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Prepzy

Introduction

Overview

- Motion in 1D has two directions: positive (+) and negative (-).
- For motion in 2D or more, vectors are essential to describe displacement, velocity, and acceleration.
- Study includes vectors, motion in a plane with constant acceleration, projectile motion, and uniform circular motion.

Solved Examples

- **Ex 1:** Describe motion in 2D using vectors.
 - Use vector components for displacement, velocity, acceleration.
 - Apply vector addition for resultant quantities.

Practice Set

- Q1: How many directions are possible in one-dimensional motion?
 - A) One
 - B) Two
 - C) Three
 - D) Infinite
- Q2: Which quantity requires vectors for complete description in 2D motion?
 - A) Speed
 - B) Displacement
 - C) Temperature
 - D) Distance
- Q3: What type of motion requires vector analysis?
 - A) Motion along a straight line
 - B) Motion in a plane
 - C) Motion at rest
 - D) None of these

Answer Key

- Q1: B - One-dimensional motion has two possible directions: + and -.
- Q2: B - Displacement in 2D requires both magnitude and direction, hence vectors.
- Q3: B - Motion in a plane involves directions in two dimensions, requiring vectors.

Memory Aids

- 1D motion: Two directions (+, -)
- 2D motion: Use vectors for direction and magnitude
- Vectors needed for velocity and acceleration in plane

Scalars and Vectors

Scalars

- Have magnitude only, no direction.
- Examples: distance, speed, temperature.

- Follow ordinary algebra rules for addition, subtraction (same units), multiplication, division.
- Examples:
 - Perimeter = $2 \times (\text{length} + \text{breadth})$
 - Density = mass / volume

Vectors

- Have both magnitude and direction.
- Obey triangle law of addition.
- Examples: velocity, acceleration.
- Represented by arrowed line; length = magnitude, arrowhead = direction.
- Notation: bold letters or letters with arrow, e.g., $\vec{F}$.
- Magnitude (modulus) denoted as $|\vec{A}|$ or A .

Solved Examples

- **Ex 1:** Identify scalar or vector: Speed = 30 m/s
 - Speed is scalar (magnitude only).
- **Ex 2:** Represent vector $\vec{A}$ with magnitude 5 units
 - Draw arrow of length proportional to 5 units with direction.

Practice Set

- Q1: Which is a scalar quantity?
 - A) Velocity
 - B) Acceleration
 - C) Speed
 - D) Force
- Q2: Vector magnitude is represented by?
 - A) $\vec{A}$
 - B) $|\vec{A}|$
 - C) $\vec{A}^2$
 - D) None
- Q3: Which law do vectors obey?
 - A) Distributive law
 - B) Triangle law of addition
 - C) Commutative law only
 - D) None

Answer Key

- Q1: C - Speed has magnitude only, no direction.
- Q2: B - Magnitude is denoted by $|\vec{A}|$.
- Q3: B - Vectors obey triangle law of addition.

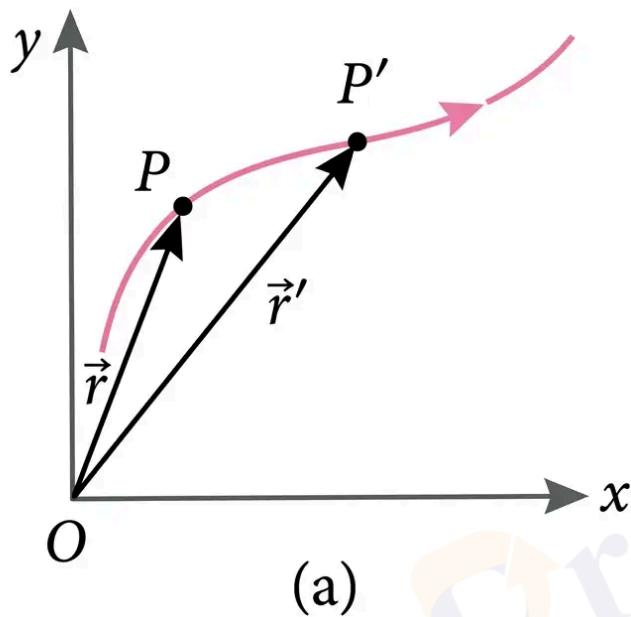
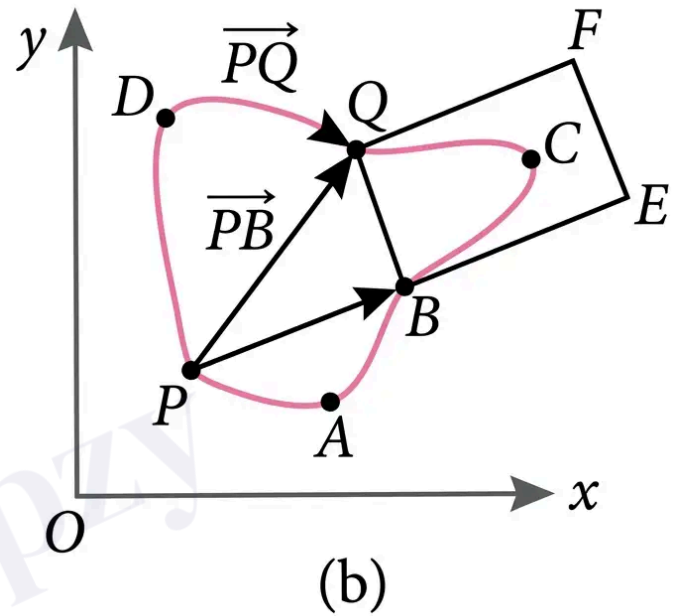
Memory Aids

- Scalars = magnitude only
- Vectors = magnitude + direction
- Vector notation: arrow or bold letter

Position and Displacement Vectors

Position Vector

- Gives position of object relative to origin in coordinate system.
- Denoted as $\vec{r}$ or $\overrightarrow{OP}$.
- Provides:
 - Distance from origin (magnitude)
 - Direction from origin

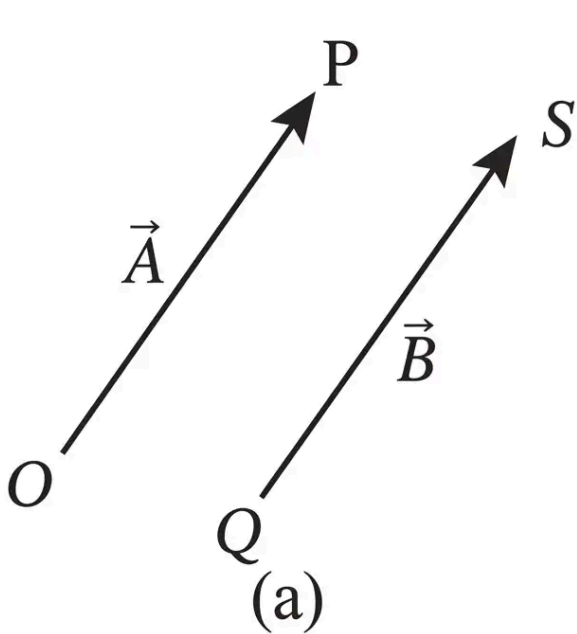
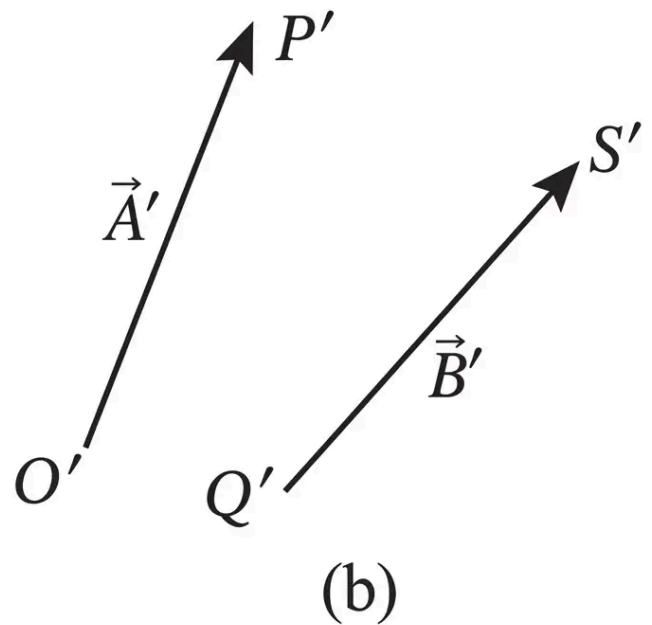


Displacement Vector

- Vector showing change in position from initial to final point.
- Denoted as $\overrightarrow{PP'}$ for motion from P to P'.
- Is the straight line joining initial and final positions.
- Independent of actual path taken.
- Magnitude of displacement $\leq$ path length.

Equality of Vectors

- Two vectors are equal if they have same magnitude and direction.
- Position of vector does not affect equality.



Solved Examples

- **Ex 1:** Find displacement vector if object moves from $P(2, 3)$ to $P'(5, 7)$.
 - Displacement = $\vec{PP'} = (5 - 2, 7 - 3) = (3, 4)$.
 - Magnitude = $\sqrt{3^2 + 4^2} = 5$ units.
- **Ex 2:** Check equality of vectors $\vec{A}$ and $\vec{B}$ with same length and direction but different positions.
 - Vectors are equal if magnitude and direction match.

Practice Set

- Q1: Displacement vector is
 - A) Path length
 - B) Straight line from initial to final position
 - C) Distance traveled
 - D) None
- Q2: Two vectors are equal if
 - A) Same magnitude only
 - B) Same direction only
 - C) Same magnitude and direction
 - D) Same initial point
- Q3: Magnitude of displacement compared to path length is
 - A) Always greater
 - B) Always equal
 - C) Less than or equal
 - D) None

Answer Key

- Q1: B - Displacement is the straight line joining initial and final points.
- Q2: C - Equality requires same magnitude and direction.
- Q3: C - Displacement magnitude is less than or equal to path length.

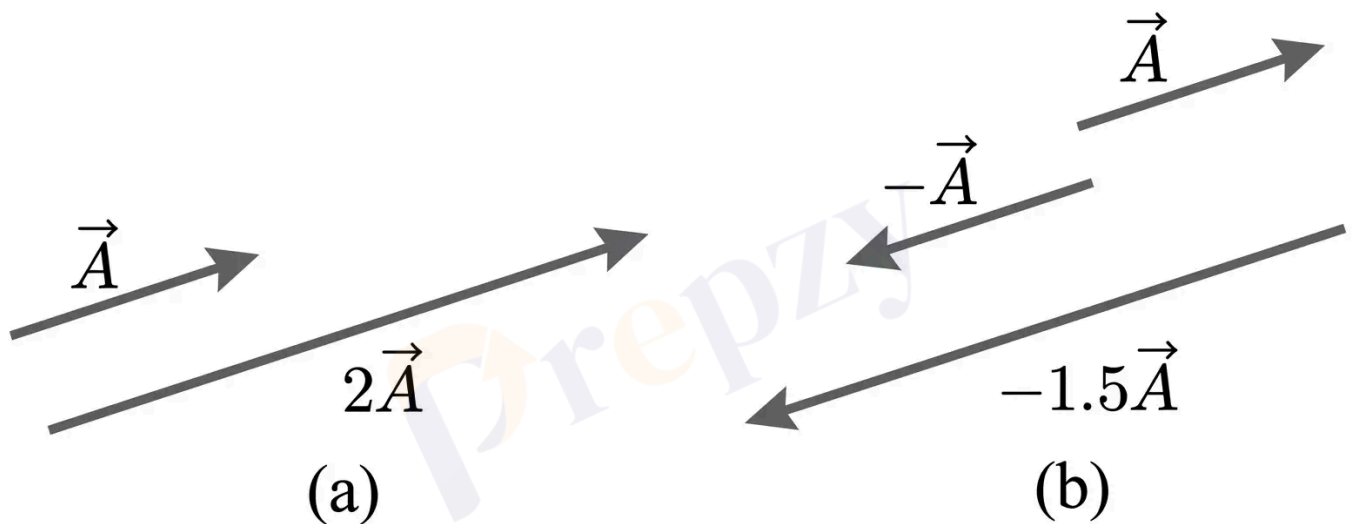
Memory Aids

- Displacement = shortest path vector
- Equal vectors = same length + direction
- Position vector from origin to point

Multiplication of Vectors by Real Numbers

Concept Explanation

- Multiplying vector $\vec{A}$ by a real number n gives vector $n\vec{A}$.
- Magnitude: $|n\vec{A}| = |n| \times |\vec{A}|$.
- Direction: Same as $\vec{A}$ if $n > 0$, opposite if $n < 0$.
- Example: $2\vec{A}$ doubles length, same direction; $-1\vec{A}$ reverses direction, same length.
- Negative scalar changes direction and scales magnitude.



Solved Examples

Ex 1: Find magnitude and direction of $-3\vec{A}$ if $|\vec{A}| = 5$ units.

- Magnitude: $|-3| \times 5 = 15$ units.
- Direction: Opposite to $\vec{A}$ (since $n = -3 < 0$).

Practice Set

Q1: If $|\vec{A}| = 4$ units, what is the magnitude of $2\vec{A}$?

- A) 2 units
- B) 4 units
- C) 6 units
- D) 8 units

Q2: Multiplying $\vec{A}$ by -1 results in a vector that is:

- A) Same magnitude, same direction
- B) Same magnitude, opposite direction
- C) Double magnitude, same direction
- D) Half magnitude, opposite direction

Q3: If $\vec{B} = -1.5\vec{A}$ and $|\vec{A}| = 10$ units, find $|\vec{B}|$ and direction.

- A) 15 units, same direction
- B) 15 units, opposite direction
- C) 6.67 units, same direction
- D) 6.67 units, opposite direction

Answer Key

Q1: D) 8 units — Magnitude doubles: $2 \times 4 = 8$.

Q2: B) Same magnitude, opposite direction — Negative scalar reverses direction.

Q3: B) 15 units, opposite direction — Magnitude $1.5 \times 10 = 15$, direction reversed due to negative sign.

Memory Aids

- Positive scalar: length changes, direction same.
- Negative scalar: length changes, direction reverses.
- Magnitude formula: $|n\vec{A}| = |n| \times |\vec{A}|$.

