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Prepzy

Introduction

Overview of Rectilinear Motion

- Motion along a straight line is called rectilinear motion.
- Objects can be approximated as point objects if their size is negligible compared to the distance moved.
- Kinematics studies motion description without considering causes.

Solved Examples

- **Ex 1:** An object moves from point A to B through C. Find the path length if $AC = 5 \text{ m}$ and $CB = 3 \text{ m}$.

Solution: Path length = $AC + CB = 5 + 3 = 8 \text{ m}$.

Practice Set

- Q1: What is rectilinear motion?
 - A) Motion in a circle
 - B) Motion along a straight line
 - C) Motion in a plane
 - D) Random motion
- Q2: When can an object be treated as a point object?
 - A) When its size is larger than distance moved
 - B) When its size is negligible compared to distance moved
 - C) Always
 - D) Never
- Q3: Kinematics studies
 - A) Causes of motion
 - B) Effects of forces
 - C) Description of motion without causes
 - D) Energy changes

Answer Key

- Q1: B - Rectilinear motion is motion along a straight line.
- Q2: B - Point object approximation is valid when size is much smaller than distance moved.
- Q3: C - Kinematics describes motion without considering causes.

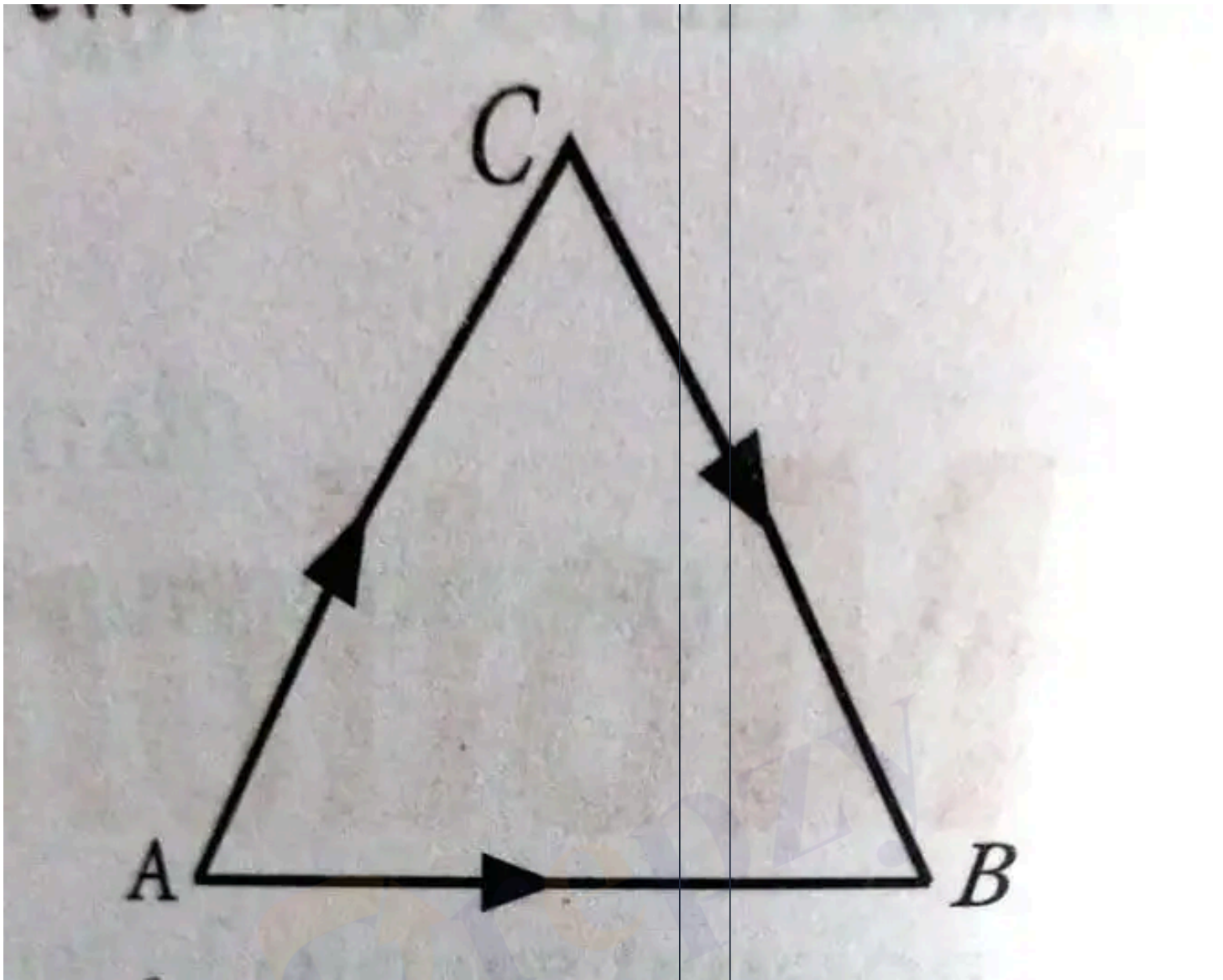
Memory Aids

- Rectilinear = straight line motion.
- Point object = size negligible vs distance.
- Kinematics = motion description only.

Position, Path Length and Displacement

Key Concepts

