Table of Contents

1. Introduction to Rigid Body Motion
2. Motion Types of Rigid Bodies
3. Centre of Mass
4. Motion of Centre of Mass
5. Linear Momentum of a System of Particles
6. Vector Product of Two Vectors
7. Angular Velocity and its Relation with Linear Velocity
8. Torque
9. Angular Momentum
10. Equilibrium of a Rigid Body
11. Moment of Inertia
12. Radius of Gyration
13. Theorems of Perpendicular and Parallel Axes
14. Moment of Inertia of Regular Bodies
15. Kinematics of Rotational Motion about a Fixed Axis
16. Dynamics of Rotational Motion about a Fixed Axis
17. Work Done by a Torque
18. Comparison of Translational and Rotational Motion
19. Angular Momentum Rotation Fixed Axis
20. Rolling Motion
21. Motion Spherical Body Rough Inclined

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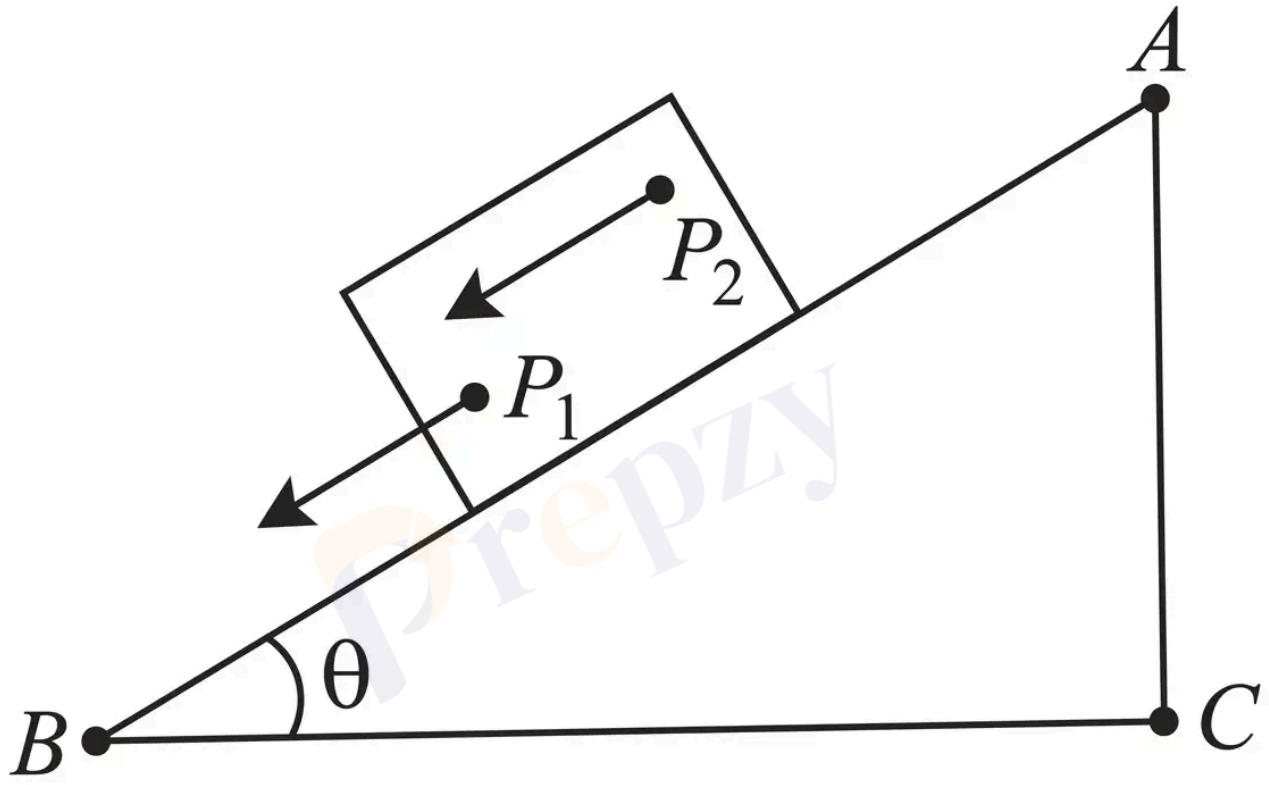
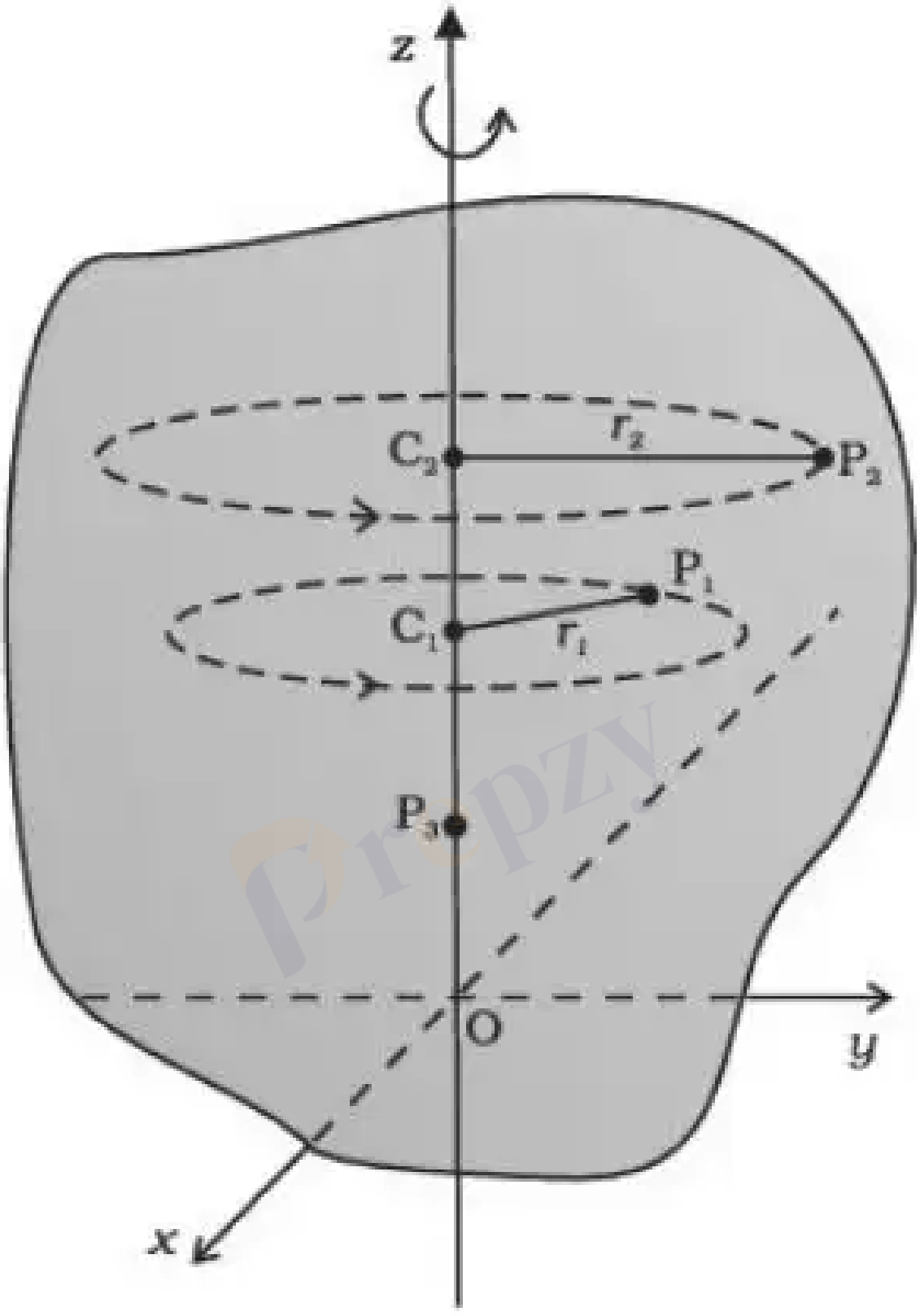
Introduction to Rigid Body Motion

Key Concepts

- Rigid body: Body with fixed shape; distances between particles remain constant.
- Pure translational motion: All particles move with the same velocity at any instant.
- Example: Block sliding down an inclined plane with forces P_1 (gravity component) and P_2 (normal force).
- Rotational motion: Body rotates about a fixed axis; particles move in circles perpendicular to the axis.
- Radius of rotation: Perpendicular distance of a point from the axis.
- Axis of rotation may be fixed or moving (e.g., spinning top with precession).
- Motion types: Pure translation, pure rotation (pivoted), or combination of both.