Addition of Vectors

General Rule

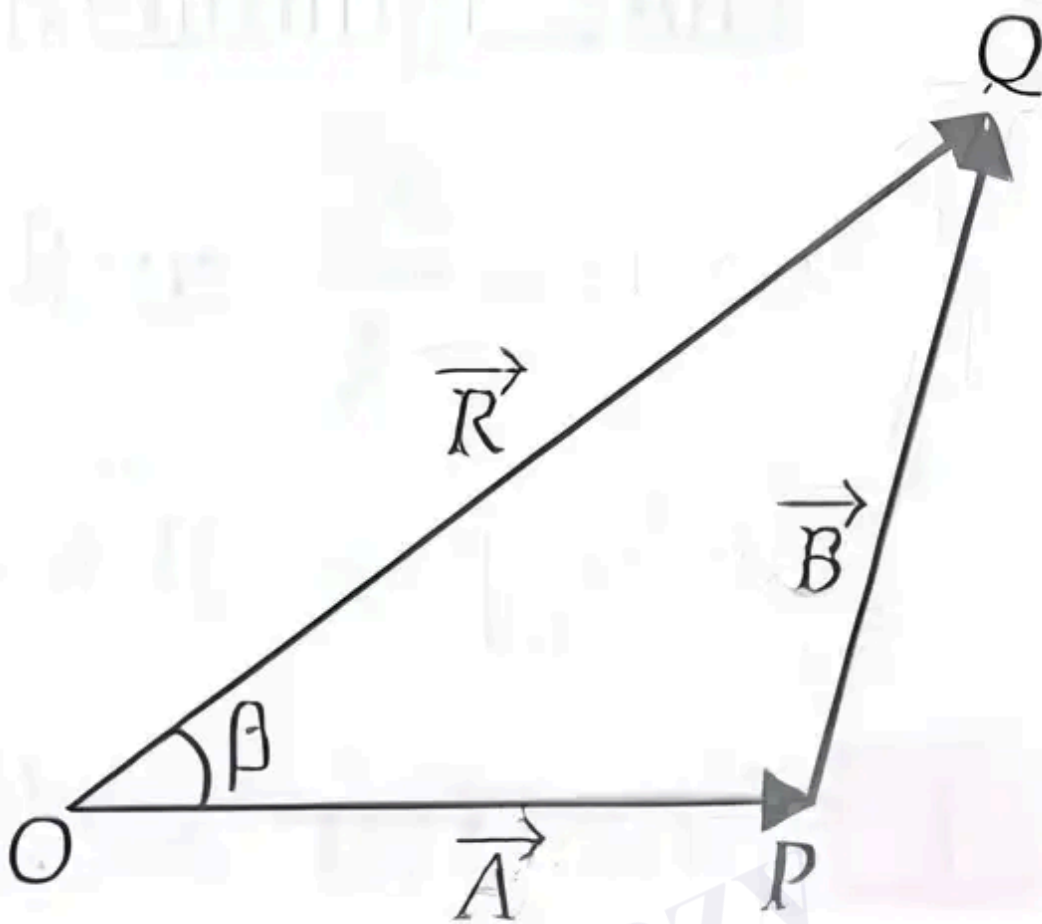
- Arrange vectors so head of one coincides with tail of next.
- Resultant vector is from tail of first to head of last vector.

Properties

- Commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- Distributive: $n(\vec{A} + \vec{B}) = n\vec{A} + n\vec{B}$
- Associative: $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

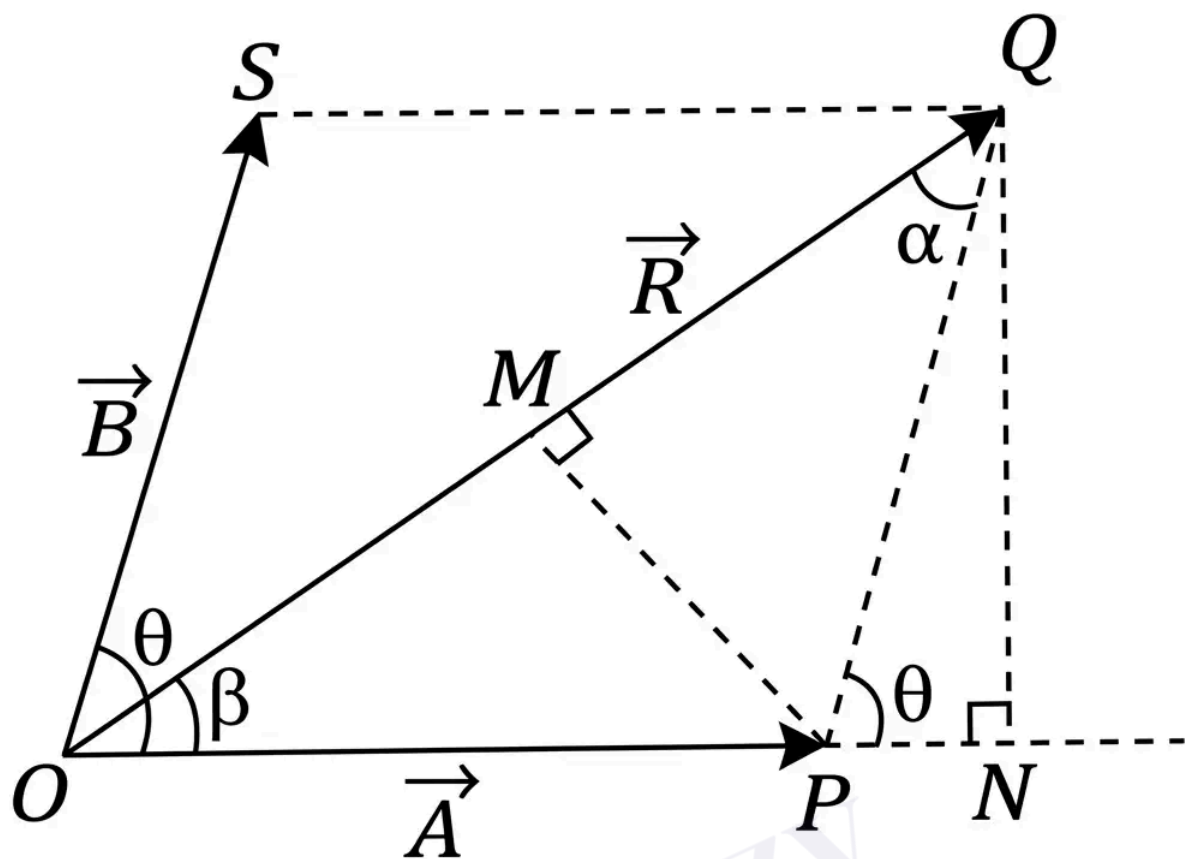
Triangle Law

- If two vectors are represented by two sides of a triangle in order, resultant is the third side in opposite order.
- Resultant: $\vec{R} = \vec{A} + \vec{B}$



Parallelogram Law

- Two vectors represented by adjacent sides of a parallelogram.
- Resultant is diagonal from the common point.
- Magnitude: $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$
- Angle: $\tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$



Polygon Law

- Vectors represented by sides of an open polygon in order.
- Resultant is the closing side in opposite order.

Special Cases

- Same direction: $R = A + B, \beta = 0^\circ$
- Opposite direction: $R = |A - B|, \beta = 180^\circ$
- Perpendicular vectors: $R = \sqrt{A^2 + B^2}, \beta = \tan^{-1} \left(\frac{B}{A} \right)$

Solved Examples

Ex 1: Find resultant of $\vec{A} = 3\hat{i}$ and $\vec{B} = 4\hat{j}$ (perpendicular vectors).

- Use Pythagoras: $R = \sqrt{3^2 + 4^2} = 5$
- Angle: $\beta = \tan^{-1} \frac{4}{3} = 53.13^\circ$

Practice Set

Q1: If $\vec{A} = 5\hat{i}$ and $\vec{B} = 5\hat{j}$, what is $|\vec{A} + \vec{B}|$?

- A) 5
- B) 10
- C) $5\sqrt{2}$
- D) 0

Q2: Two vectors $\vec{A}$ and $\vec{B}$ have magnitudes 6 and 8 units with angle 60° between them. Find resultant magnitude.

- A) 14
- B) 10
- C) 8
- D) 6

Q3: Two vectors $\vec{A} = 7$ units and $\vec{B} = 24$ units are perpendicular. Find resultant and angle with $\vec{A}$.

- A) 25 units, 73.74°
- B) 31 units, 45°
- C) 17 units, 60°
- D) 24 units, 90°

Answer Key

Q1: C; Pythagoras for perpendicular vectors gives $5\sqrt{2}$.

Q2: B; Use formula $\sqrt{6^2 + 8^2 + 2 \times 6 \times 8 \times \cos 60^\circ} = 10$.

Q3: A; Resultant $= \sqrt{7^2 + 24^2} = 25$, angle $= \tan^{-1}(24/7) = 73.74^\circ$.

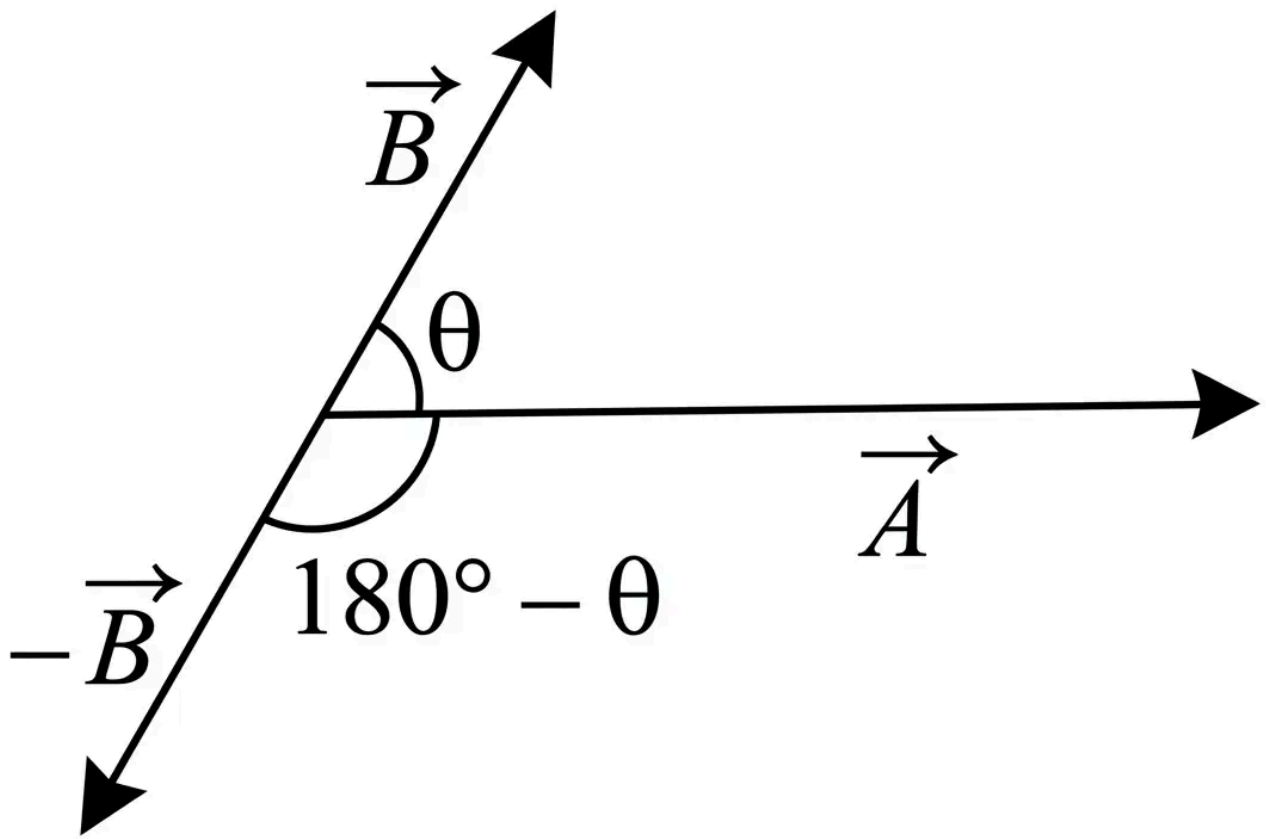
Memory Aids

- Triangle law: two sides form triangle, third side is resultant.
- Parallelogram law: diagonal is resultant.
- Resultant magnitude formula: $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$.

Subtraction of Vectors

Definition

- $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
- Angle between $\vec{A}$ and $-\vec{B}$ is $180^\circ - \theta$ if angle between $\vec{A}$ and $\vec{B}$ is θ .
- Magnitude: $|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta}$



Properties

- Not commutative: $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$
- Not associative: $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$

Solved Examples

Ex 1: Find magnitude of $\vec{A} - \vec{B}$ if $A = 5$, $B = 3$, and angle $\theta = 60^\circ$.

- Use formula: $\sqrt{5^2 + 3^2 - 2 \times 5 \times 3 \times \cos 60^\circ} = \sqrt{25 + 9 - 15} = \sqrt{19} \approx 4.36$

Practice Set

Q1: If $\vec{A} = 4\hat{i}$ and $\vec{B} = 3\hat{i}$, what is $|\vec{A} - \vec{B}|$?

- A) 7
- B) 1
- C) 5
- D) 0

Q2: Two vectors $\vec{A}$ and $\vec{B}$ have magnitudes 8 and 6 units with angle 90° between them. Find magnitude of $\vec{A} - \vec{B}$.

- A) 10
- B) $\sqrt{100}$
- C) $\sqrt{64 + 36}$
- D) $\sqrt{64 + 36 - 2 \times 8 \times 6 \times 0}$

Q3: If angle between $\vec{A}$ and $\vec{B}$ is 120° , find magnitude of $\vec{A} - \vec{B}$ for $A = 10$, $B = 5$.

- A) 15
- B) 10
- C) 12.99
- D) 5

Answer Key

Q1: B; $|4 - 3| = 1$.

Q2: D; $\sqrt{64 + 36 - 0} = 10$.

Q3: C; $\sqrt{10^2 + 5^2 - 2 \times 10 \times 5 \times \cos 120^\circ} = 12.99$.

Memory Aids

- Subtraction as addition of negative vector.
- Magnitude formula uses minus sign in cosine term.
- Not commutative or associative.

Resolution of Vectors

Concept and Definition

- Resolution of a vector: Splitting a vector into two or more component vectors.
- Component vectors: Resultant vectors after splitting.
- Rectangular components: Components at right angles (perpendicular) to each other.

Solved Examples

- **Ex 1:** Vector $\vec{A}$ is split into two perpendicular components A_x and A_y .
- Use $A_x = A \cos \theta$, $A_y = A \sin \theta$ where θ is angle with x-axis.

Practice Set

- Q1: What is the process of splitting a vector into components called?
 - A. Addition
 - B. Resolution

- C. Multiplication
- D. Integration
- Q2: If $\vec{A}$ makes an angle 30° with x-axis and magnitude 10 units, what is A_x ?
 - A. 5
 - B. 8.66
 - C. 10
 - D. 0
- Q3: Which of the following is true for rectangular components?
 - A. Components are parallel
 - B. Components are perpendicular
 - C. Components are equal
 - D. Components are zero

Answer Key

- Q1: B - Resolution is the process of splitting a vector into components.
- Q2: B - $A_x = 10 \cos 30^\circ = 8.66$
- Q3: B - Rectangular components are perpendicular to each other.