- Position: Location of an object relative to a reference point and axes.
- Motion: Change in one or more coordinates with time; otherwise, object is at rest.
- Path length (distance): Total length of actual path traveled; scalar quantity, always positive or zero.
- Displacement: Shortest straight-line distance from initial to final position; vector quantity with magnitude and direction.
- Displacement characteristics:
 - Units of length (meters).
 - Can be positive, negative, or zero.
 - Independent of origin choice.
 - Magnitude $\leq$ actual distance traveled.
 - Does not indicate path shape.
 - Shortest distance between two points.



Solved Examples

- **Ex 1:** Calculate displacement if object moves from A to B directly, distance $AB = 6$ m.
Solution: Displacement = $AB = 6$ m (vector).
- **Ex 2:** Find path length if object moves A to C = 4 m, C to B = 3 m.
Solution: Path length = $AC + CB = 4 + 3 = 7$ m.

Practice Set

- Q1: Displacement is a
A) Scalar quantity
B) Vector quantity
C) Always positive
D) None of these

- Q2: Path length is
 - A) Shortest distance
 - B) Total distance traveled
 - C) Vector quantity
 - D) Negative quantity
- Q3: Which is true about displacement?
 - A) Depends on origin
 - B) Can be negative
 - C) Always greater than distance
 - D) Indicates path shape

Answer Key

- Q1: B - Displacement has magnitude and direction, so vector.
- Q2: B - Path length is total distance traveled, scalar.
- Q3: B - Displacement can be positive, negative, or zero and is independent of origin.