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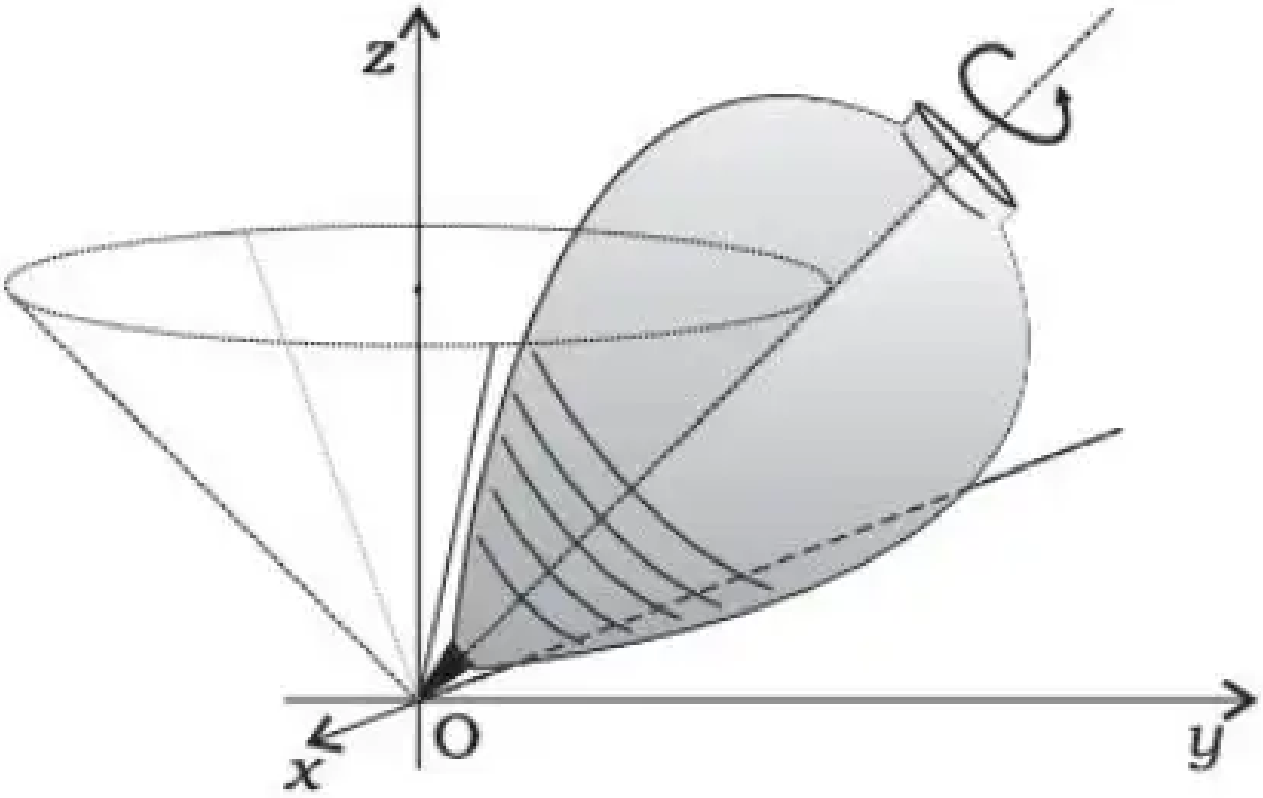


Solved Examples

Ex 1

- Block of mass 5 kg slides down an inclined plane at angle $\theta = 30^\circ$.
- Find the component of gravitational force down the slope: $P_1 = mg\sin\theta = 5 \times 9.8 \times \sin 30^\circ = 24.5\,\mathrm{N}$.

Ex 2

- Point P_1 at radius $r_1 = 0.2\,m$ rotates about z-axis with angular velocity $\omega = 10\,\mathrm{rad/s}$.
- Calculate linear speed: $v = r\omega = 0.2 \times 10 = 2\,\mathrm{m/s}$.

Practice Set

Q1

In pure translational motion, the velocity of all particles is:

- A) Different
- B) Zero
- C) Same
- D) Varies with position

Q2

A rigid body rotates about a fixed axis. The path of each particle is:

- A) Straight line
- B) Ellipse
- C) Circle
- D) Parabola

Q3

A spinning top's axis sweeps out a cone. This motion is called:

- A) Precession
- B) Nutation
- C) Translation
- D) Rotation

Answer Key

- Q1: C - All particles have the same velocity in pure translation.
- Q2: C - Particles move in circles around the axis in rotation.
- Q3: A - The axis movement around vertical is precession.