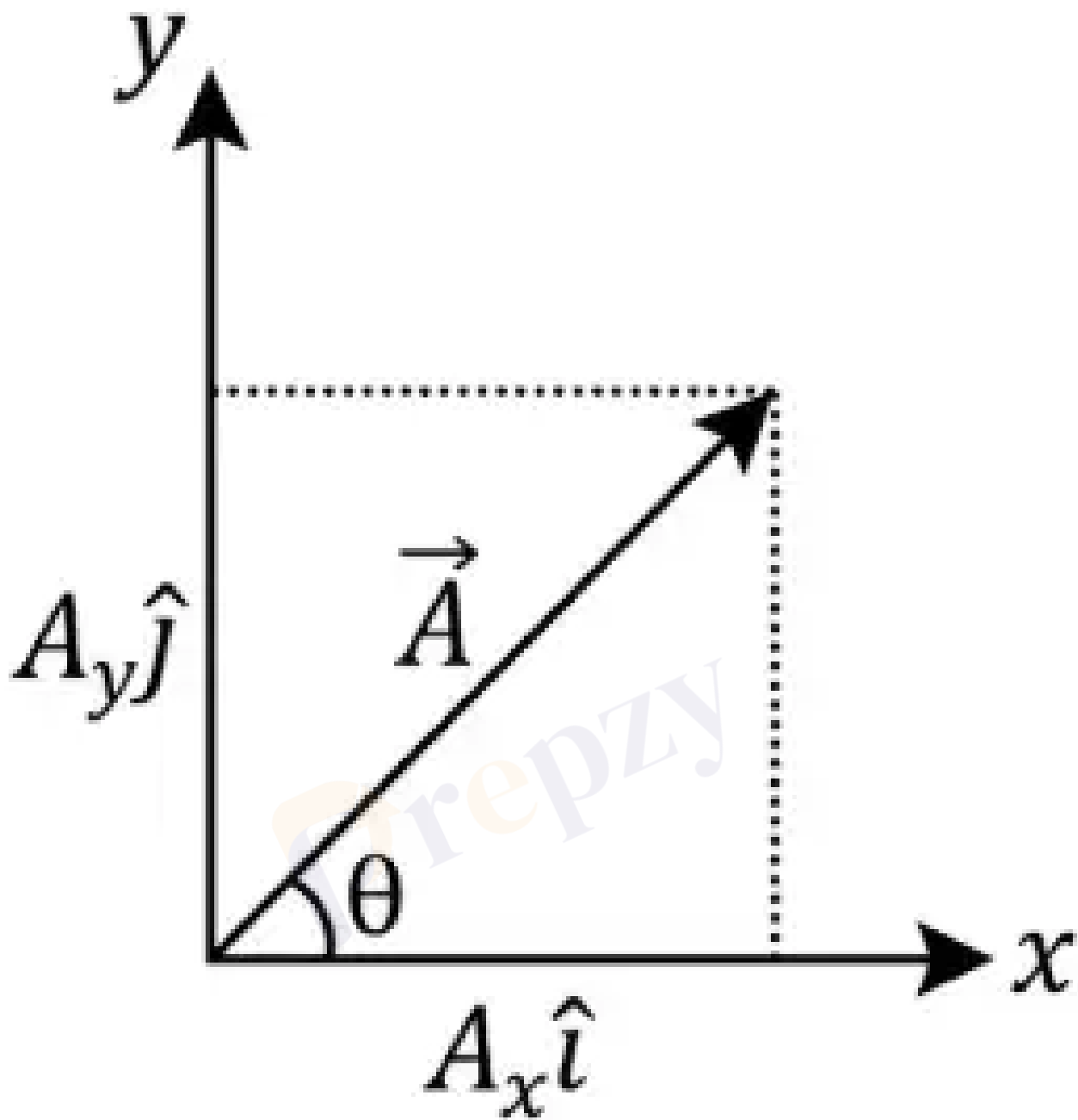
Memory Aids

- Remember: $A_x = A \cos \theta$, $A_y = A \sin \theta$
- Rectangular components form right angles.
- Resolution = splitting vector into perpendicular parts.

Rectangular Components in a Plane

Key Formulas and Concepts

- Vector $\vec{A}$ makes angle θ with x-axis.
- Components: $\vec{A} = A_x \hat{i} + A_y \hat{j}$
- $A_x = A \cos \theta$, $A_y = A \sin \theta$
- Magnitude: $A = \sqrt{A_x^2 + A_y^2}$
- Direction: $\tan \theta = \frac{A_y}{A_x}$



Solved Examples

- **Ex 1:** Find A_x and A_y if $A = 5$ units and $\theta = 60^\circ$.
- $A_x = 5 \cos 60^\circ = 2.5$, $A_y = 5 \sin 60^\circ = 4.33$

Practice Set

- Q1: If $A = 10$ units and $\theta = 45^\circ$, what is A_y ?
 - A. 7.07
 - B. 5

- C. 10
- D. 0
- Q2: The magnitude of $\vec{A}$ is 13 units, $A_x = 5$. Find A_y .
 - A. 12
 - B. 8
 - C. 10
 - D. 7
- Q3: If $\tan \theta = 1$, what is the angle θ ?
 - A. 30°
 - B. 45°
 - C. 60°
 - D. 90°

Answer Key

- Q1: A - $A_y = 10 \sin 45^\circ = 7.07$
- Q2: B - $A_y = \sqrt{13^2 - 5^2} = 12$ (Check: actually $\sqrt{169 - 25} = \sqrt{144} = 12$)
- Q3: B - $\tan 45^\circ = 1$

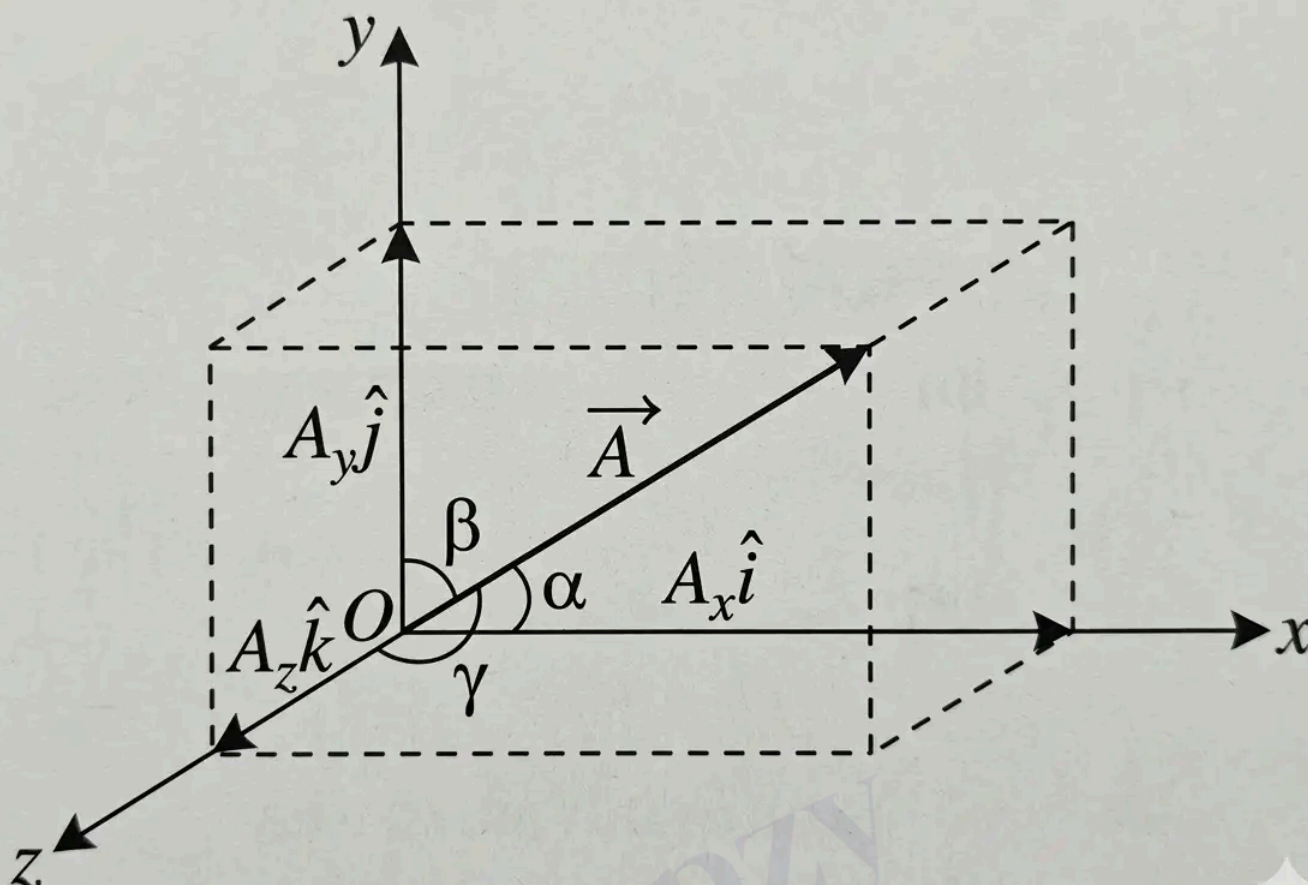
Memory Aids

- Use Pythagoras: $A^2 = A_x^2 + A_y^2$
- $\tan \theta = \frac{A_y}{A_x}$ for direction
- Cosine for x-component, sine for y-component

Rectangular Components in Space

Key Formulas and Concepts

- Vector $\vec{A}$ makes angles α, β, γ with x, y, z axes.
- Components: $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
- $A_x = A \cos \alpha, A_y = A \cos \beta, A_z = A \cos \gamma$
- Magnitude: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$
- Direction cosines: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$



Solved Examples

- **Ex 1:** Given $A = 10$, $\alpha = 60^\circ$, $\beta = 45^\circ$, find A_x and A_y .
- $A_x = 10 \cos 60^\circ = 5$, $A_y = 10 \cos 45^\circ = 7.07$

Practice Set

- Q1: If $A = 5$ and $\cos \alpha = 0.6$, find A_x .
 - A. 3
 - B. 5
 - C. 0.6
 - D. 8.3
- Q2: Which equation represents direction cosines?
 - A. $\cos \alpha + \cos \beta + \cos \gamma = 1$
 - B. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
 - C. $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 1$
 - D. $\tan \alpha + \tan \beta + \tan \gamma = 1$
- Q3: Find magnitude A if $A_x = 3$, $A_y = 4$, $A_z = 12$.
 - A. 13
 - B. 14
 - C. 15
 - D. 16

Answer Key

- Q1: $A - A_x = 5 \times 0.6 = 3$
- Q2: B - Direction cosines satisfy $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
- Q3: C - $A = \sqrt{3^2 + 4^2 + 12^2} = 13$

Memory Aids

- Direction cosines squared sum to 1.
- Use cosine of angles for components.
- Magnitude from 3D Pythagoras.

Vector Addition Analytical Method

Component-wise Addition

- Vectors $\vec{A}$ and $\vec{B}$ in x-y plane: $\vec{A} = A_x \hat{i} + A_y \hat{j}$, $\vec{B} = B_x \hat{i} + B_y \hat{j}$
- Resultant vector $\vec{R} = \vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} = R_x \hat{i} + R_y \hat{j}$
- Each component of $\vec{R}$ is sum of corresponding components of $\vec{A}$ and $\vec{B}$
- In 3D, $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$, $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$
- Resultant in 3D: $\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} + (A_z + B_z) \hat{k} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$
- Method extends to any number of vectors for addition or subtraction

Solved Examples

- **Ex 1:** Given $\vec{A} = 3\hat{i} + 4\hat{j}$, $\vec{B} = 1\hat{i} + 2\hat{j}$, find $\vec{R} = \vec{A} + \vec{B}$.
 - Calculate components: $R_x = 3 + 1 = 4$, $R_y = 4 + 2 = 6$
 - Result: $\vec{R} = 4\hat{i} + 6\hat{j}$
- **Ex 2:** For 3D vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 1\hat{i} + 0\hat{j} + 5\hat{k}$, find $\vec{R} = \vec{A} + \vec{B}$.
 - Calculate components: $R_x = 2 + 1 = 3$, $R_y = 3 + 0 = 3$, $R_z = 4 + 5 = 9$
 - Result: $\vec{R} = 3\hat{i} + 3\hat{j} + 9\hat{k}$

Practice Set

- Q1: If $\vec{A} = 5\hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$, what is R_y of $\vec{R} = \vec{A} + \vec{B}$?
 - A. 6
 - B. 8
 - C. 2
 - D. 4
- Q2: For vectors $\vec{A} = 1\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 4\hat{i} + 0\hat{j} + 1\hat{k}$, find R_z .
 - A. 4
 - B. 3
 - C. 1
 - D. 2
- Q3: Given $\vec{A} = 2\hat{i} + 3\hat{j}$, $\vec{B} = -1\hat{i} + 4\hat{j}$, and $\vec{C} = 3\hat{i} - 2\hat{j}$, find R_x of $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.
 - A. 4
 - B. 3
 - C. 2
 - D. 5

Answer Key

- Q1: B. 8 — Sum of y-components: $2 + 4 = 6$ (Correction: $2 + 4 = 6$, so correct answer is A. 6)
- Q2: D. 4 — Sum of z-components: $3 + 1 = 4$
- Q3: A. 4 — Sum of x-components: $2 + (-1) + 3 = 4$