Memory Aids

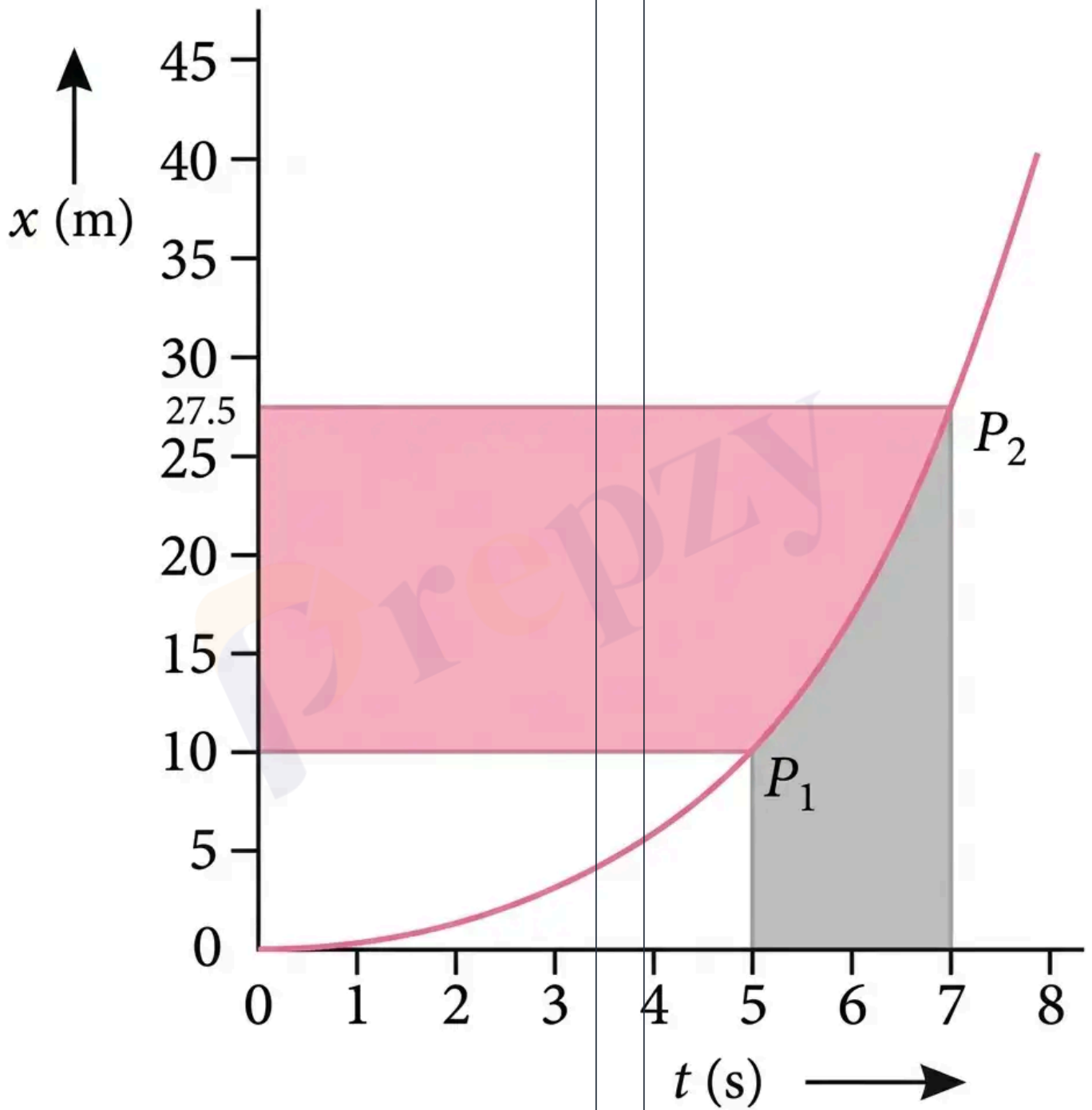
- Distance = scalar, path length actual traveled.
- Displacement = vector, shortest path.
- Displacement sign shows direction.

Average Velocity and Average Speed

Key Concepts

- Average velocity: Displacement divided by time interval.
 - Formula: $\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$
 - Graphically, slope of line joining initial and final positions on x-t graph.
 - Can be positive, negative, or zero.
- Average speed: Total path length divided by total time.
 - Formula: $v_{av} = \frac{\text{Total path length}}{\text{Total time}}$
 - For straight line motion, average speed = average velocity.
 - For varying speeds and distances:

- $v_{av} = \frac{s_1+s_2+s_3+\dots}{t_1+t_2+t_3+\dots}$
- or $v_{av} = \frac{s_1+s_2+s_3}{\frac{s_1}{v_1}+\frac{s_2}{v_2}+\frac{s_3}{v_3}}$
- or $v_{av} = \frac{v_1t_1+v_2t_2+v_3t_3}{t_1+t_2+t_3}$



Solved Examples

- **Ex 1:** Calculate average velocity between $t = 5s$ and $t = 7s$ if $x_1 = 10m$, $x_2 = 27.5m$.
Solution: $\bar{v} = \frac{27.5-10}{7-5} = 8.75 \text{ m/s}$.