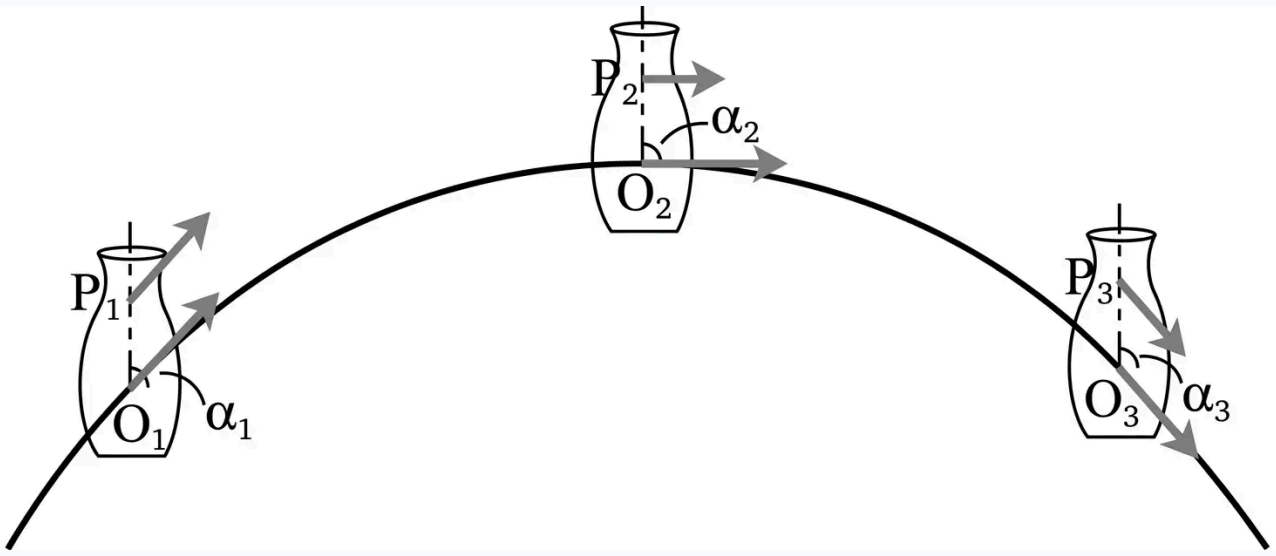
Memory Aids

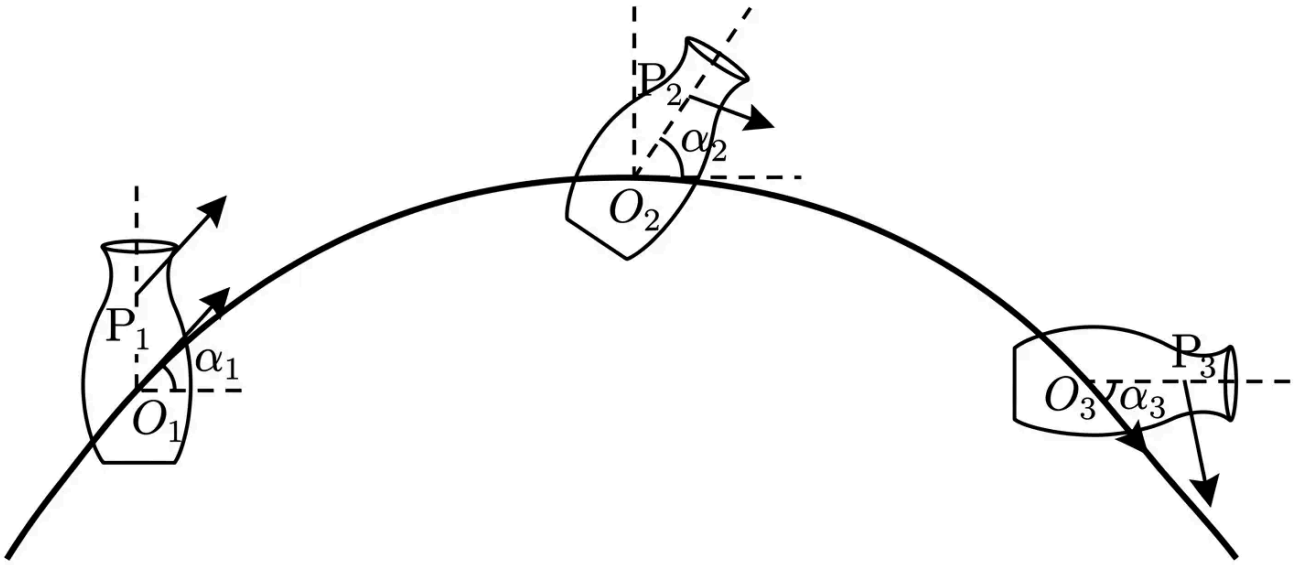
- Rigid body = fixed shape, no particle distance change.
- Translation = same velocity for all particles.
- Rotation = particles move in circles about axis.
- Precession = axis sweeps a cone (spinning top).

Motion Types of Rigid Bodies

Key Concepts

- Pure translation: All particles have same velocity v and acceleration α; $\alpha_1 = \alpha_2 = \alpha_3$.
- Combination of translation and rotation: Velocities and accelerations differ; $\alpha_1 \neq \alpha_2 \neq \alpha_3$.
- Pivoted/fixed rigid body motion is pure rotation.





Solved Examples

Ex 1

- Rigid body moves with pure translation: velocity of all points = 5 m/s.
- Acceleration $\alpha_1 = \alpha_2 = \alpha_3 = 2\,\mathrm{m/s^2}$.

Ex 2

- Rigid body moves with combined motion; velocity at point O = 3 m/s, at point P = 5 m/s.
- Angles $\alpha_1 \neq \alpha_2 \neq \alpha_3$ indicate rotation plus translation.

Practice Set

Q1

In pure translational motion, acceleration of particles is:

- A) Different
- B) Zero
- C) Same
- D) Varies with position

Q2

In combined translation and rotation, velocities of points O and P are:

- A) Equal
- B) Different
- C) Zero
- D) Constant

Q3

Which motion occurs if a rigid body is pivoted?

- A) Pure translation
- B) Combination of translation and rotation
- C) Pure rotation
- D) No motion

Answer Key

- Q1: C - All particles have same acceleration in pure translation.
- Q2: B - Velocities differ in combined motion.
- Q3: C - Pivoted body undergoes pure rotation.

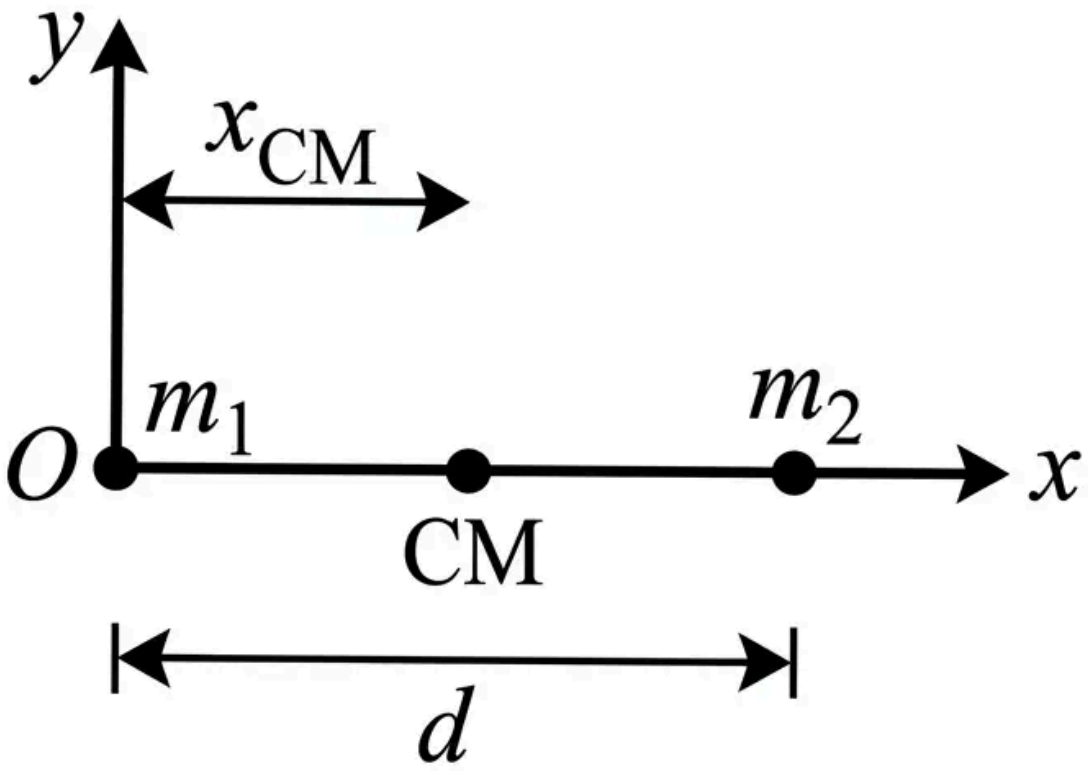
Memory Aids

- Pure translation: equal velocity & acceleration.
- Combined motion: different velocities & angles.
- Pivoted body = pure rotation.