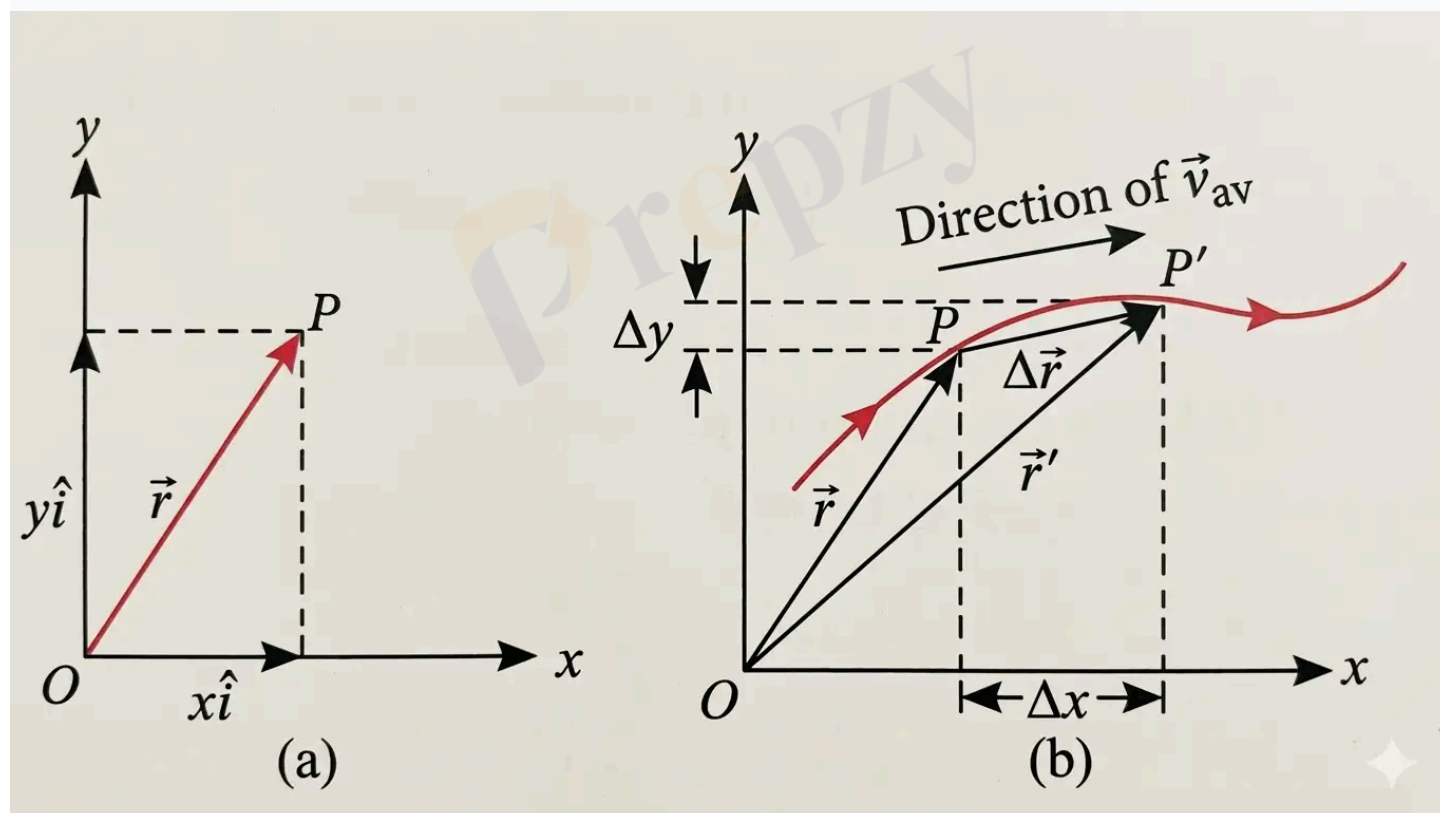
Memory Aids

- Resultant components = sum of individual components
- Use $R_x = \sum A_x, R_y = \sum A_y, R_z = \sum A_z$
- Extend method to any number of vectors

Position Vector

Concepts

- Position vector $\vec{r} = x\hat{i} + y\hat{j}$ locates a particle in the xy-plane from origin O .
- Components: x along x-axis, y along y-axis.
- Represents the particle's position as a vector from origin to point P .



Solved Examples

- **Ex 1:** Find position vector for $x = 3\text{ m}, y = 4\text{ m}$.
 - $\vec{r} = 3\hat{i} + 4\hat{j}\text{ m}$

Practice Set

- Q1: Position vector of point $P(2, 5)$ is?

- A) $2\hat{i} + 5\hat{j}$
- B) $5\hat{i} + 2\hat{j}$
- C) $2\hat{i} - 5\hat{j}$
- D) $-2\hat{i} + 5\hat{j}$
- Q2: If $\vec{r} = 4\hat{i} + 3\hat{j}$, magnitude is?
 - A) 5
 - B) 7
 - C) 1
 - D) 12
- Q3: For $\vec{r} = 6\hat{i} - 8\hat{j}$, direction angle θ with x-axis is?
 - A) 53.13°
 - B) 126.87°
 - C) 36.87°
 - D) 143.13°

Answer Key

- Q1: A) $2\hat{i} + 5\hat{j}$ - Direct components from coordinates.
- Q2: A) 5 - Magnitude $= \sqrt{4^2 + 3^2} = 5$.
- Q3: B) 126.87° - $\theta = \tan^{-1}(-8/6)$ in 2nd quadrant.

Memory Aids

- Position vector $= x\hat{i} + y\hat{j}$.
- Magnitude $= \sqrt{x^2 + y^2}$.
- Direction $\theta = \tan^{-1}(y/x)$.

Displacement Vector

Concepts

- Displacement $\Delta\vec{r} = \vec{r}' - \vec{r} = \hat{i}\Delta x + \hat{j}\Delta y$.
- Directed from initial point P to final point P' .
- Represents change in position vector.

Solved Examples

- **Ex 1:** Particle moves from $(2, 3)$ to $(5, 7)$.
 - $\Delta\vec{r} = (5 - 2)\hat{i} + (7 - 3)\hat{j} = 3\hat{i} + 4\hat{j}$

Practice Set

- Q1: Displacement from $(1, 2)$ to $(4, 6)$ is?
 - A) $3\hat{i} + 4\hat{j}$
 - B) $4\hat{i} + 6\hat{j}$
 - C) $1\hat{i} + 2\hat{j}$
 - D) $-3\hat{i} - 4\hat{j}$
- Q2: If $\Delta\vec{r} = 5\hat{i} - 12\hat{j}$, magnitude is?
 - A) 13

- B) 7
- C) 17
- D) 10
- Q3: Displacement vector direction for $\Delta x = -3$, $\Delta y = 4$ is?
 - A) 126.87°
 - B) 53.13°
 - C) 36.87°
 - D) 143.13°

Answer Key

- Q1: A) $3\hat{i} + 4\hat{j}$ - Difference of coordinates.
- Q2: A) 13 - Magnitude $= \sqrt{5^2 + (-12)^2} = 13$.
- Q3: A) 126.87° - $\tan^{-1}(4/-3)$ in 2nd quadrant.

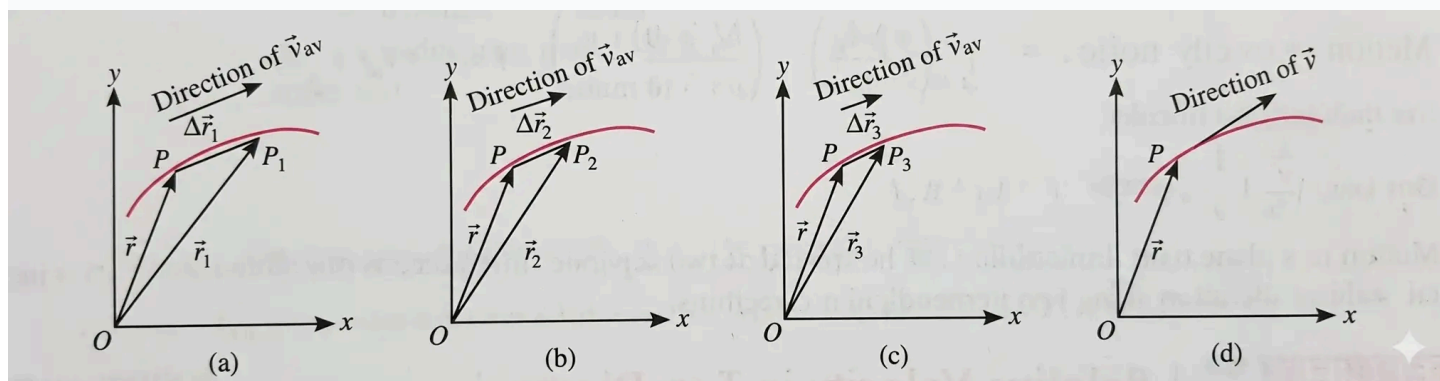
Memory Aids

- Displacement = final position - initial position.
- Component form: $\Delta \vec{r} = \hat{i}\Delta x + \hat{j}\Delta y$.
- Direction $\theta = \tan^{-1}(\Delta y/\Delta x)$.

Velocity

Concepts

- Average velocity $\vec{V}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \hat{i} \frac{\Delta x}{\Delta t} + \hat{j} \frac{\Delta y}{\Delta t} = \vec{V}_{x,av}\hat{i} + \vec{V}_{y,av}\hat{j}$.
- Instantaneous velocity $\vec{V} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \hat{i} \frac{dx}{dt} + \hat{j} \frac{dy}{dt} = v_x\hat{i} + v_y\hat{j}$.
- Magnitude $v = \sqrt{v_x^2 + v_y^2}$, direction $\theta = \tan^{-1}(v_y/v_x)$.
- Velocity vector is tangent to the path and points in direction of motion.



Solved Examples

- **Ex 1:** Particle moves from $(1, 2)$ to $(4, 6)$ in 2 s.
 - $\vec{V}_{av} = \frac{(3\hat{i} + 4\hat{j})}{2} = 1.5\hat{i} + 2\hat{j} \text{ m/s}$
- **Ex 2:** Instantaneous velocity components: $v_x = 3 \text{ m/s}$, $v_y = 4 \text{ m/s}$.
 - Magnitude $v = 5 \text{ m/s}$, direction $\theta = 53.13^\circ$.

Practice Set

- Q1: Average velocity for displacement $4\hat{i} + 3\hat{j}$ in 1 s is?
 - A) $4\hat{i} + 3\hat{j}$ m/s
 - B) $3\hat{i} + 4\hat{j}$ m/s
 - C) $7\hat{i} + 0\hat{j}$ m/s
 - D) $1\hat{i} + 1\hat{j}$ m/s
- Q2: Instantaneous velocity components $v_x = 6 \text{ m/s}$, $v_y = 8 \text{ m/s}$. Magnitude is?
 - A) 10 m/s
 - B) 14 m/s
 - C) 2 m/s
 - D) 4 m/s
- Q3: Direction of velocity with $v_x = -3 \text{ m/s}$, $v_y = 3 \text{ m/s}$ is?
 - A) 45°
 - B) 135°
 - C) 225°
 - D) 315°

Answer Key

- Q1: A) $4\hat{i} + 3\hat{j}$ m/s - Average velocity = displacement/time.
- Q2: A) 10 m/s - $\sqrt{6^2 + 8^2} = 10$.
- Q3: B) 135° - $\tan^{-1}(3/-3)$ in 2nd quadrant.

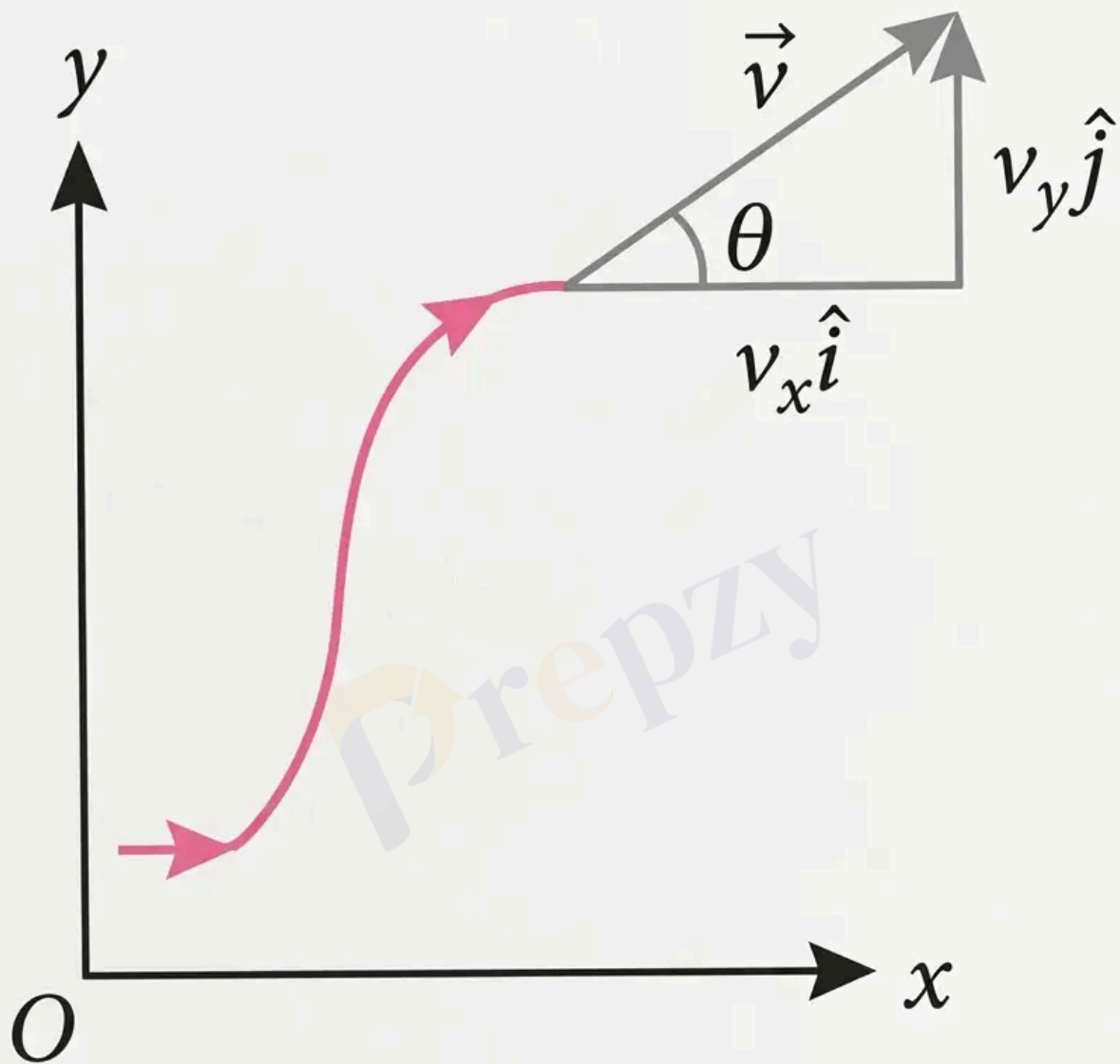
Memory Aids

- $\vec{V}_{av} = \frac{\Delta \vec{r}}{\Delta t}$.
- Instantaneous velocity = derivative of position vector.
- Magnitude and direction from components.

Acceleration

Concepts

- Average acceleration $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \hat{i} \frac{\Delta v_x}{\Delta t} + \hat{j} \frac{\Delta v_y}{\Delta t} = a_{x,av} \hat{i} + a_{y,av} \hat{j}$.
- Instantaneous acceleration is limit as $\Delta t \rightarrow 0$.
- Acceleration vector shows rate of change of velocity in magnitude and direction.