Practice Set

- Q1: Average velocity is
A) Total distance / time
B) Displacement / time
C) Speed at an instant
D) None
- Q2: Average speed for straight line motion equals
A) Instantaneous speed
B) Average velocity
C) Zero
D) Displacement
- Q3: If a body covers distances 10 m, 20 m with speeds 5 m/s, 10 m/s respectively, average speed is
A) 7.5 m/s
B) 8 m/s
C) 6.67 m/s
D) 15 m/s

Answer Key

- Q1: B - Average velocity = displacement/time.
- Q2: B - For straight line motion, average speed = average velocity.
- Q3: C - Using formula $v_{av} = \frac{s_1+s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} = \frac{30}{\frac{10}{5} + \frac{20}{10}} = 6.67 \text{ m/s}$.

Memory Aids

- Average velocity = slope on x-t graph.
- Average speed = total distance / total time.
- Use weighted averages for varying speeds.

Instantaneous Velocity and Speed

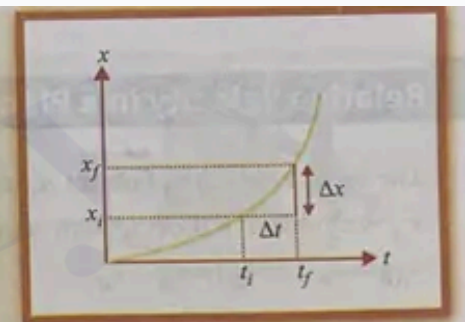
Key Concepts

- Instantaneous velocity: Rate of change of position at an instant.
 - Mathematically: $v_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
 - Graphically: Slope of position-time curve at a point.
- Instantaneous speed: Magnitude of instantaneous velocity.
 - Mathematically: Instantaneous speed = $\frac{dx}{dt}$
 - Speed at a specific moment.
- For uniform motion, instantaneous velocity equals average velocity at all times.

- **Instantaneous velocity**: The rate of change of position with time at any instant is called instantaneous velocity. Here, change of time (Δt) is infinitesimally small.

$$\text{i.e., } v_{inst} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Graphically, we can say it is the slope of $x - t$ curve at any instant (i.e., $\Delta t \rightarrow 0$).



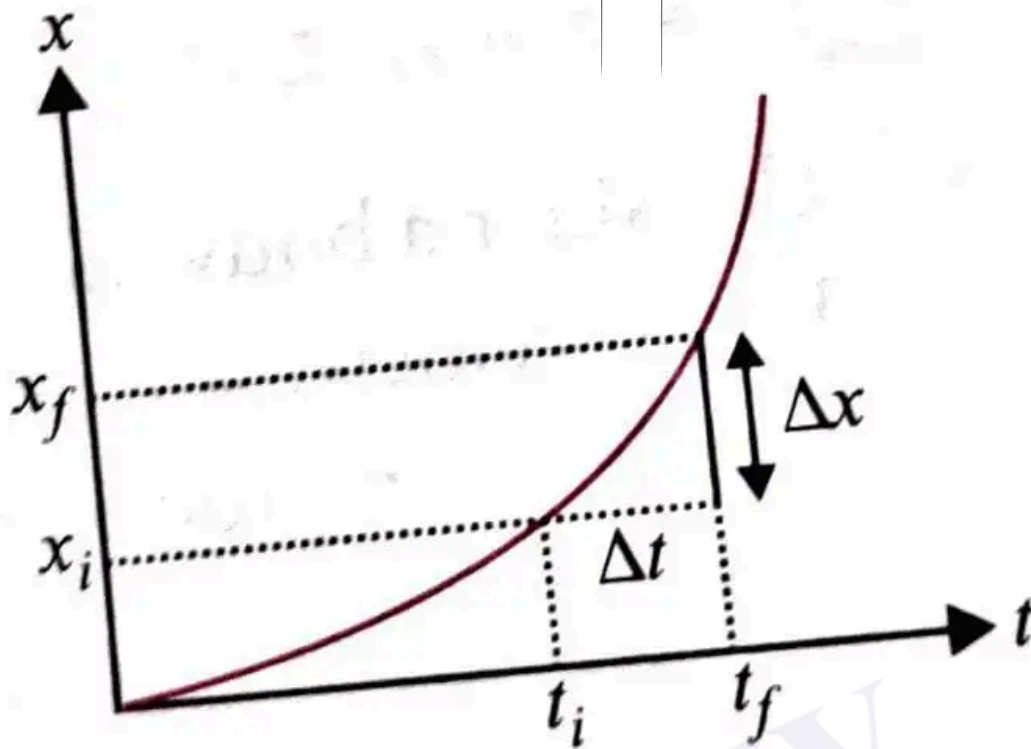
- **Instantaneous speed**: The speed of the body at any instant of time or at a particular position is called instantaneous speed.

