.22 = 37.72 m

Step 3: Difference in circumference = 2 pi (37.72) - 2 pi (36.5) = 2 pi (1.22) = 7.66 m

Therefore, the stagger is 7.66 meters.

Practice Set

- Level 1: Define stagger in a relay race.
- Level 2: Calculate stagger for a 200 m track with lane width 1 m.
- Level 3: Derive a general formula for stagger length for lane n on a track with lane width w and inner radius r.

Answer Key

- Level 1: Stagger is the distance between starting points of adjacent lanes to equalize running distance.
- Level 2: Stagger = 2 pi d7 1 d7 (n-1) = 2 pi d7 1 = 6.28 m for second lane.
- Level 3: Stagger for lane n = 2 pi w (n-1).

Quick Reference

Stagger length = difference in arc lengths = 2 pi d7 lane width d7 (lane number - 1)

Glossary

- **Stagger:** The offset distance between starting points of adjacent lanes.
- **Lane width:** The width of each lane on the track.
- **Arc length:** The length of a curved segment of a circle.

Perimeter of Shapes

The perimeter of a shape is the total length around its boundary. For polygons, it is the sum of the lengths of all sides.

Concept Explanation

Perimeter is the distance traveled by a point moving around the shape once.

Formula Derivation

- Square with side a: Perimeter = 4a
- Equilateral triangle with side a: Perimeter = 3a
- Rectangle with length a and width b: Perimeter = 2(a + b)

Worked Illustrations

Examples of calculating perimeter for square, equilateral triangle, and rectangle.

Solved Examples

Calculate the perimeter of a rectangle with length 5 units and width 3 units.

Perimeter = 2(5 + 3) = 2(8) = 16 units.

Practice Set

- Level 1: Find the perimeter of a square with side 7 units.
- Level 2: Find the perimeter of an equilateral triangle with side 9 units.
- Level 3: Find the perimeter of a rectangle with length 12 units and width 5 units.

Answer Key

- Level 1: 28 units
- Level 2: 27 units
- Level 3: 34 units

Quick Reference

Perimeter formulas: Square = 4a, Equilateral triangle = 3a, Rectangle = 2(a + b)

Glossary

- **Perimeter:** Total length around a shape.

- **Side length:** Length of one side of a polygon.

Circumference of a Circle

The circumference is the perimeter of a circle, the total distance around the circle.

Concept Explanation

For a circle with radius r, the circumference C is proportional to the diameter d = 2r.

Formula Derivation

The ratio of circumference to diameter is constant for all circles, denoted by pi:

C = pi d = 2 pi r

Worked Illustrations

For circles of different sizes, the ratio C/d = pi remains constant.

Solved Examples

Find the circumference of a circle with radius 7 units.

C = 2 pi r = 2 d7 3.1416 d7 7 = 43.9824 units

Practice Set

- Level 1: Find circumference of circle with radius 5 units.
- Level 2: Find circumference of circle with diameter 20 units.
- Level 3: Derive the formula for circumference using the concept of pi.

Answer Key

- Level 1: 31.416 units
- Level 2: 62.832 units
- Level 3: C = pi d = 2 pi r

Quick Reference

Circumference C = 2 pi r

Glossary

- **Circumference:** Perimeter of a circle.
- **Radius:** Distance from center to any point on the circle.
- **Diameter:** Twice the radius, distance across the circle through the center.
- **Pi (pi):** Constant ratio of circumference to diameter, approximately 3.1416.

Arc Length of a Circle

The arc length is the length of a portion of the circumference subtended by a central angle.

Concept Explanation

For a circle of radius r, an arc subtending an angle theta degrees at the center has length proportional to theta/360.

Formula Derivation

Arc length l = 2 pi r d7 theta/360

Worked Illustrations

Calculate arc length for r = 10 units and theta = 90 degrees:

l = 2 pi d7 10 d7 90/360 = 5 pi b1 15.708 units.

Solved Examples

Find the length of an arc subtending 60 degrees in a circle of radius 12 units.

l = 2 pi d7 12 d7 60/360 = 4 pi b1 12.566 units.

Practice Set

- Level 1: Find arc length for radius 7 units and angle 45 degrees.
- Level 2: Find arc length for radius 15 units and angle 120 degrees.
- Level 3: Prove the arc length formula using proportionality.

Answer Key

- Level 1: l = 2 pi d7 7 d7 45/360 = 7 pi / 4 b1 5.498 units.
- Level 2: l = 2 pi d7 15 d7 120/360 = 10 pi b1 31.416 units.
- Level 3: Arc length is proportional to central angle; full circumference corresponds to 360 degrees.

Quick Reference

Arc length l = 2 pi r d7 theta/360

Glossary

- **Arc:** Part of the circumference between two points.
- **Central angle:** Angle subtended at the center by the arc.

Area of Polygons

Area measures the amount of space enclosed by a polygon.

Concept Explanation

Area is measured in square units, based on unit squares.

Formula Derivation

- Square with side a: Area = a squared
- Rectangle with length a and width b: Area = ab
- Parallelogram with base b and height h: Area = bh
- Triangle with base b and height h: Area = 1/2 bh

Worked Illustrations

Transforming parallelograms into rectangles to find area.

Solved Examples

Find area of parallelogram with base 8 units and height 5 units.

Area = 8 d7 5 = 40 square units.

Practice Set

- Level 1: Find area of square with side 6 units.
- Level 2: Find area of triangle with base 10 units and height 7 units.
- Level 3: Find area of parallelogram with base 12 units and height 9 units.