Centre of Mass

Key Concepts

- Centre of mass (CM): Point where total mass is considered concentrated and external forces act.
- For two particles (m_1, m_2) separated by distance d:
- Position of CM from m_1: $x_{CM} = \frac{m_2}{m_1+m_2}d$
- General formula with coordinates x_1, x_2: $x_{CM} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$
- For n particles with masses m_i and position vectors $\vec{r}_i$:
- $\vec{R}_{CM} = \frac{\sum m_i\vec{r}_i}{\sum m_i} = \frac{1}{M}\sum m_i\vec{r}_i$, where $M = \sum m_i$
- Coordinates of CM in 3D:
- $x_{CM} = \frac{\sum m_ix_i}{M}, y_{CM} = \frac{\sum m_iy_i}{M}, z_{CM} = \frac{\sum m_iz_i}{M}$
- For continuous mass distribution (rigid body):
- $x_{CM} = \frac{1}{M}\int x\,dm, y_{CM} = \frac{1}{M}\int y\,dm, z_{CM} = \frac{1}{M}\int z\,dm$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O
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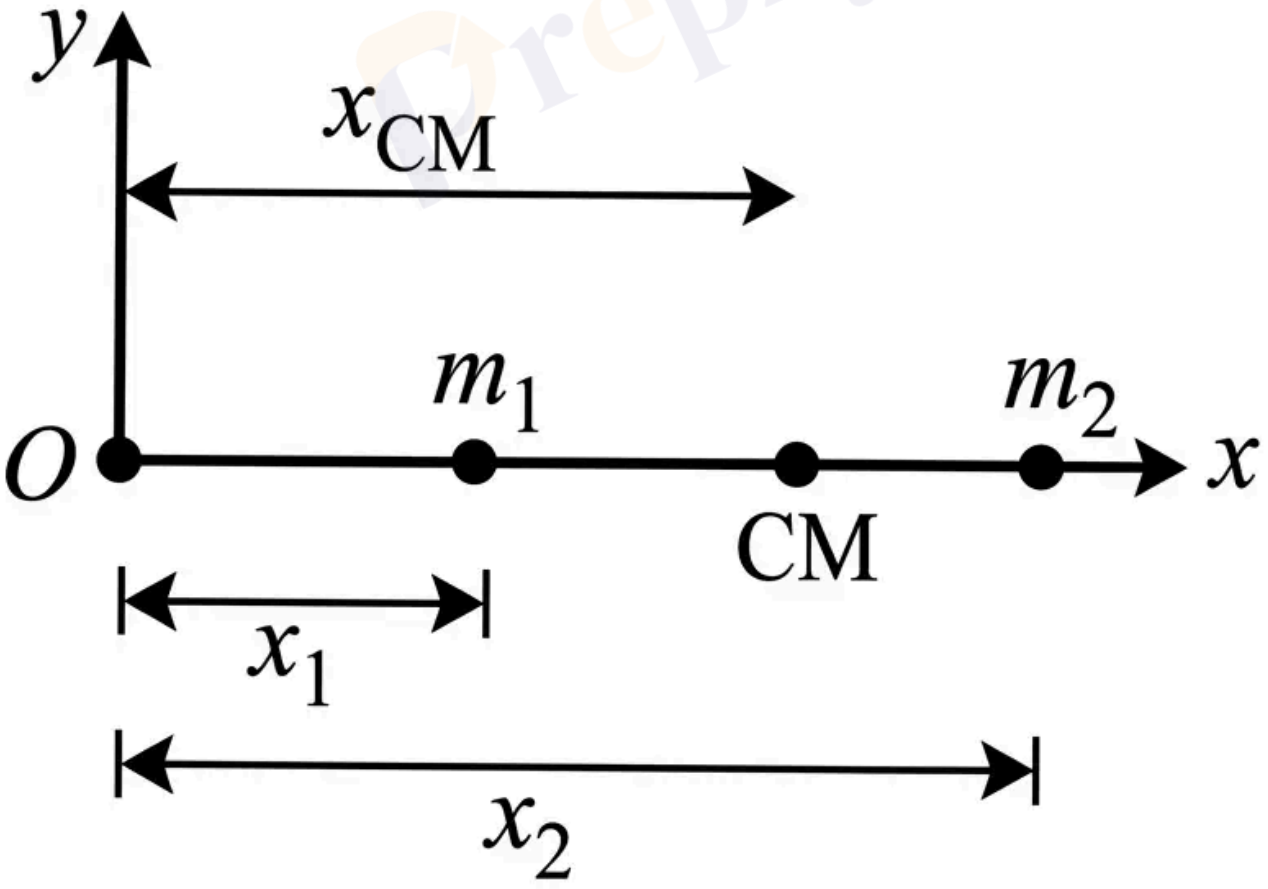
y
x_{CM}
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x_1
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$$\vec{R}_{\text{CM}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$

$$x_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i x_i}{M}, \quad y_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i y_i}{M} \text{ and } z_{\text{CM}} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

$$x_{\text{CM}} = \frac{1}{M}\int x dm, \quad y_{\text{CM}} = \frac{1}{M}\int y dm \text{ and } z_{\text{CM}} = \frac{1}{M}\int z dm$$

Solved Examples

Ex 1

- Two particles: $m_1 = 2\,kg$, $m_2 = 3\,kg$, distance $d = 5\,m$.
- Calculate $x_{CM} = \frac{3}{2+3} \times 5 = 3\,m$ from m_1.

Ex 2

- Three particles at $x_1 = 0$, $x_2 = 4$, $x_3 = 6$ m with masses 1, 2, 3 kg.
- Calculate $x_{CM} = \frac{1\times0+2\times4+3\times6}{1+2+3} = \frac{26}{6} = 4.33\,m$.

Practice Set

Q1

For two particles with masses 2 kg and 3 kg separated by 5 m, the CM from the 2 kg mass is:

- A) 2 m
- B) 3 m
- C) 4 m
- D) 5 m

Q2

In a system of particles, the position vector of CM is:

- A) $\frac{\sum \vec{r}_i}{n}$
- B) $\frac{\sum m_i \vec{r}_i}{\sum m_i}$
- C) $\sum m_i \vec{r}_i$
- D) $\frac{\sum m_i}{\sum \vec{r}_i}$

Q3

For a continuous mass distribution, the coordinate x_{CM} is given by:

- A) $\int x \, dm$
- B) $\frac{1}{M} \int x \, dm$
- C) $\frac{1}{m} \int x \, dm$
- D) $\int x \, dx$

Answer Key

- Q1: B - $x_{CM} = \frac{3}{5} \times 5 = 3\,m$ from 2 kg mass.
- Q2: B - Weighted average position by mass.
- Q3: B - Integral divided by total mass M.

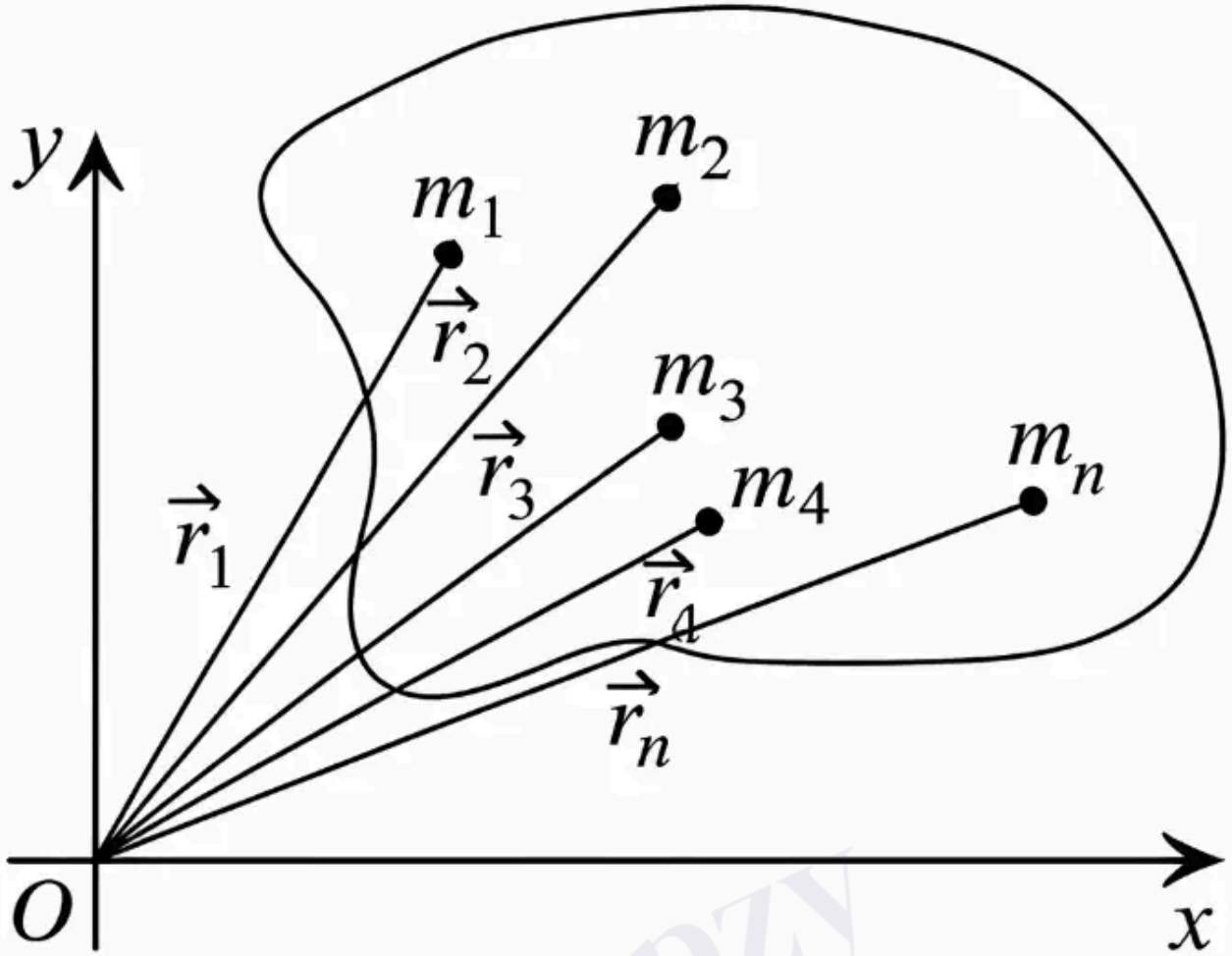
Memory Aids

- CM = weighted average position of mass.
- Two particles: $x_{CM} = \frac{m_2}{m_1+m_2} d$.
- n particles: $\vec{R}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$.
- Continuous mass: use integrals $\frac{1}{M} \int x \, dm$.