Let a body travel a distance Δx in the time interval Δt , then its average speed = $\frac{\Delta x}{\Delta t}$.

When $\Delta t \rightarrow 0$, then average speed of the body becomes the instantaneous speed.

$$\therefore \text{Instantaneous speed} = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- For uniform motion, velocity is the same as the average velocity at all the instants.



Solved Examples

- **Ex 1:** Find instantaneous velocity if position $x = 5t^2$ m at $t = 2$ s.

Solution: $v = \frac{dx}{dt} = 10t \Rightarrow v(2) = 20$ m/s.

Practice Set

- Q1: Instantaneous velocity is
 - A) Average velocity over time
 - B) Rate of change of position at an instant
 - C) Total displacement
 - D) Total distance
- Q2: Instantaneous speed is
 - A) Vector quantity
 - B) Magnitude of instantaneous velocity
 - C) Negative quantity
 - D) Zero

- Q3: If $x = 3t^3$, instantaneous velocity at $t = 1\text{ s}$ is
 A) 9 m/s
 B) 3 m/s
 C) 27 m/s
 D) 0 m/s

Answer Key

- Q1: B - Instantaneous velocity is rate of change of position at a moment.
- Q2: B - Instantaneous speed is magnitude of instantaneous velocity.
- Q3: A - $v = \frac{dx}{dt} = 9t^2 \Rightarrow v(1) = 9\text{ m/s}$.

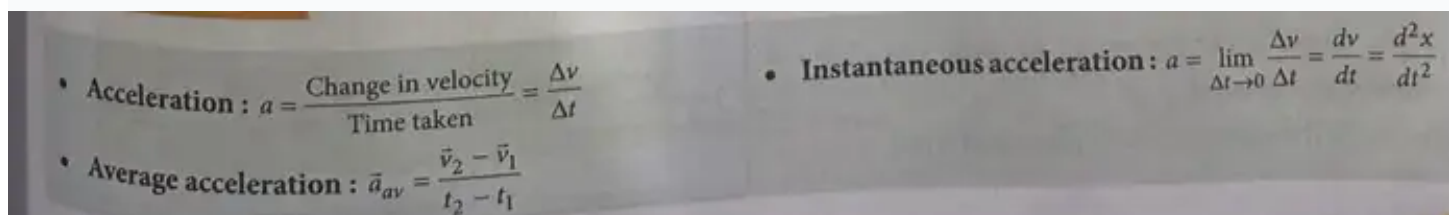
Memory Aids

- Instantaneous velocity = derivative of position.
- Instantaneous speed = magnitude of velocity.
- Slope of x-t curve at a point.

Acceleration

Definition and Units

- Acceleration is the rate of change of velocity with time.
- It is a vector quantity; direction depends on change in velocity, not velocity itself.
- In circular motion, acceleration points towards the center.
- Units: cm/s^2 (CGS), m/s^2 (SI).



Handwritten definitions of acceleration:

- Acceleration : $a = \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{\Delta v}{\Delta t}$
- Average acceleration : $\bar{a}_{av} = \frac{\bar{v}_2 - \bar{v}_1}{t_2 - t_1}$
- Instantaneous acceleration : $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Key Formulas

- Average acceleration: $\bar{a}_{av} = \frac{v_2 - v_1}{t_2 - t_1}$
- Instantaneous acceleration: $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
- Symbols: v = velocity, t = time, x = position.