Answer Key

- Level 1: 36 sq. units
- Level 2: 35 sq. units
- Level 3: 108 sq. units

Quick Reference

Area formulas: Square = a squared, Rectangle = ab, Parallelogram = bh, Triangle = 1/2 bh

Glossary

- **Area:** Space enclosed by a shape.
- **Base:** A chosen side of a polygon, often used with height.
- **Height:** Perpendicular distance from base to opposite vertex or side.

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Area of Triangles

The area of a triangle can be found using base and height or Heron's formula.

Concept Explanation

Area = 1/2 d7 base d7 height

Formula Derivation

Two congruent triangles form a parallelogram, so area of triangle is half the area of parallelogram.

Worked Illustrations

Using right triangles and general triangles to find area.

Solved Examples

Find area of triangle with sides 3, 4, 5 units using Heron's formula.

Semi-perimeter s = (3+4+5)/2 = 6

Area = square root of s(s-3)(s-4)(s-5) = square root of 6 d7 3 d7 2 d7 1 = square root of 36 = 6 sq. units.

Practice Set

- Level 1: Find area of triangle with base 8 units and height 5 units.
- Level 2: Use Heron's formula to find area of triangle with sides 7, 8, 9 units.
- Level 3: Prove the median divides a triangle into two equal areas.

Answer Key

- Level 1: 20 sq. units
- Level 2: Semi-perimeter s=12, Area = square root of 12(12-7)(12-8)(12-9) = square root of 12 d7 5 d7 4 d7 3 = square root of 720 b1 26.83 sq. units
- Level 3: Theorem states median divides triangle into two equal areas by equal base segments and equal height.

Quick Reference

Area of triangle = 1/2 bh, Heron's formula = square root of s(s-a)(s-b)(s-c)

Glossary

- **Heron's formula:** Formula to find area from side lengths.
- **Median:** Line segment from vertex to midpoint of opposite side.

Area of Cyclic Quadrilaterals

A cyclic quadrilateral is a four-sided figure with all vertices on a circle.

Concept Explanation

Area depends on side lengths and semi-perimeter.

Formula Derivation

Brahmagupta's formula for cyclic quadrilateral with sides a, b, c, d and semi-perimeter s = (a+b+c+d)/2:

Area = square root of (s-a)(s-b)(s-c)(s-d)

Worked Illustrations

Verification for rectangles and isosceles trapeziums.

Solved Examples

Calculate area of cyclic quadrilateral with sides 3, 4, 5, 6 units.

Semi-perimeter s = (3+4+5+6)/2 = 9

Area = square root of (9-3)(9-4)(9-5)(9-6) = square root of 6 d7 5 d7 4 d7 3 = square root of 360 b1 18.97 sq. units.

Practice Set

- Level 1: Define cyclic quadrilateral.
- Level 2: Use Brahmagupta's formula for given sides.
- Level 3: Prove Brahmagupta's formula generalizes Heron's formula.

Answer Key

- Level 1: Quadrilateral with vertices on a circle.

- Level 2: Calculate area using formula.
- Level 3: Show setting one side to zero reduces formula to Heron's formula.

Quick Reference

Brahmagupta's formula: square root of (s-a)(s-b)(s-c)(s-d)

Glossary

- **Cyclic quadrilateral:** Quadrilateral inscribed in a circle.
- **Semi-perimeter:** Half the perimeter.

Area of a Circle

The area of a circle is the space enclosed by its circumference.

Concept Explanation

Area is proportional to the square of the radius.

Formula Derivation

Area A = pi r squared, where r is the radius.

Archimedes showed area equals half the product of circumference and radius:

A = 1/2 d7 C d7 r = 1/2 d7 2 pi r d7 r = pi r squared

Worked Illustrations

Visualizing circle slices rearranged into parallelogram shape to understand area formula.

Solved Examples

Find area of circle with radius 10 units.

A = pi d7 10 squared = 100 pi b1 314.16 sq. units

Practice Set

- Level 1: Find area of circle with radius 7 units.
- Level 2: Calculate area of semicircle with radius 14 units.
- Level 3: Derive area formula using limit of regular polygons.

Answer Key

- Level 1: 49 pi b1 153.94 sq. units
- Level 2: 1/2 pi d7 14 squared = 98 pi b1 307.88 sq. units
- Level 3: Use polygon approximation and limit process.

Quick Reference

Area of circle = pi r squared

Glossary

Prepzy

- **Pi (pi):** Ratio of circumference to diameter.
- **Radius:** Distance from center to circle edge.

Area of Sectors and Segments of a Circle

A sector is the region bounded by two radii and the arc between them. A segment is the region bounded by an arc and its chord.

Concept Explanation

Area of sector is proportional to the central angle.

Formula Derivation

Area of sector with central angle theta degrees:

Area = pi r squared d7 theta/360

Worked Illustrations

Area of semicircle (theta = 180 degrees) is half the area of the circle.

Area of quarter circle (theta = 90 degrees) is one-fourth the area of the circle.

Solved Examples

Find area of sector with radius 6 units and angle 60 degrees.

Area = pi d7 6 squared d7 60/360 = 6 pi b1 18.85 sq. units

Practice Set

- Level 1: Find area of sector with radius 5 units and angle 90 degrees.
- Level 2: Find area of sector with radius 10 units and angle 45 degrees.
- Level 3: Calculate area of segment given radius and chord length.

Answer Key

- Level 1: 1/4 pi d7 5 squared = 25 pi / 4 b1 19.63 sq. units
- Level 2: pi d7 10 squared d7 45/360 = 100 pi / 8 = 12.5 pi b1 39.27 sq. units
- Level 3: Use sector area minus triangle area formulas.

Quick Reference

Area of sector = pi r squared d7 theta/360

Glossary

- **Sector:** Region bounded by two radii and an arc.
- **Segment:** Region bounded by an arc and its chord.