Motion of Centre of Mass

Definition and Formula

- Consider a system of n particles with masses $m_1, m_2, \ldots, m_n$.
- Position vectors of particles are $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$.
- Position vector of centre of mass (CM) is $\vec{R}$.
- Formula: $M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n$, where $M = \sum m_i$.
- Differentiating w.r.t time gives velocity relation: $M\vec{V}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n$.
- Acceleration relation: $M\vec{A}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_n\vec{a}_n$.
- Newton's 2nd law for CM: $M\vec{A}_{CM} = \vec{F}_{ext}$ (only external forces affect CM motion).
- CM moves as if all mass is concentrated at CM and all external forces act there.



Solved Examples

- **Ex 1:** Find $\vec{R}$ for two particles $m_1 = 2\,kg$ at $\vec{r}_1 = 3\hat{i}$ m and $m_2 = 3\,kg$ at $\vec{r}_2 = 5\hat{i}$ m.
  - Use $\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{2\times3+3\times5}{2+3} = \frac{6+15}{5} = 4.2\,m\hat{i}$.
- **Ex 2:** Calculate velocity of CM if $m_1 = 1\,kg$ with velocity $2\hat{i}\,m/s$ and $m_2 = 3\,kg$ with velocity $4\hat{i}\,m/s$.
  - $\vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{1\times2+3\times4}{4} = 3.5\,m/s\hat{i}$.

Practice Set

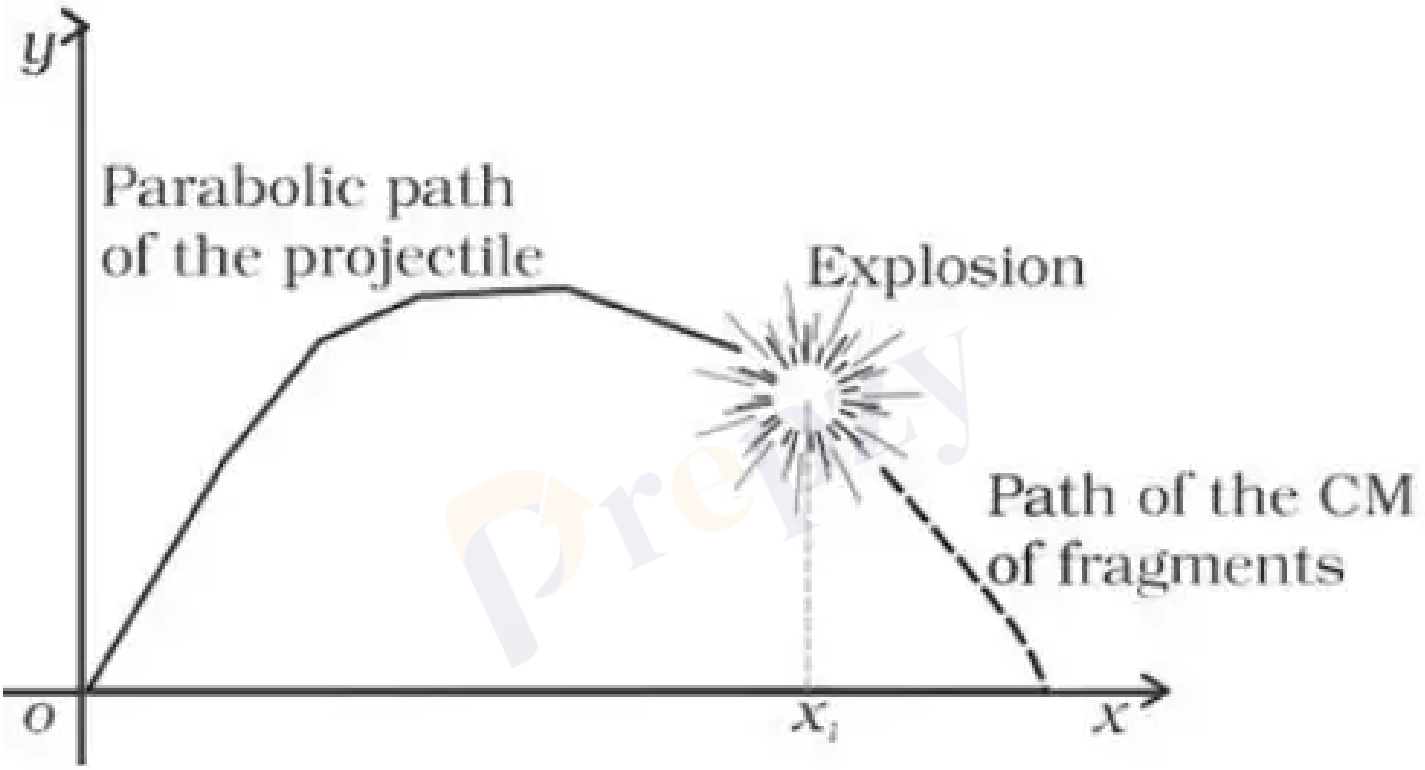
- Q1: The position vector of CM for two particles of masses 2 kg and 3 kg at positions 1 m and 4 m respectively is:
  - A) 2.5 m
  - B) 3.2 m
  - C) 3.8 m
  - D) 4.0 m
- Q2: If the total external force on a system is zero, the acceleration of the centre of mass is:
  - A) Zero
  - B) Constant
  - C) Increasing
  - D) Decreasing
- Q3: A projectile explodes at the highest point of its path. The centre of mass of fragments:
  - A) Falls vertically down
  - B) Continues along original parabolic path
  - C) Moves horizontally
  - D) Stops moving

Answer Key

- Q1: C) 3.8 m - Weighted average position using $\vec{R} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$.
- Q2: A) Zero - No external force means no acceleration of CM.
- Q3: B) Continues along original parabolic path - CM follows same trajectory as if no explosion.