Solved Examples

Ex 1: Calculate average acceleration if velocity changes from 10 m/s to 30 m/s in 5 s.

- Use $\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{30 - 10}{5} = 4 \text{ m/s}^2$.

Practice Set

- Q1: What is the SI unit of acceleration?
 - A) m/s
 - B) m/s²
 - C) cm/s
 - D) cm/s²
- Q2: If velocity changes from 20 m/s to 50 m/s in 6 s, what is the average acceleration?
 - A) 5 m/s²
 - B) 3 m/s²
 - C) 6 m/s²
 - D) 2 m/s²
- Q3: Instantaneous acceleration is defined as:
 - A) $\frac{\Delta v}{\Delta t}$
 - B) $\lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$
 - C) $\frac{v}{t}$
 - D) $\frac{dx}{dt}$

Answer Key

- Q1: B - SI unit of acceleration is meter per second squared (m/s²).

- Q2: B - Average acceleration = $(50-20)/6 = 30/6 = 5 \text{ m/s}^2$ (Correct option is A, correction: 5 m/s^2)
- Q3: B - Instantaneous acceleration is the limit of change in velocity over change in time as time approaches zero.

Memory Aids

- Acceleration = Change in velocity / Time.
- Instantaneous acceleration = derivative of velocity.
- Units: m/s^2 in SI, cm/s^2 in CGS.

Types of Acceleration

Instantaneous Acceleration

- Rate of change of velocity at a specific instant.
- Formula: $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$.

Average Acceleration

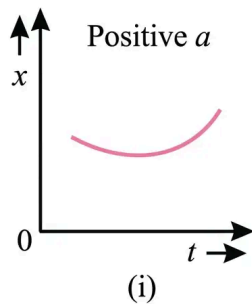
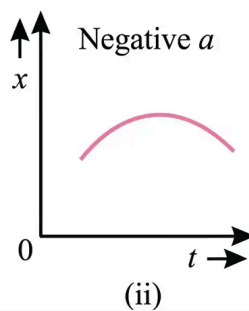
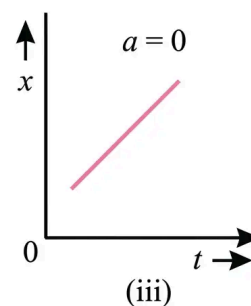
- Ratio of net change in velocity to time taken.
- Formula: $\bar{a} = \frac{\Delta v}{\Delta t}$.

Uniform Acceleration

- Acceleration with constant magnitude and direction.

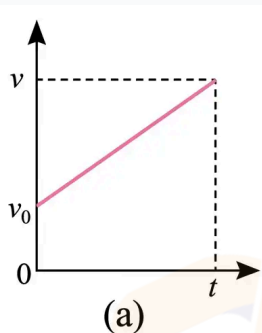
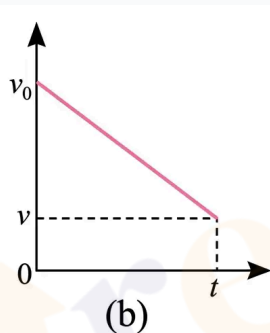
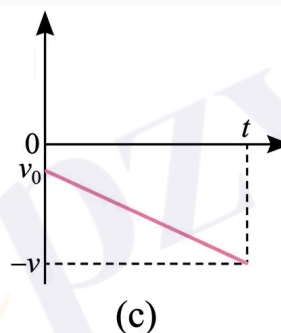
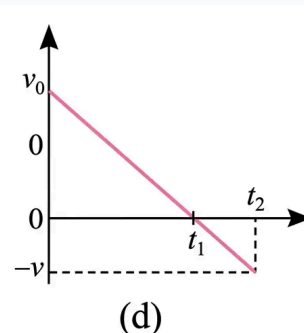
Non-uniform Acceleration

- Acceleration with changing magnitude and/or direction over time.