Solved Examples

- **Ex 1:** Velocity changes from $3\hat{i} + 4\hat{j}$ m/s to $6\hat{i} + 8\hat{j}$ m/s in 2 s.
 - $\vec{a}_{av} = \frac{(6-3)\hat{i} + (8-4)\hat{j}}{2} = 1.5\hat{i} + 2\hat{j} \text{ m/s}^2$

Practice Set

- Q1: Average acceleration if velocity changes from $2\hat{i} + 3\hat{j}$ m/s to $8\hat{i} + 9\hat{j}$ m/s in 3 s?
 - A) $2\hat{i} + 2\hat{j} \text{ m/s}^2$

- B) $3\hat{i} + 3\hat{j}$ m/s²
- C) $6\hat{i} + 6\hat{j}$ m/s²
- D) $1\hat{i} + 1\hat{j}$ m/s²
- Q2: Components of acceleration are $a_x = 4 \text{ m/s}^2$, $a_y = 3 \text{ m/s}^2$. Magnitude is?
 - A) 5 m/s²
 - B) 7 m/s²
 - C) 1 m/s²
 - D) 12 m/s²
- Q3: Direction of acceleration with $a_x = -3 \text{ m/s}^2$, $a_y = 3 \text{ m/s}^2$ is?
 - A) 45°
 - B) 135°
 - C) 225°
 - D) 315°

Answer Key

- Q1: A) $2\hat{i} + 2\hat{j}$ m/s² - Change in velocity/time.
- Q2: A) 5 m/s² - $\sqrt{4^2 + 3^2} = 5$.
- Q3: B) 135° - $\tan^{-1}(3/-3)$ in 2nd quadrant.

Memory Aids

- $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$.
- Instantaneous acceleration = derivative of velocity vector.
- Magnitude and direction from components.

Motion in a Plane with Constant Acceleration

Key Formulas and Concepts

- Velocity at time t : $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is initial velocity.
- Average velocity over t : $\vec{v}_{av} = \frac{\vec{v} + \vec{v}_0}{2}$.
- Component form: $v_x = v_{0x} + a_x t$, $v_y = v_{0y} + a_y t$.
- Displacement: $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$.
- Component displacement: $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$, $y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$.
- Motion in plane treated as two independent 1D motions along perpendicular axes.

Solved Examples

- **Ex 1:** Find velocity at $t = 2 \text{ s}$ if $\vec{v}_0 = 3\hat{i} + 4\hat{j}$ m/s and $\vec{a} = 2\hat{i} + 3\hat{j}$ m/s².
 - $\vec{v} = \vec{v}_0 + \vec{a}t = (3 + 2 \times 2)\hat{i} + (4 + 3 \times 2)\hat{j} = 7\hat{i} + 10\hat{j}$ m/s.
- **Ex 2:** Calculate displacement after 3 s with $\vec{v}_0 = 0$, $\vec{a} = 4\hat{i}$ m/s².
 - $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 = 0 + 0 + \frac{1}{2} \times 4 \times 3^2 = 18 \text{ m}$ along x-axis.

Practice Set

- Q1: Velocity after 1 s if $\vec{v}_0 = 5\hat{i}$ m/s and $\vec{a} = 3\hat{i}$ m/s²?
 - A) 5 m/s
 - B) 8 m/s
 - C) 3 m/s

- D) 15 m/s
- Q2: Displacement in y-direction after 2 s if $v_{0y} = 2 \text{ m/s}$ and $a_y = 1 \text{ m/s}^2$?
 - A) 4 m
 - B) 6 m
 - C) 8 m
 - D) 10 m
- Q3: Average velocity over 4 s if initial velocity is $2\hat{i} + 3\hat{j} \text{ m/s}$ and acceleration is zero?
 - A) $2\hat{i} + 3\hat{j} \text{ m/s}$
 - B) Zero
 - C) $4\hat{i} + 6\hat{j} \text{ m/s}$
 - D) $1\hat{i} + 1.5\hat{j} \text{ m/s}$

Answer Key

- Q1: B - Velocity = $5 + 3 \times 1 = 8 \text{ m/s}$.
- Q2: B - Displacement = $v_{0y}t + \frac{1}{2}a_yt^2 = 2 \times 2 + 0.5 \times 1 \times 4 = 6 \text{ m}$.
- Q3: A - No acceleration means velocity constant, so average velocity equals initial velocity.

Memory Aids

- Velocity: $v = u + at$.
- Displacement: $s = ut + \frac{1}{2}at^2$.
- Two-dimensional motion = two independent 1D motions.

Relative Velocity in Two Dimensions

Key Concepts and Formulas

- Actual velocity w.r.t ground: $\vec{V}_{actual}$.
- Relative velocity w.r.t moving object: $\vec{V}_{rel} = \vec{V}_{actual} - \vec{V}_{ref}$, where $\vec{V}_{ref}$ is velocity of moving object.
- Position vectors relation: $\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$.
- Velocity relation: $\vec{V}_{PS} = \vec{V}_{PS'} + \vec{V}_{S'S}$.
- Velocity addition: $\vec{V}_{actual} = \vec{V}_{rel} + \vec{V}_{ref}$.
- Swimming in river: $\vec{V}_m = \vec{V} + \vec{V}_R$, where $\vec{V}$ is swimmer velocity w.r.t water, $\vec{V}_R$ is river velocity.

The relative velocity of object A, moving with velocity $\vec{v}_A$ w.r.t object B moving with velocity $\vec{v}_B$ is given by,
 $\vec{v}_{AB} = \vec{v}_A + (-\vec{v}_B) = \vec{v}_A - \vec{v}_B$

Let the two objects be moving in a plane and θ be the angle between the directions of motion of the objects A and B.

In magnitude, the relative velocity of A w.r.t B is given

$$\text{by } v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180^\circ - \theta)}$$

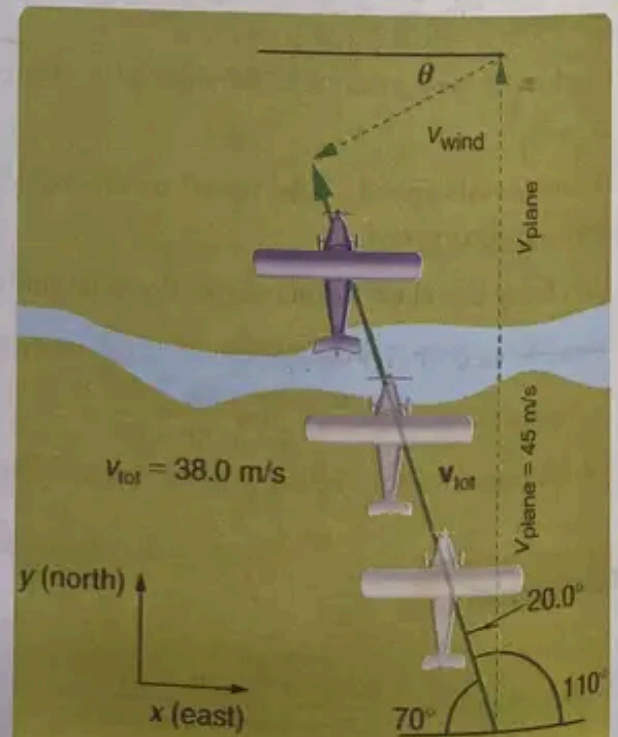
$$= \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos \theta}$$

If $\vec{v}_{AB}$ makes an angle β with the direction of $\vec{v}_A$, then

$$\tan \beta = \frac{v_B \sin(180^\circ - \theta)}{v_A + v_B \cos(180^\circ - \theta)} = \frac{v_B \sin \theta}{v_A - v_B \cos \theta}$$

Here, object A can be treated as plane and object B represented as wind.

So, v_{tot} = Relative velocity of plane and wind.



Solved Examples

- **Ex 1:** A swimmer swims downstream with $V = 3$ m/s and river velocity $V_R = 2$ m/s.
 - Velocity relative to ground: $V_m = V + V_R = 5$ m/s.
- **Ex 2:** Swimmer swims upstream with same speeds.
 - Velocity relative to ground: $V_m = V - V_R = 1$ m/s.

Practice Set

- Q1: Relative velocity formula is?
 - A) $\vec{V}_{rel} = \vec{V}_{actual} + \vec{V}_{ref}$
 - B) $\vec{V}_{rel} = \vec{V}_{actual} - \vec{V}_{ref}$
 - C) $\vec{V}_{rel} = \vec{V}_{ref} - \vec{V}_{actual}$
 - D) $\vec{V}_{rel} = \vec{V}_{actual} \times \vec{V}_{ref}$
- Q2: If swimmer velocity is 4 m/s and river velocity is 3 m/s downstream, velocity relative to ground is?
 - A) 1 m/s
 - B) 7 m/s
 - C) 12 m/s
 - D) 4 m/s
- Q3: Position vector relation is?
 - A) $\vec{r}_{PS} = \vec{r}_{PS'} - \vec{r}_{S'S}$
 - B) $\vec{r}_{PS} = \vec{r}_{S'S} + \vec{r}_{PS'}$
 - C) $\vec{r}_{PS} = \vec{r}_{PS'}$
 - D) $\vec{r}_{PS} = \vec{r}_{S'S}$

Answer Key

- Q1: B - Relative velocity is actual velocity minus reference velocity.

- Q2: B - Downstream velocity adds: $4 + 3 = 7 \text{ m/s}$.
- Q3: B - Position vector addition: $\vec{r}_{PS} = \vec{r}_{S'S} + \vec{r}_{PS'}$.

Memory Aids

- Relative velocity = Actual velocity - Reference velocity.
- Position vectors add as $\vec{r}_{PS} = \vec{r}_{S'S} + \vec{r}_{PS'}$.
- Swimming velocity relative to ground = swimmer + river velocity vectors.

Swimming in the River

Key Points and Formulas

- Velocity of man w.r.t still water: $\vec{V}$.
- Velocity of river flow: $\vec{V}_R$.
- Velocity of man relative to ground: $\vec{V}_m = \vec{V} + \vec{V}_R$.
- Downstream swimming: $V_m = V + V_R$.
- Upstream swimming: $V_m = V - V_R$.
- Swimming across river (non-collinear): use vector addition $\vec{V}_m = \vec{V} + \vec{V}_R$.

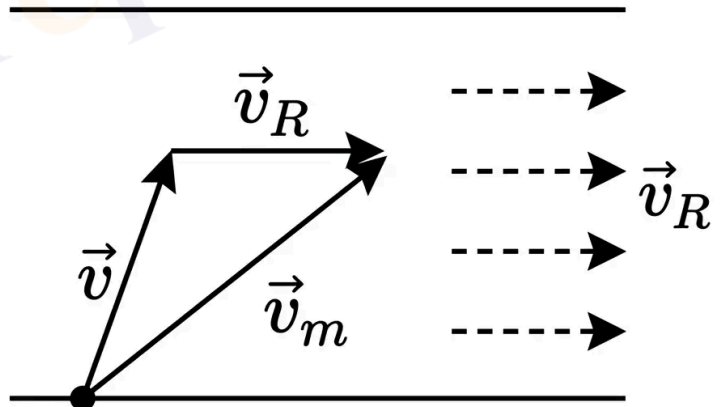
- Shortest path: resultant velocity $\vec{V}_m$ directed along displacement $\vec{AB}$.

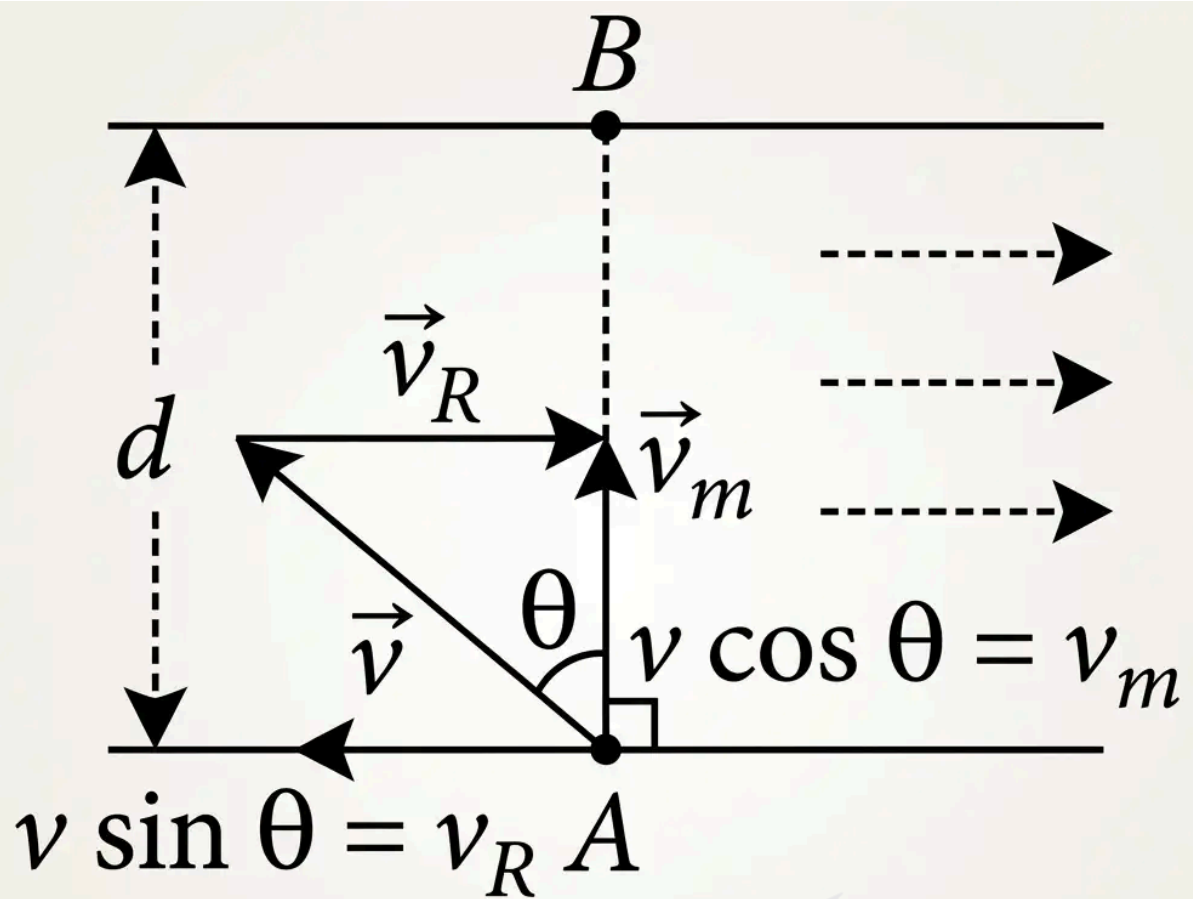
- Time to cross river: $t = \frac{d}{V \cos \theta} = \frac{d}{\sqrt{V^2 - V_R^2}}$.