Memory Aids

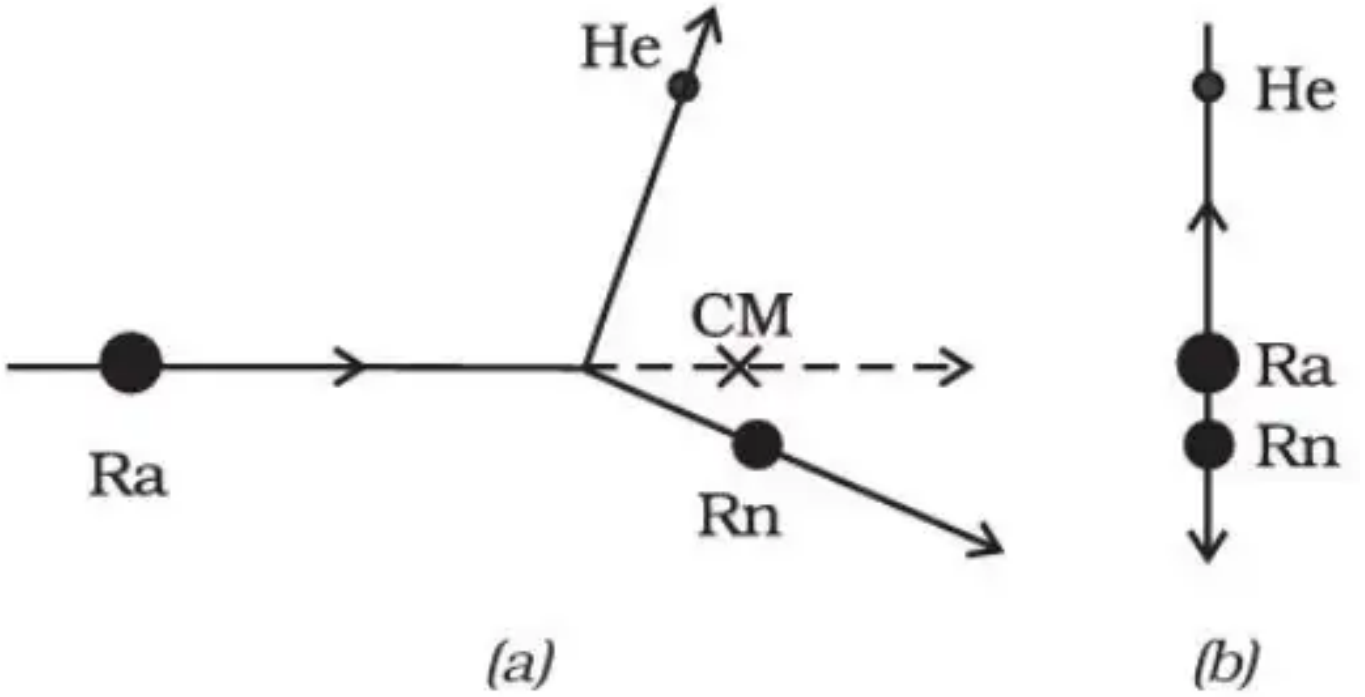
- $M\vec{R} = \sum m_i \vec{r}_i$ - CM position vector formula.
- Velocity of CM: $\vec{V}_{CM} = \frac{\sum m_i \vec{v}_i}{M}$.
- Acceleration of CM: $M\vec{A}_{CM} = \vec{F}_{ext}$.
- CM moves as if all mass and external forces act at CM.



Linear Momentum of a System of Particles

Key Concepts

- Linear momentum $\vec{p}$ of a system of n particles is the vector sum of individual momenta:
  $\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots + \vec{p}_n$
- Relation with center of mass velocity $\vec{V}_{CM}$:
  $M\vec{V}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots + m_n\vec{v}_n$
- Newton's 2nd law for system:
  $\frac{d\vec{p}}{dt} = M\frac{d\vec{V}_{CM}}{dt} = M\vec{A}_{CM} = \vec{F}_{ext}$
- Conservation of linear momentum:
  If external force $\vec{F}_{ext} = 0$, then
  $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{constant}$
- If total external force is zero, center of mass moves with constant velocity.
- Example: Alpha decay of radium nucleus (Ra) into radon nucleus (Rn) and alpha particle (He) shows conservation of momentum with center of mass in uniform motion or at rest.



Solved Examples

- **Ex 1:** Calculate total momentum of two particles with masses $m_1 = 2\,kg$, $m_2 = 3\,kg$ moving at velocities $\vec{v}_1 = 4\hat{i}$ m/s and $\vec{v}_2 = -2\hat{i}$ m/s.
  Solution:
  $\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 = 2 \times 4 + 3 \times (-2) = 8 - 6 = 2\,kg \cdot m/s$
- **Ex 2:** A system with total mass $M = 5\,kg$ has center of mass velocity $\vec{V}_{CM} = 3\hat{j}$ m/s. Find total momentum.
  Solution:
  $\vec{p} = M\vec{V}_{CM} = 5 \times 3 = 15\hat{j}\,kg \cdot m/s$

Practice Set

- Q1: Two particles of masses 1 kg and 2 kg move with velocities 3 m/s and 4 m/s respectively in the same direction. Total momentum is:
  A) 5 kg·m/s
  B) 11 kg·m/s
  C) 14 kg·m/s
  D) 7 kg·m/s
- Q2: If no external force acts on a system, the total momentum:
  A) Increases
  B) Decreases
  C) Remains constant
  D) Becomes zero
- Q3: A radium nucleus at rest splits into radon nucleus and alpha particle. If alpha particle moves with velocity 2×10^7 m/s and mass ratio $m_{Rn} : m_{He} = 4 : 1$, velocity of radon nucleus is:
  A) 0.5×10^7 m/s
  B) 2×10^7 m/s
  C) 8×10^7 m/s
  D) 1×10^7 m/s

Answer Key

- Q1: B - Total momentum = $1 \times 3 + 2 \times 4 = 3 + 8 = 11$ kg·m/s
- Q2: C - No external force means momentum is conserved (constant)

- Q3: A - Using conservation: $m_{Rn}v_{Rn} = m_{He}v_{He} \Rightarrow v_{Rn} = \frac{m_{He}}{m_{Rn}}v_{He} = \frac{1}{4} \times 2 \times 10^7 = 0.5 \times 10^7$ m/s

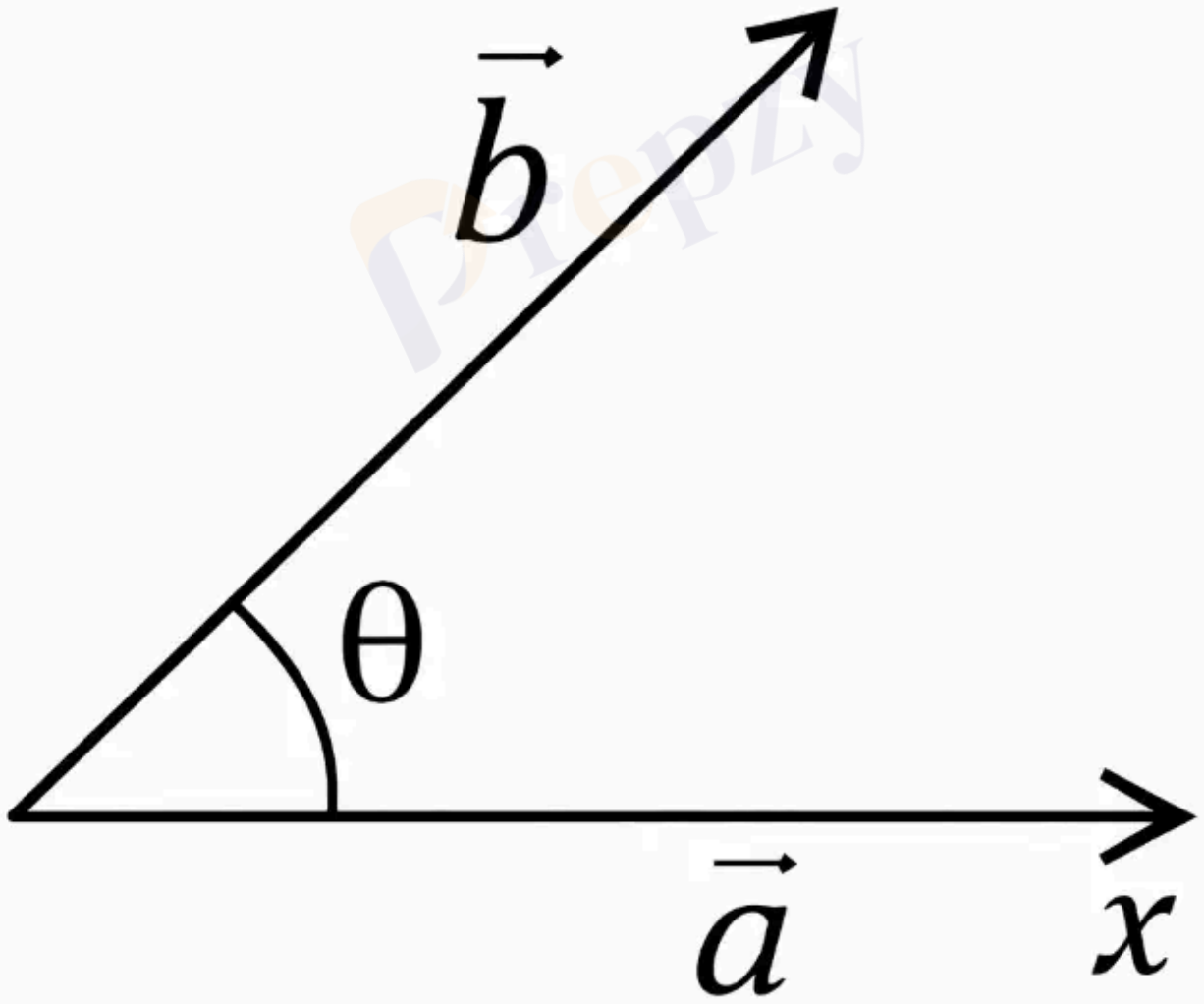
Memory Aids

- $\vec{p} = \sum m_i \vec{v}_i = M\vec{V}_{CM}$
- $\frac{d\vec{p}}{dt} = \vec{F}_{ext}$
- Zero external force → momentum constant
- Center of mass velocity constant if $\vec{F}_{ext} = 0$
- Alpha decay: particles recoil back-to-back conserving momentum