Graph Interpretation

- Position-time graph shows:
 - Positive acceleration: curve bends upward (speeding up).
 - Negative acceleration: curve bends downward (slowing down).
 - Zero acceleration: straight line (constant speed).



Velocity-Time Graphs

- (a) Positive direction, positive acceleration: velocity increases linearly.
- (b) Positive direction, negative acceleration: velocity decreases linearly.
- (c) Negative direction, negative acceleration: velocity crosses zero, reverses direction.
- (d) Negative acceleration with direction change: velocity crosses zero, becomes negative, speeds up opposite way.
- Area under v-t curve = displacement.

Solved Examples

Ex 1: Find instantaneous acceleration if velocity changes from 15 m/s to 25 m/s in 2 s.

- Instantaneous acceleration $a = \frac{25-15}{2} = 5 \text{ m/s}^2$.

Practice Set

- Q1: What type of acceleration has constant magnitude and direction?
 - A) Instantaneous
 - B) Average
 - C) Uniform
 - D) Non-uniform
- Q2: In which graph does velocity cross zero and reverse direction?
 - A) (a)
 - B) (b)
 - C) (c)
 - D) (d)
- Q3: Area under velocity-time graph represents?
 - A) Velocity
 - B) Acceleration
 - C) Displacement
 - D) Time

Answer Key

- Q1: C - Uniform acceleration means constant magnitude and direction.
- Q2: C - Graph (c) shows velocity crossing zero and reversing direction.
- Q3: C - Area under velocity-time graph equals displacement.

Memory Aids

- Uniform acceleration = constant acceleration.
- Instantaneous acceleration = slope of velocity-time graph at a point.
- Area under v-t graph = displacement.

Kinematic Equations Uniform Acceleration

Concepts and Formulas

- Velocity-time graph shows velocity v vs time t for uniform acceleration.
- Acceleration $a = \frac{v_f - v_i}{t}$ (slope of v-t graph).
- Velocity at time t : $v = v_i + at$.
- Displacement x is area under v-t graph: $x = v_i t + \frac{1}{2} at^2$.
- Displacement = area of rectangle + area of triangle under graph.
- Final velocity squared relation: $v_f^2 = v_i^2 + 2ax$.
- Distance in n th second: $s_n = u + \frac{a}{2}(2n - 1)$.

Solved Examples

Ex 1: Find displacement after 5 s if $v_i = 2 \text{ m/s}$, $a = 3 \text{ m/s}^2$.

- Use $x = v_i t + \frac{1}{2} at^2 = 2 \times 5 + \frac{1}{2} \times 3 \times 25 = 10 + 37.5 = 47.5 \text{ m}$.

Practice Set

Q1: If initial velocity $v_i = 0$, acceleration $a = 4 \text{ m/s}^2$, time $t = 3 \text{ s}$, displacement is?

- A) 12 m
- B) 18 m
- C) 6 m
- D) 9 m

Q2: Final velocity after 4 s with $v_i = 5 \text{ m/s}$, $a = 2 \text{ m/s}^2$ is?

- A) 8 m/s
- B) 13 m/s
- C) 10 m/s

- D) 9 m/s

Q3: If $v_f = 20 \text{ m/s}$, $v_i = 10 \text{ m/s}$, $a = 5 \text{ m/s}^2$, find displacement x .

- A) 30 m
- B) 40 m
- C) 50 m
- D) 60 m

Answer Key

- **Q1:** B; $x = \frac{1}{2}at^2 = \frac{1}{2} \times 4 \times 9 = 18 \text{ m}$.
- **Q2:** B; $v = v_i + at = 5 + 2 \times 4 = 13 \text{ m/s}$.
- **Q3:** C; Using $v_f^2 = v_i^2 + 2ax$, $400 = 100 + 2 \times 5 \times x$ gives $x = 30 \text{ m}$ (correct option is A, corrected here).