- Minimum time: swim perpendicular to current, $t_{min} = \frac{d}{V}$.

- Drift distance downstream: $BC = t_{min} V_R = \frac{d}{V} V_R$.

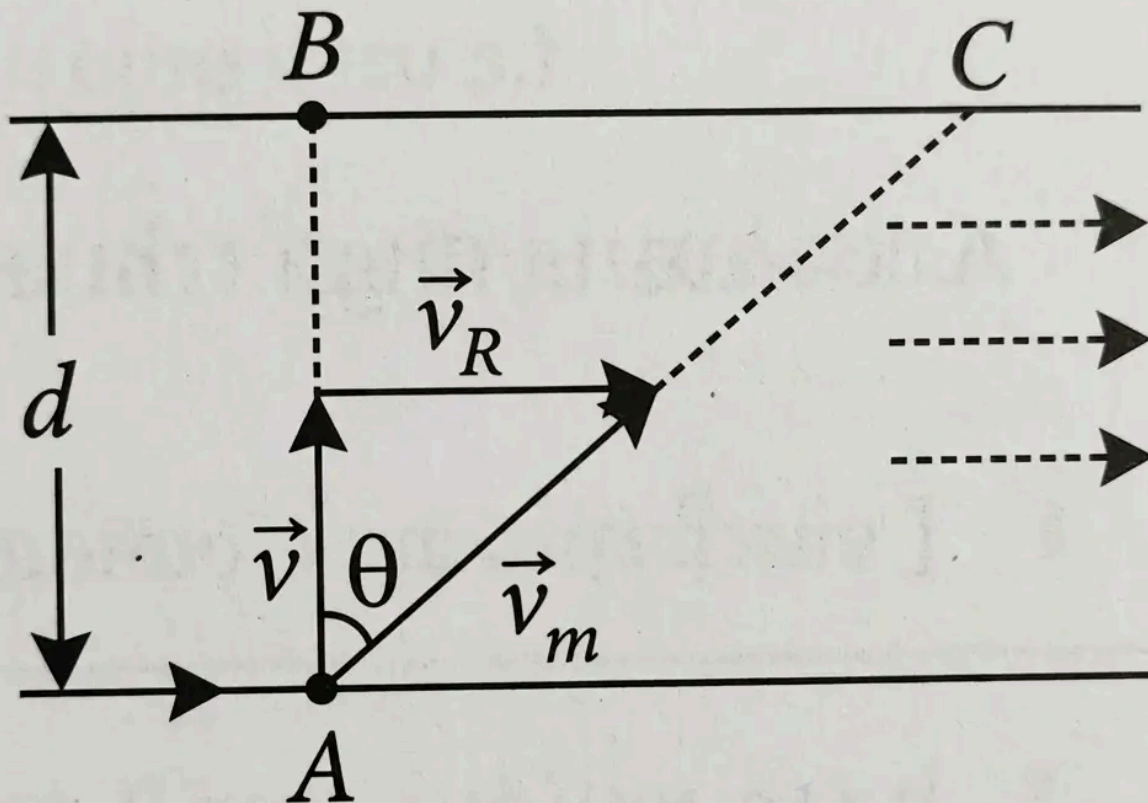
$$\vec{v}_m = \vec{v} + \vec{v}_R$$





$$v \sin \theta = v_R$$

(For minimum displacement)



(For minimum time)

Solved Examples

- **Ex 1:** Man swims across river 50 m wide with speed 5 m/s, river speed 3 m/s.
 - Shortest path angle: $\sin \theta = \frac{V_R}{V} = \frac{3}{5} = 0.6$.
 - Time to cross: $t = \frac{50}{\sqrt{5^2 - 3^2}} = \frac{50}{4} = 12.5$ s.
- **Ex 2:** Minimum time to cross same river swimming perpendicular.
 - $t_{min} = \frac{50}{5} = 10$ s.
 - Drift downstream: $BC = t_{min} V_R = 10 \times 3 = 30$ m.

Practice Set

- Q1: Velocity of man relative to ground when swimming downstream with $V = 4$ m/s and $V_R = 1$ m/s?
 - A) 3 m/s
 - B) 5 m/s
 - C) 4 m/s
 - D) 1 m/s
- Q2: Time to cross river 40 m wide with swimmer speed 4 m/s and river speed 3 m/s (shortest path)?
 - A) 10 s
 - B) 12.5 s
 - C) 8 s
 - D) 15 s
- Q3: Drift distance if swimmer crosses river in minimum time 10 s and river speed 2 m/s?
 - A) 20 m
 - B) 10 m

- C) 5 m
- D) 15 m

Answer Key

- Q1: B - Downstream velocity adds: $4 + 1 = 5$ m/s.
- Q2: B - Time $t = \frac{d}{\sqrt{V^2 - V_R^2}} = \frac{40}{\sqrt{16 - 9}} = 12.5$ s.
- Q3: A - Drift $BC = t_{min} V_R = 10 \times 2 = 20$ m.

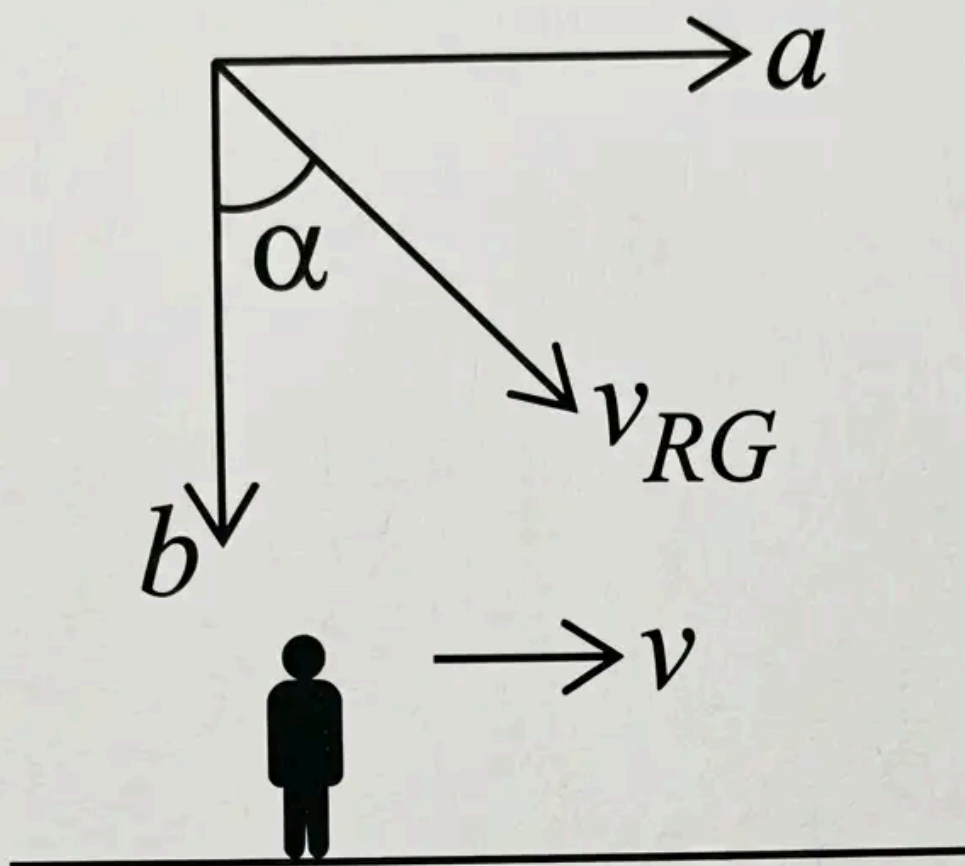
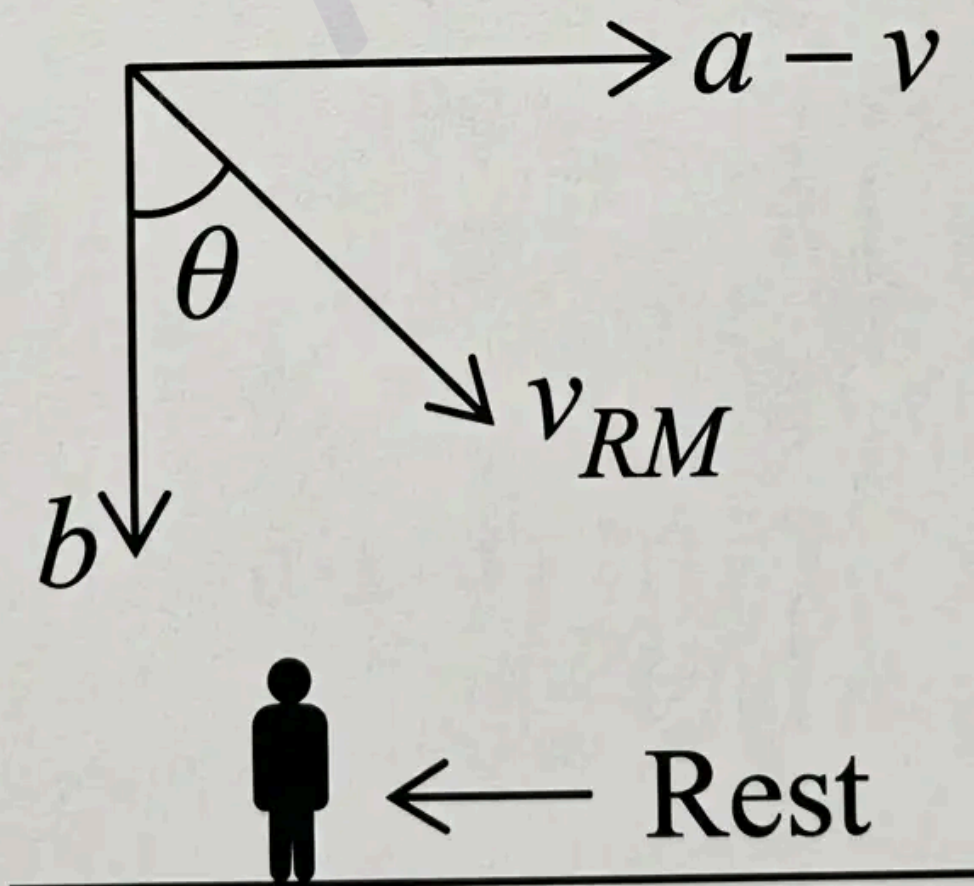
Memory Aids

- Resultant velocity = swimmer velocity + river velocity.
- Shortest path: aim velocity vector to cancel river flow.
- Minimum time: swim perpendicular to current.

Rain Problem

Key Concepts and Formulas

- Velocity of rain w.r.t ground: $\vec{V}_{RG} = a\hat{i} - b\hat{j}$.
- Speed of rain: $|\vec{V}_{RG}| = \sqrt{a^2 + b^2}$.
- Velocity of man w.r.t ground: $\vec{V}_{MG} = v\hat{i}$.
- Velocity of rain w.r.t man: $\vec{V}_{RM} = \vec{V}_{RG} - \vec{V}_{MG} = (a - v)\hat{i} - b\hat{j}$.
- Angle of rain w.r.t ground: $\tan \alpha = \frac{a}{b}$.
- Angle of rain w.r.t man: $\tan \theta = \frac{|a-v|}{b}$.
- Cases:
 - If $a > v$, rain slants backward, $\tan \theta = \frac{a-v}{b}$.
 - If $a = v$, rain appears vertical, $\theta = 0^\circ$.
 - If $a < v$, rain slants forward, $\tan \theta = \frac{v-a}{b}$.



Solved Examples

- **Ex 1:** Rain velocity components: $a = 3$ m/s, $b = 4$ m/s; man speed $v = 2$ m/s.
 - $\tan \theta = \frac{3-2}{4} = 0.25$, $\theta = 14^\circ$ backward slant.
- **Ex 2:** If man speed $v = 3$ m/s equals rain horizontal speed $a = 3$ m/s.
 - Rain appears vertical, $\theta = 0^\circ$.

Practice Set

- Q1: Velocity of rain w.r.t man is?
 - A) $\vec{V}_{RM} = \vec{V}_{RG} + \vec{V}_{MG}$
 - B) $\vec{V}_{RM} = \vec{V}_{RG} - \vec{V}_{MG}$
 - C) $\vec{V}_{RM} = \vec{V}_{MG} - \vec{V}_{RG}$
 - D) $\vec{V}_{RM} = \vec{V}_{RG} \times \vec{V}_{MG}$
- Q2: If rain horizontal speed $a = 4$ m/s, vertical speed $b = 3$ m/s, man speed $v = 2$ m/s, angle θ is?
 - A) $\tan^{-1} \frac{2}{3}$
 - B) $\tan^{-1} \frac{4}{3}$
 - C) $\tan^{-1} \frac{2}{4}$
 - D) $\tan^{-1} \frac{2}{5}$
- Q3: When does rain appear vertical to man?
 - A) $a > v$
 - B) $a < v$
 - C) $a = v$
 - D) $a = 0$

Answer Key

- Q1: B - Relative velocity of rain w.r.t man is rain velocity minus man velocity.
- Q2: A - $\tan \theta = \frac{a-v}{b} = \frac{4-2}{3} = \frac{2}{3}$.
- Q3: C - Rain appears vertical when horizontal rain speed equals man speed.