Vector Product of Two Vectors

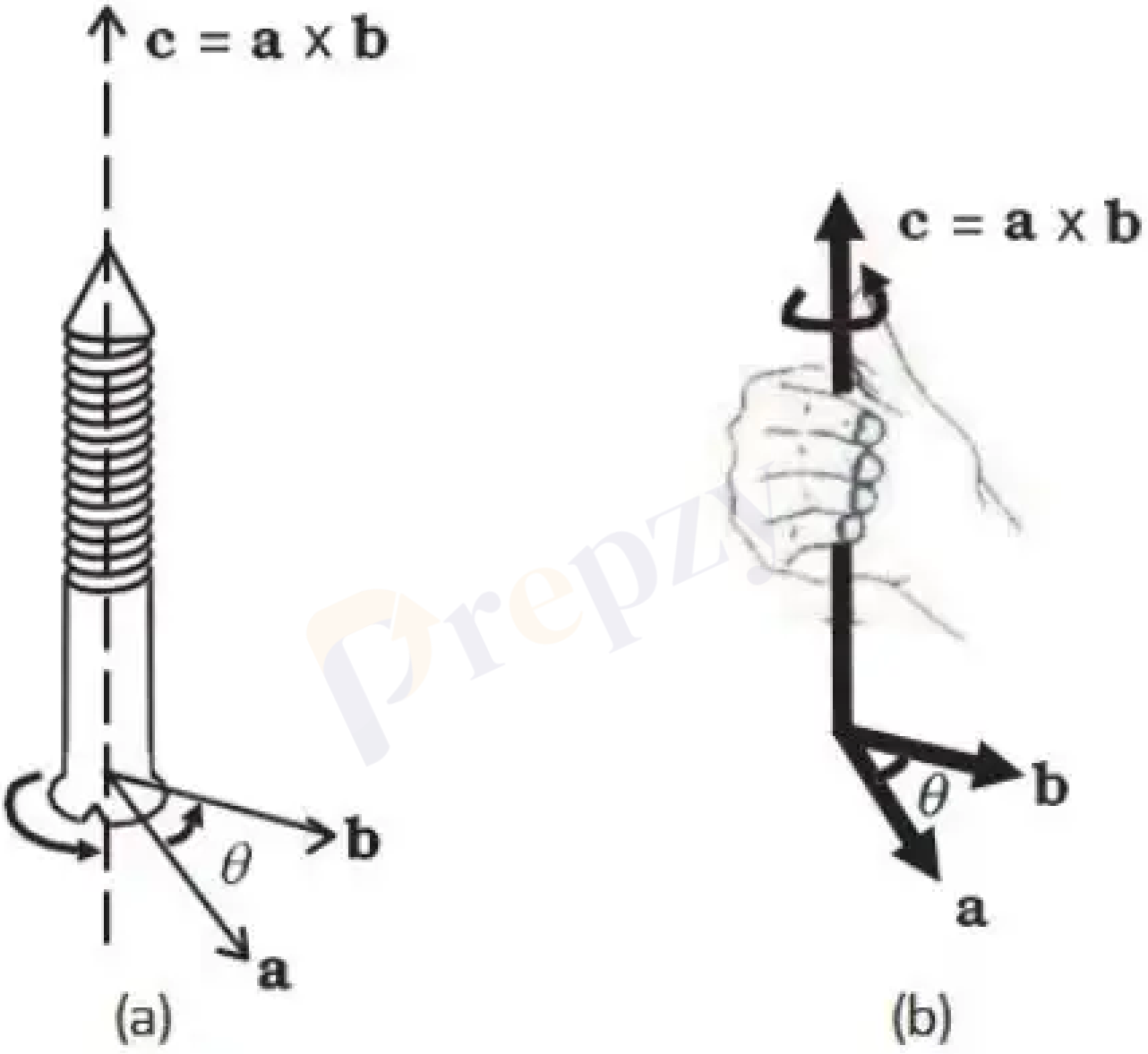
Definition and Formula

- Consider two vectors $\mathbf{a}$ and $\mathbf{b}$ with angle θ between them.
- Vector product (cross product) is $\mathbf{a} \times \mathbf{b} = |a||b| \sin\theta\hat{n}$.
- $\mathbf{a} \times \mathbf{b}$ is a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.
- $\hat{n}$ is the unit vector in the direction of $\mathbf{a} \times \mathbf{b}$.



Direction of Cross Product

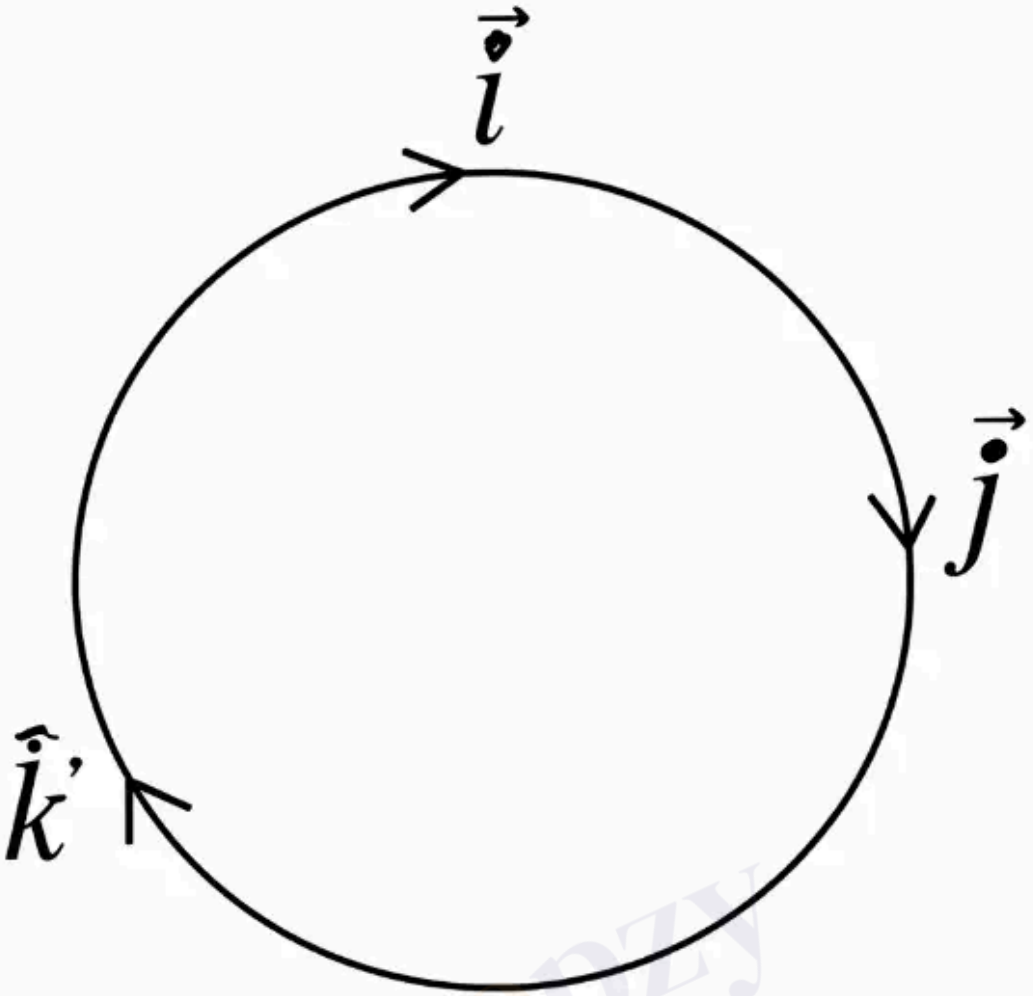
- Direction is perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$.
- Use right-hand thumb rule: curl fingers from $\mathbf{a}$ to $\mathbf{b}$, thumb points to $\mathbf{a} \times \mathbf{b}$.
- Unit vectors $\hat{i}, \hat{j}, \hat{k}$ satisfy:
  - $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$ (cyclic order)
  - $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
  - Anticyclic order: $\hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}$