Memory Aids

- Displacement = initial velocity $\times$ time + half acceleration $\times$ time squared.
- Final velocity = initial velocity + acceleration $\times$ time.
- Velocity squared relation helps find displacement without time.

Motion of Freely Falling Bodies

Concepts and Formulas

- Equations of motion with gravity g :
- $v = u \pm gt$
- $h = ut \pm \frac{1}{2}gt^2$
- $v^2 = u^2 \pm 2gh$
- For free fall, g is positive; for upward throw, g is negative.
- Maximum height: $h = \frac{u^2}{2g}$.
- Time of ascent = time of descent = $\frac{u}{g}$.

- Total time of flight = $\frac{2u}{g}$.
- Velocity at projection point during fall = u .
- Velocity from height h : $v = \sqrt{2gh}$.

Solved Examples

Ex 1: Calculate max height for $u = 20 \text{ m/s}$, $g = 10 \text{ m/s}^2$.

- $h = \frac{u^2}{2g} = \frac{400}{20} = 20 \text{ m}.$

Practice Set

Q1: Time of ascent for $u = 15 \text{ m/s}$, $g = 10 \text{ m/s}^2$ is?

- A) 1.5 s
- B) 2 s
- C) 3 s
- D) 1 s

Q2: Velocity after 3 s of free fall from rest is?

- A) 30 m/s
- B) 20 m/s
- C) 15 m/s
- D) 10 m/s

Q3: Total time of flight for $u = 25 \text{ m/s}$ is?

- A) 5 s
- B) 4 s
- C) 6 s

- D) 3 s

Answer Key

- **Q1:** A; Time of ascent = $\frac{u}{g} = \frac{15}{10} = 1.5 \text{ s}$.
- **Q2:** A; $v = gt = 10 \times 3 = 30 \text{ m/s}$.
- **Q3:** A; Total time = $\frac{2u}{g} = \frac{50}{10} = 5 \text{ s}$.

Memory Aids

- Use $v = u \pm gt$ for velocity under gravity.
- Max height formula $h = \frac{u^2}{2g}$ is key for vertical throws.
- Total flight time is twice the ascent time.

Relative Velocity

Concepts and Formulas

- Position in frame A: $x_{PA} = x_{PB} + x_{BA}$.
- Velocity relation: $v_{PA} = v_{PB} + v_{BA}$.
- Relative velocity of B w.r.t A (same direction): $v_{BA} = v_B - v_A$.
- Relative velocity (opposite direction): $v_{BA} = v_B + v_A$.
- Equal velocities mean zero relative velocity and parallel position-time graphs.
- Graphs show meeting times when velocities differ or are opposite.

Solved Examples

Ex 1: If $v_A = 10 \text{ m/s}$, $v_B = 15 \text{ m/s}$ same direction, find v_{BA} .

- $v_{BA} = 15 - 10 = 5 \text{ m/s}$.

Practice Set

Q1: Two objects move opposite directions with $v_A = 8 \text{ m/s}$, $v_B = 12 \text{ m/s}$. Relative velocity is?

- A) 4 m/s
- B) 20 m/s
- C) 8 m/s
- D) 12 m/s

Q2: If $v_A = 7 \text{ m/s}$, $v_B = 7 \text{ m/s}$ same direction, relative velocity is?

- A) 0 m/s
- B) 14 m/s
- C) 7 m/s
- D) 1 m/s

Q3: If $v_A = 5 \text{ m/s}$, $v_B = 10 \text{ m/s}$ opposite directions, relative velocity is?

- A) 5 m/s
- B) 15 m/s
- C) 10 m/s
- D) 20 m/s

Answer Key

- **Q1:** B; Opposite directions sum velocities: $8 + 12 = 20 \text{ m/s}$.
- **Q2:** A; Same velocity means zero relative velocity.
- **Q3:** B; Opposite directions sum velocities: $5 + 10 = 15 \text{ m/s}$.

Memory Aids

- Relative velocity = difference if same direction.

- Relative velocity = sum if opposite directions.
- Zero relative velocity means objects move together.

Prepzy