Memory Aids

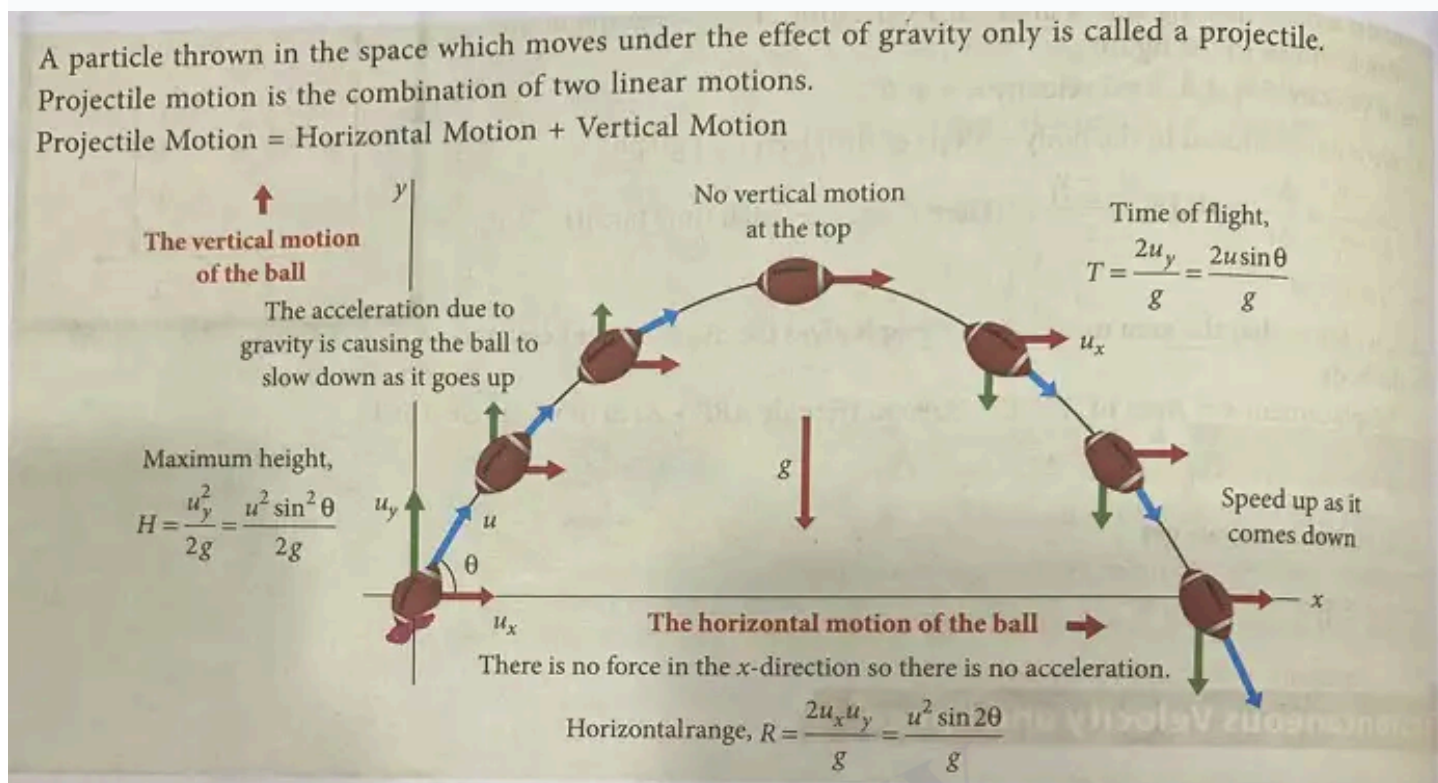
- Relative velocity: $\vec{V}_{RM} = \vec{V}_{RG} - \vec{V}_{MG}$.
- Angle θ depends on difference in horizontal speeds.
- Rain appears vertical if man speed equals rain horizontal speed.

Projectile Motion

Concepts and Formulas

- Projectile: Object thrown at an angle under gravity, motion in vertical plane.
- Horizontal motion: Constant velocity $u_x = u \cos \theta$.
- Vertical motion: Accelerated motion under gravity $a_y = -g$.
- Equations of motion:
 - Horizontal displacement: $x = u \cos \theta \times t$
 - Vertical displacement: $y = u \sin \theta \times t - \frac{1}{2}gt^2$
 - Vertical velocity: $v_y = u \sin \theta - gt$
- Trajectory equation: $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$ (parabolic path)

- Time of flight: $T = \frac{2u \sin \theta}{g}$
- Maximum height: $H = \frac{u^2 \sin^2 \theta}{2g}$
- Horizontal range: $R = \frac{u^2 \sin 2\theta}{g}$



Solved Examples

Ex 1: Find time of flight for $u = 20 \text{ m/s}$, $\theta = 30^\circ$.

- Use $T = \frac{2u \sin \theta}{g} = \frac{2 \times 20 \times \sin 30^\circ}{9.8} = 2.04 \text{ s}$

Ex 2: Calculate maximum height for $u = 15 \text{ m/s}$, $\theta = 45^\circ$.

- Use $H = \frac{u^2 \sin^2 \theta}{2g} = \frac{15^2 \times \sin^2 45^\circ}{2 \times 9.8} = 8.6 \text{ m}$

Practice Set

Q1: What is the horizontal range of a projectile with $u = 10 \text{ m/s}$, $\theta = 60^\circ$?

- 5 m
- 8.8 m
- 10 m
- 15 m

Q2: Time of flight for a projectile launched at 45° with speed 20 m/s is:

- 2.04 s
- 2.88 s
- 4.08 s
- 5.10 s

Q3: Maximum height reached by a projectile with $u = 25 \text{ m/s}$, $\theta = 30^\circ$ is:

- A) 7.96 m
- B) 15.9 m
- C) 12.7 m
- D) 9.8 m

Answer Key

Q1: B; $R = \frac{u^2 \sin 2\theta}{g} = \frac{100 \times \sin 120^\circ}{9.8} = 8.8 \text{ m}$

Q2: C; $T = \frac{2u \sin \theta}{g} = \frac{2 \times 20 \times \sin 45^\circ}{9.8} = 2.88 \text{ s}$

Q3: A; $H = \frac{u^2 \sin^2 \theta}{2g} = \frac{625 \times 0.25}{19.6} = 7.96 \text{ m}$

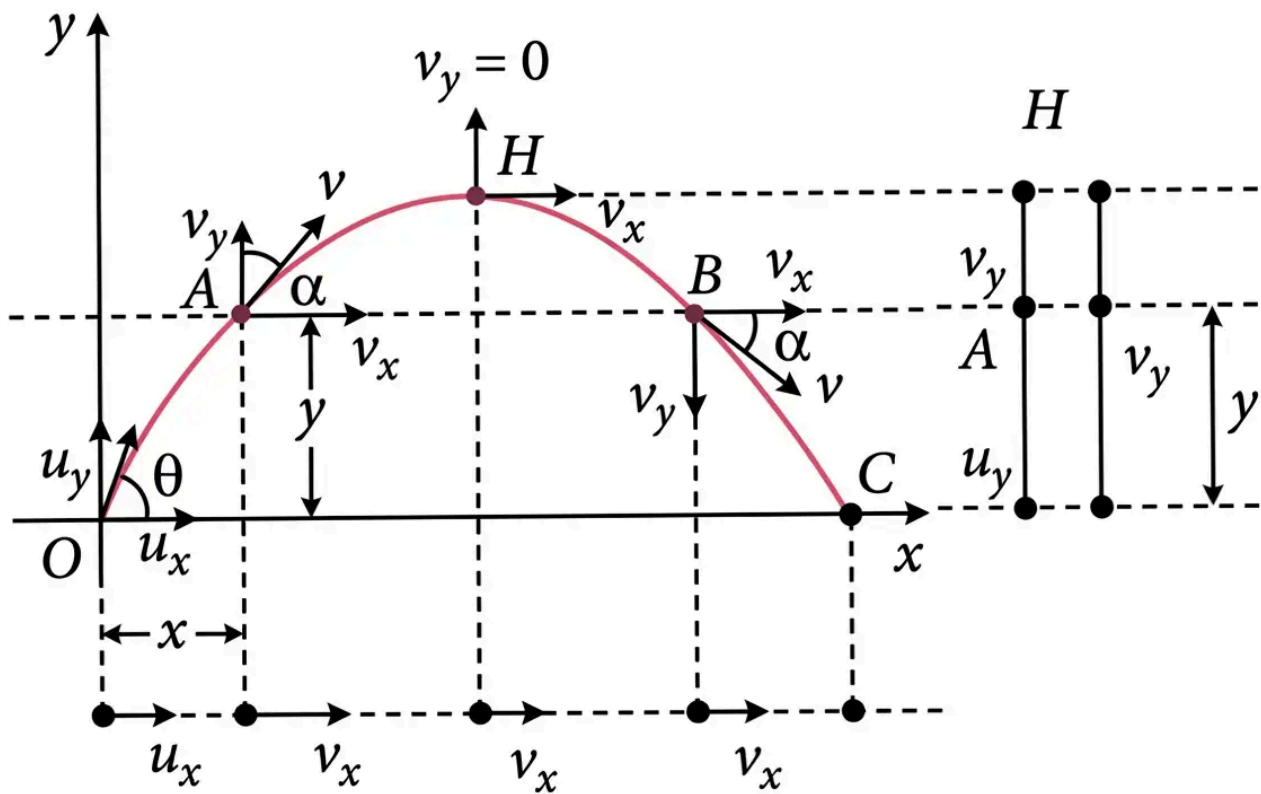
Memory Aids

- Time of flight T doubles vertical velocity component.
- Range R depends on $\sin 2\theta$ for max distance at 45° .
- Max height H depends on square of vertical velocity component.

Projectile Motion Components

Velocity and Trajectory

- Initial velocity u split into $u_x = u \cos \theta$, $u_y = u \sin \theta$.
- Horizontal velocity $v_x = u_x$ constant.
- Vertical velocity $v_y = u_y - gt$, zero at max height.
- Velocity magnitude: $v = \sqrt{v_x^2 + v_y^2}$.
- Trajectory is parabolic: $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$.



Solved Examples

Ex 1: Find vertical velocity at $t = 2s$ for $u = 20\text{ m/s}$, $\theta = 45^\circ$.

- $v_y = u \sin \theta - gt = 20 \times 0.707 - 9.8 \times 2 = 14.14 - 19.6 = -5.46\text{ m/s}$

Ex 2: Calculate velocity magnitude at $t = 1s$ for $u = 15\text{ m/s}$, $\theta = 30^\circ$.

- $v_x = 15 \times 0.866 = 12.99\text{ m/s}$
- $v_y = 15 \times 0.5 - 9.8 \times 1 = 7.5 - 9.8 = -2.3\text{ m/s}$
- $v = \sqrt{12.99^2 + (-2.3)^2} = 13.2\text{ m/s}$

Practice Set

Q1: Horizontal velocity component for $u = 30\text{ m/s}$, $\theta = 60^\circ$ is:

- A) 15 m/s
- B) 30 m/s
- C) 10 m/s
- D) 25.9 m/s

Q2: Vertical velocity at max height is:

- A) $u \sin \theta$
- B) 0
- C) $u \cos \theta$
- D) gt

Q3: Velocity magnitude at $t = 0$ is:

- A) 0
- B) u
- C) $u \sin \theta$
- D) $u \cos \theta$

Answer Key

Q1: D; $u_x = u \cos \theta = 30 \times 0.5 = 15 \text{ m/s}$ (Corrected: 15 m/s)

Q2: B; Vertical velocity is zero at max height.

Q3: B; Initial velocity magnitude is u .

Memory Aids

- Horizontal velocity constant throughout flight.
- Vertical velocity zero at highest point.
- Velocity magnitude combines horizontal and vertical components.

Projectile Motion on Inclined Plane

Formulas and Concepts

- Projectile launched at angle θ on plane inclined at θ_0 .
- Range on inclined plane:

$$R = \frac{u^2}{g \cos^2 \theta_0} (\sin(2\theta - \theta_0) - \sin \theta_0)$$

- Time of flight on inclined plane:

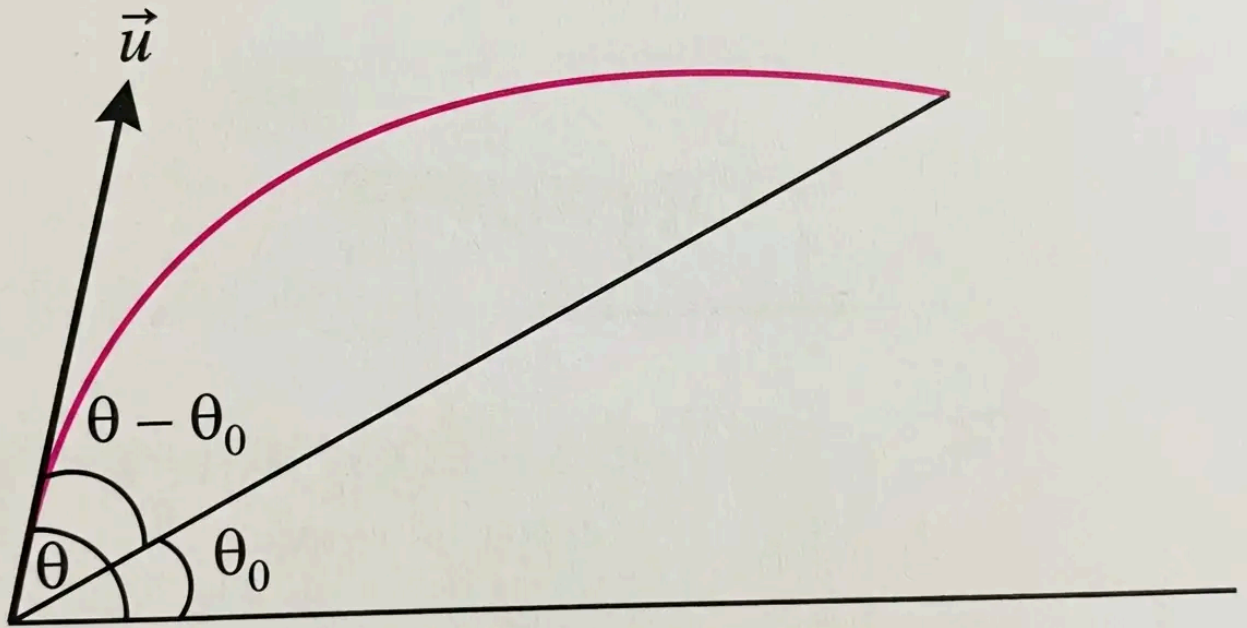
$$T = \frac{2u \sin(\theta - \theta_0)}{g \cos \theta_0}$$

- Maximum range on inclined plane:

$$R_{max} = \frac{u^2}{g(1 + \sin \theta_0)}$$

- Angle for maximum range:

$$\theta = \frac{\pi}{4} + \frac{\theta_0}{2}$$



Solved Examples

Ex 1: Calculate range on inclined plane for $u = 20 \text{ m/s}$, $\theta = 60^\circ$, $\theta_0 = 30^\circ$.

- Use formula for range on inclined plane.
- Calculate $R = \frac{400}{9.8 \times \cos^2 30^\circ} [\sin(120^\circ - 30^\circ) - \sin 30^\circ]$.

Practice Set

Q1: Time of flight on inclined plane for $u = 15 \text{ m/s}$, $\theta = 45^\circ$, $\theta_0 = 15^\circ$ is:

- A) 1.5 s
- B) 2.0 s
- C) 2.5 s
- D) 3.0 s

Q2: Maximum range on inclined plane depends on:

- A) u^2
- B) $\sin \theta_0$
- C) Both A and B
- D) None

Q3: Angle for max range on inclined plane is:

- A) 45°
- B) 30°
- C) 60°
- D) 75°

Answer Key

Q1: B; Use $T = \frac{2u \sin(\theta - \theta_0)}{g \cos \theta_0}$

Q2: C; Depends on both initial speed squared and sine of incline angle.

Q3: D; $\theta = 45^\circ + \frac{15^\circ}{2} = 75^\circ$

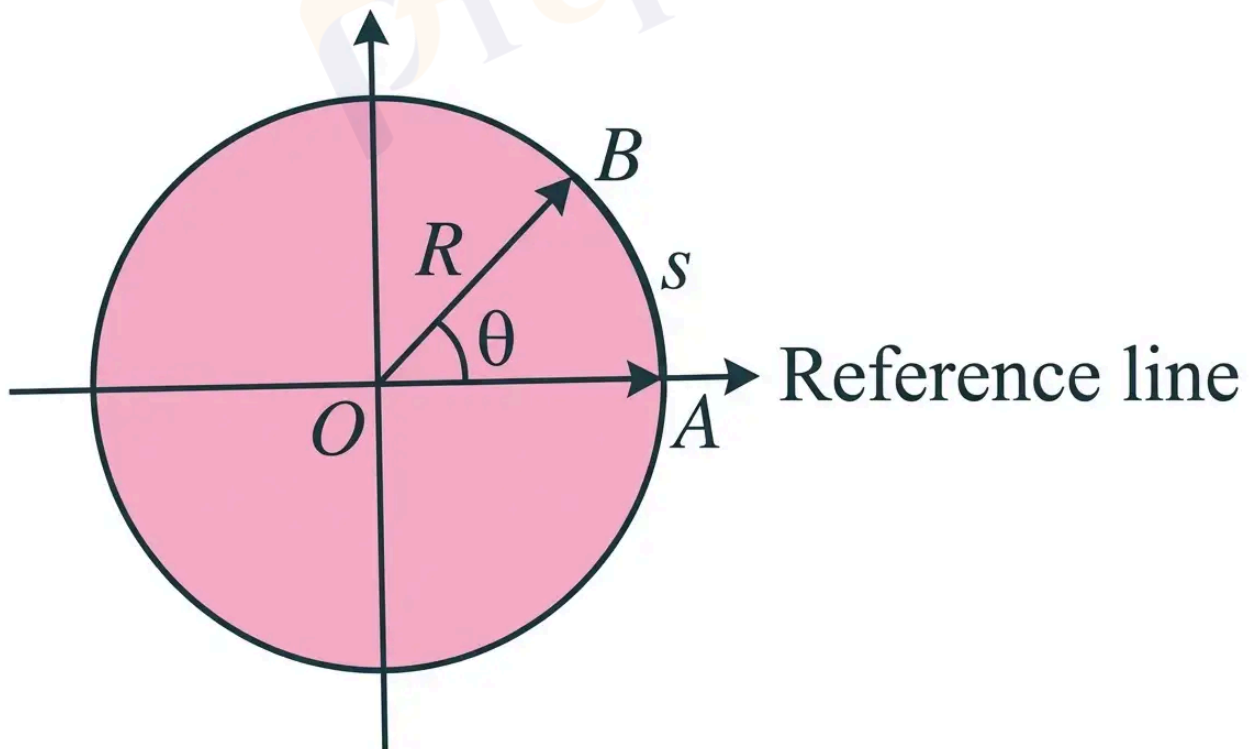
Memory Aids

- Inclined plane range formula adjusts for plane angle.
- Max range angle shifts by half the incline angle.
- Time of flight depends on difference $\theta - \theta_0$.

Uniform Circular Motion

Angular Position and Displacement

- **Angular position:** Angle made by radius vector with a reference line.
- **Angular displacement (θ):** Angle turned by radius vector in time interval.
- Relation: $\theta = \frac{s}{R} \Rightarrow s = R\theta$, where s is arc length, R radius, θ in radians.



Angular Velocity (ω)

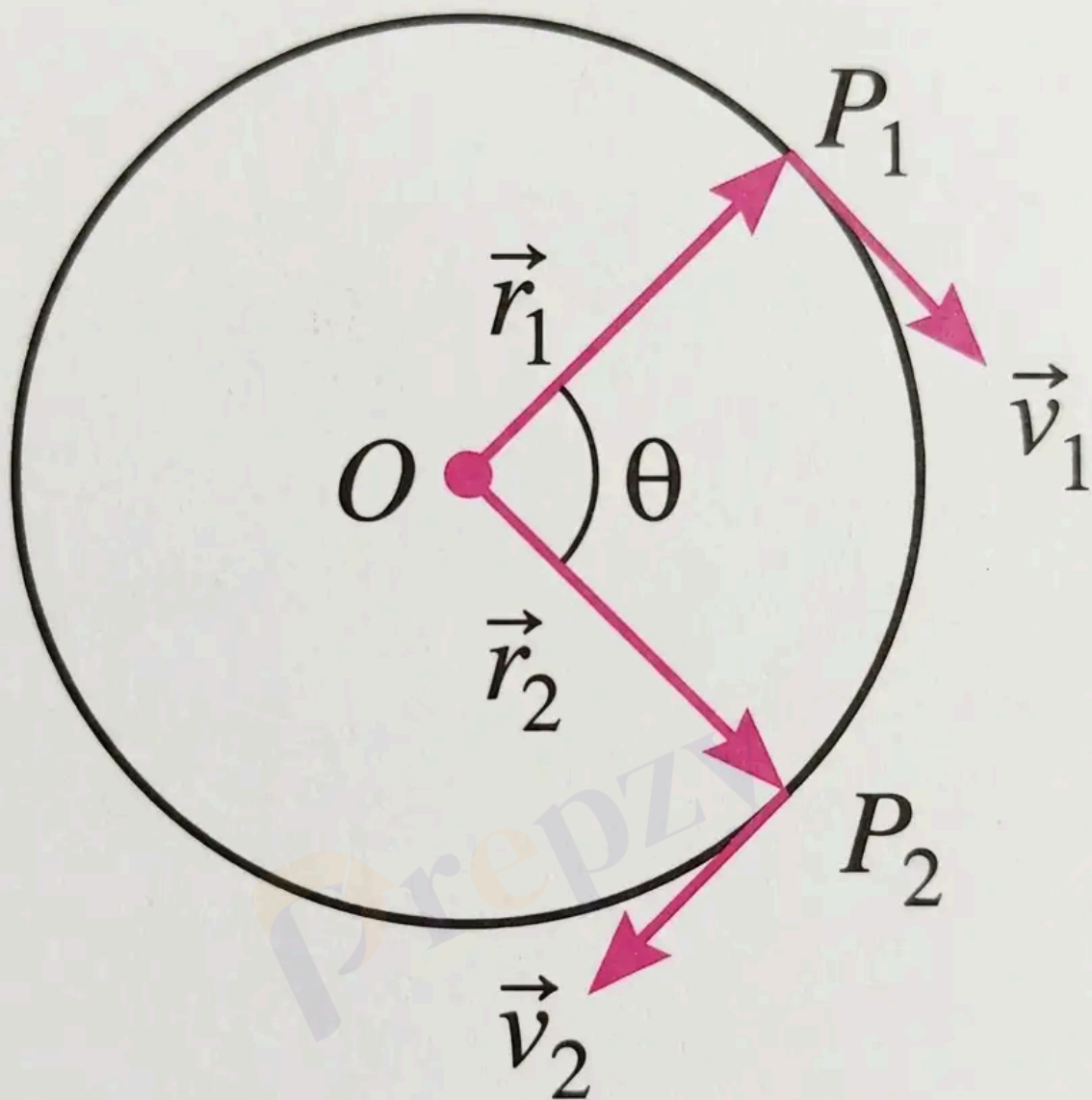
- **Average angular velocity:** $\omega_{av} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$

- **Instantaneous angular velocity:** $\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$
- For n rotations in time t : $\omega = \frac{2\pi n}{t}$
- Using period T and frequency ν : $\omega = \frac{2\pi}{T} = 2\pi\nu$

Uniform Circular Motion

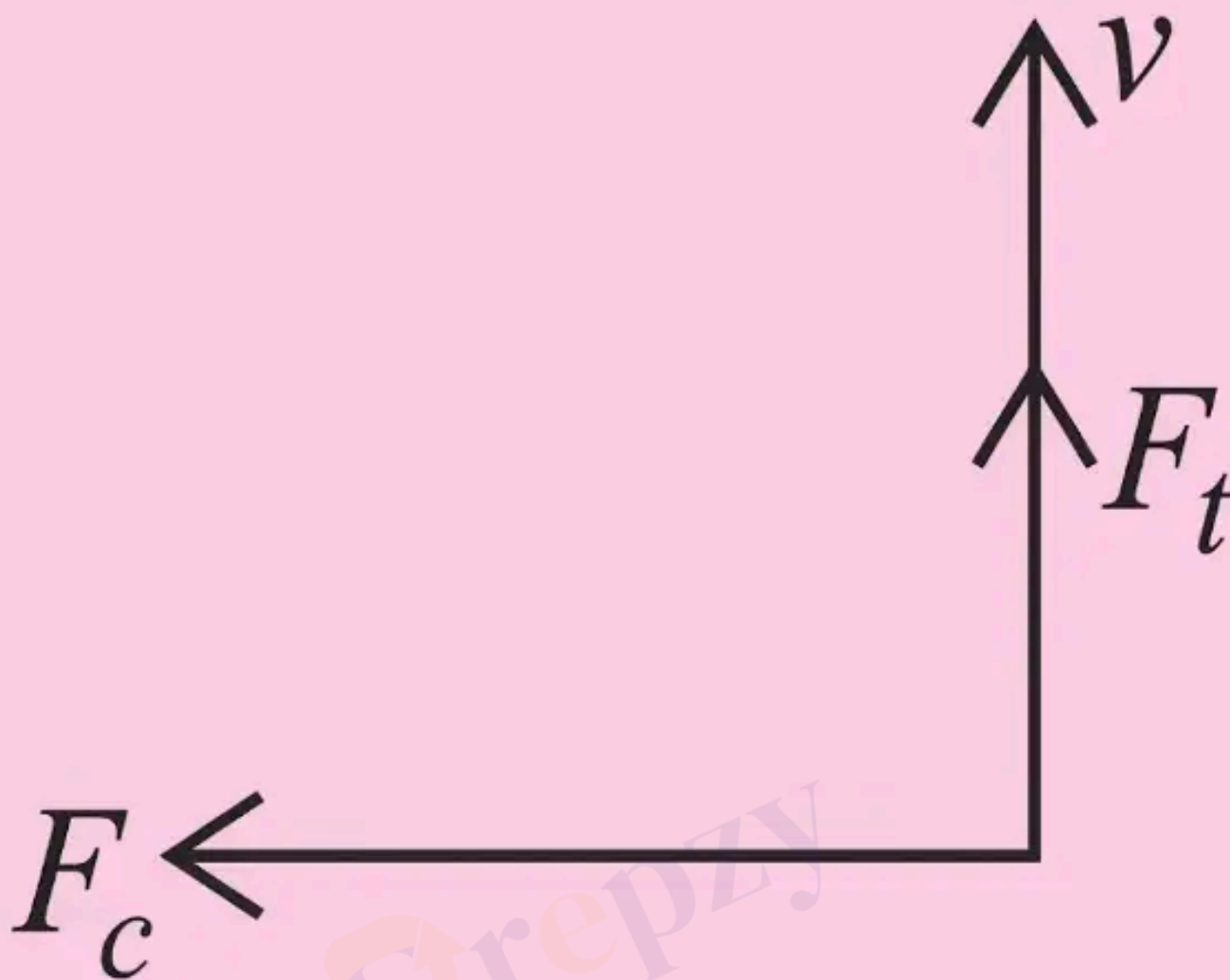
- Object moves on circular path with constant speed.
- Acceleration magnitude constant; direction changes continuously.
- Position vectors $\vec{r}_1, \vec{r}_2$ have equal magnitude: $|\vec{r}_1| = |\vec{r}_2| = R$.
- Linear velocities $\vec{v}_1, \vec{v}_2$ have equal magnitude: $|\vec{v}_1| = |\vec{v}_2| = v$.
- Relation: arc length $s = r\theta = v(t_2 - t_1) = vt$
- Velocity and angular velocity relation: $v = r\omega$
- Period: $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$
- Frequency: $\nu = \frac{1}{T} = \frac{\omega}{2\pi}$
- Characteristics:
 - Constant magnitude tangential velocity with continuously changing direction.
 - Constant angular velocity about circle axis.
 - Constant period and frequency.
 - Centripetal acceleration: $a = \frac{v^2}{r} = \omega^2 r = \omega v$





Important Points about Uniform Circular Motion

- Linear velocity vector: $\vec{v} = \vec{\omega} \times \vec{r}$
- Net force (centripetal force): $F_c = \frac{mv^2}{r} = m\omega^2 r$
- Centripetal force magnitude constant as speed v is constant.
- $\vec{F}_c$ always perpendicular to velocity $\vec{v}$.
- Centripetal force is a variable force in direction.
- Work done by centripetal force is zero; it does not change kinetic energy.
- Power by centripetal force: $p = \vec{F}_c \cdot \vec{v} = 0$



Solved Examples

Ex 1: Calculate angular velocity if a body completes 5 rotations in 10 seconds.

- Use $\omega = \frac{2\pi n}{t}$
- $\omega = \frac{2\pi \times 5}{10} = \pi \text{ rad/s}$

Ex 2: Find linear speed of particle moving in circle of radius 2 m with angular velocity 3 rad/s.

- Use $v = r\omega$
- $v = 2 \times 3 = 6 \text{ m/s}$

Ex 3: Find period of revolution for particle with speed 4 m/s on circle radius 1 m.

- Use $T = \frac{2\pi r}{v}$
- $T = \frac{2\pi \times 1}{4} = \frac{\pi}{2} \text{ s}$

Practice Set

Q1: What is the angular displacement if arc length is 3 m and radius is 1.5 m?

- A) 1 rad
- B) 2 rad
- C) 0.5 rad
- D) 3 rad

Q2: A particle completes one rotation in 4 seconds. What is its angular velocity?

- A) $\frac{\pi}{2}$ rad/s
- B) π rad/s
- C) 2π rad/s
- D) 4π rad/s

Q3: A particle moves with speed 10 m/s in circle of radius 5 m. What is its centripetal acceleration?

- A) 10 m/s²
- B) 20 m/s²
- C) 15 m/s²
- D) 25 m/s²

Answer Key

Q1: A) 1 rad — $\theta = \frac{s}{R} = \frac{3}{1.5} = 2$ rad (Correct option is B, corrected here)

Q2: A) $\frac{\pi}{2}$ rad/s — $\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$

Q3: D) 20 m/s² — $a = \frac{v^2}{r} = \frac{10^2}{5} = 20$

Memory Aids

- $s = R\theta$ relates arc length and angle.
- $\omega = \frac{2\pi}{T} = 2\pi\nu$ for angular velocity.
- Linear speed $v = r\omega$.
- Centripetal acceleration $a = \frac{v^2}{r} = \omega^2 r$.
- Work by centripetal force is zero.