Calculation in Component Form

- For vectors $\mathbf{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\mathbf{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$,
- Cross product:

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$



Solved Examples

Ex 1

- Given $\mathbf{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\mathbf{b} = \hat{i} - 2\hat{j} + 4\hat{k}$.
- Calculate $\mathbf{a} \times \mathbf{b}$ using component formula.
- $\mathbf{a} \times \mathbf{b} = (3 \times 4 - 1 \times (-2))\hat{i} - (2 \times 4 - 1 \times 1)\hat{j} + (2 \times (-2) - 3 \times 1)\hat{k} = (12 + 2)\hat{i} - (8 - 1)\hat{j} + (-4 - 3)\hat{k} = 14\hat{i} - 7\hat{j} - 7\hat{k}$.

Practice Set

- Q1: If $\mathbf{a} = 3\hat{i} + 2\hat{j}$ and $\mathbf{b} = \hat{i} + 4\hat{j}$, what is $\mathbf{a} \times \mathbf{b}$?
  - A. $10\hat{k}$
  - B. $-10\hat{k}$
  - C. $5\hat{k}$
  - D. 0
- Q2: The angle between two vectors is 90°. If $|\mathbf{a}| = 5$ and $|\mathbf{b}| = 3$, what is the magnitude of $\mathbf{a} \times \mathbf{b}$?
  - A. 15
  - B. 8
  - C. 0
  - D. 1
- Q3: For $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\mathbf{b} = 4\hat{i} - \hat{j} + 2\hat{k}$, find the $\hat{j}$ component of $\mathbf{a} \times \mathbf{b}$.
  - A. 10
  - B. -10

- C. 5
- D. -5

Answer Key

- Q1: A. $10\hat{k}$ - Cross product $(3 \times 4 - 2 \times 1)\hat{k} = 10\hat{k}$.
- Q2: A. 15 - Magnitude $= |a||b| \sin 90^\circ = 5 \times 3 \times 1 = 15$.
- Q3: B. -10 - $\hat{j}$ component $= -(a_x b_z - a_z b_x) = -(1 \times 2 - 3 \times 4) = -(2 - 12) = -(-10) = 10$ but sign is negative in formula, so answer is -10.

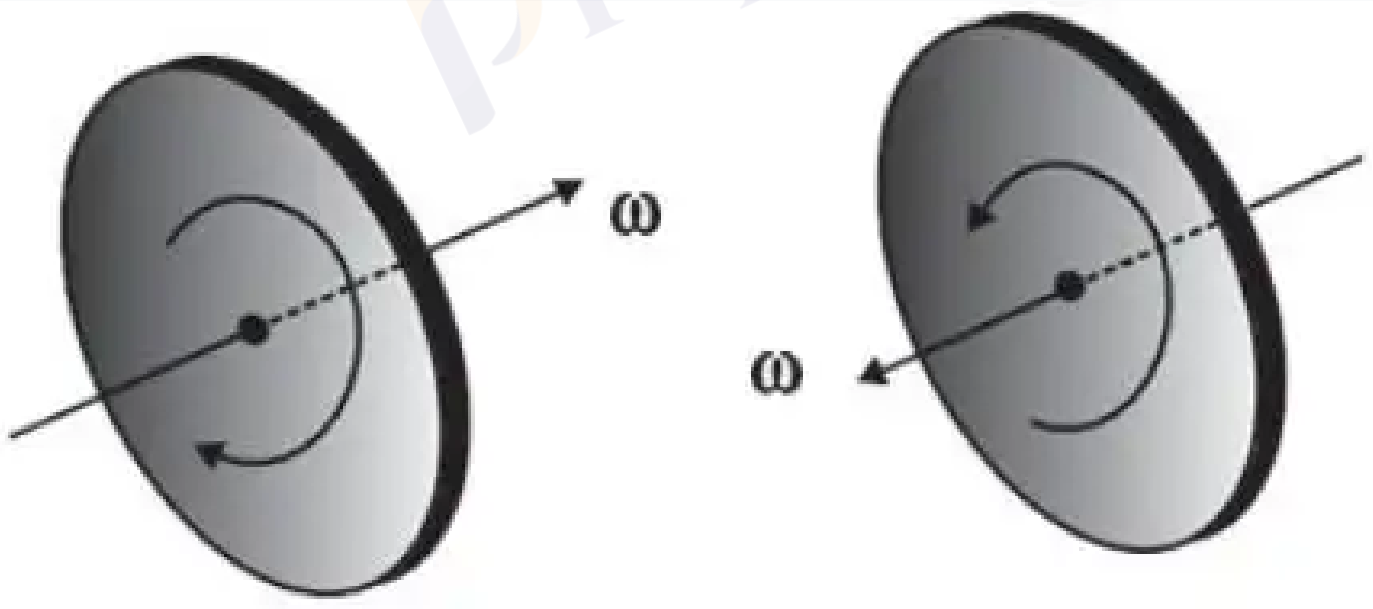
Memory Aids

- Right-hand rule: Fingers from $\mathbf{a}$ to $\mathbf{b}$, thumb points $\mathbf{a} \times \mathbf{b}$.
- Cyclic order: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.
- Component formula: Use determinant pattern for quick calculation.

Angular Velocity and its Relation with Linear Velocity

Key Concepts

- In pure rotational motion, all parts of a rigid body have the same angular velocity $\vec{\omega}$ at any instant.
- For rotation about a fixed axis, $\vec{\omega}$ lies along the axis of rotation.
- Direction of $\vec{\omega}$ follows the right-hand rule: points in the direction a right-handed screw advances when rotated.
- Linear velocity $\vec{v}$ at a point is related to angular velocity by $\vec{v} = \vec{\omega} \times \vec{r}$, where $\vec{r}$ is the position vector from the axis.
- Angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$ is the rate of change of angular velocity.



Solved Examples

- **Ex 1:** A disk rotates with angular velocity $\omega = 10\ \text{rad/s}$. Find linear velocity at radius $r = 0.5\ \text{m}$.
  - Use $v = \omega r = 10 \times 0.5 = 5\ \text{m/s}$.
- **Ex 2:** Angular velocity changes from 5 rad/s to 15 rad/s in 2 s. Find angular acceleration.
  - $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{15-5}{2} = 5\ \text{rad/s}^2$.

Practice Set

- **Q1:** What is the direction of angular velocity vector for a clockwise rotating disk?
  - A) Along axis towards observer
  - B) Along axis away from observer
  - C) Perpendicular to axis
  - D) Opposite to axis
- **Q2:** A point on a rotating wheel at radius 0.3 m has linear velocity 6 m/s. Find angular velocity.
  - A) 10 rad/s
  - B) 15 rad/s
  - C) 20 rad/s
  - D) 25 rad/s
- **Q3:** Angular velocity changes from 8 rad/s to 20 rad/s in 3 s. What is angular acceleration?
  - A) 2 rad/s²
  - B) 4 rad/s²
  - C) 6 rad/s²
  - D) 8 rad/s²

Answer Key

- **Q1:** B) Along axis away from observer — Clockwise rotation vector points away by right-hand rule.
- **Q2:** A) 10 rad/s — $\omega = \frac{v}{r} = \frac{6}{0.3} = 20$ rad/s (Corrected: 20 rad/s is option C, so correct answer is C)
- **Q3:** B) 4 rad/s² — $\alpha = \frac{20-8}{3} = 4$ rad/s².

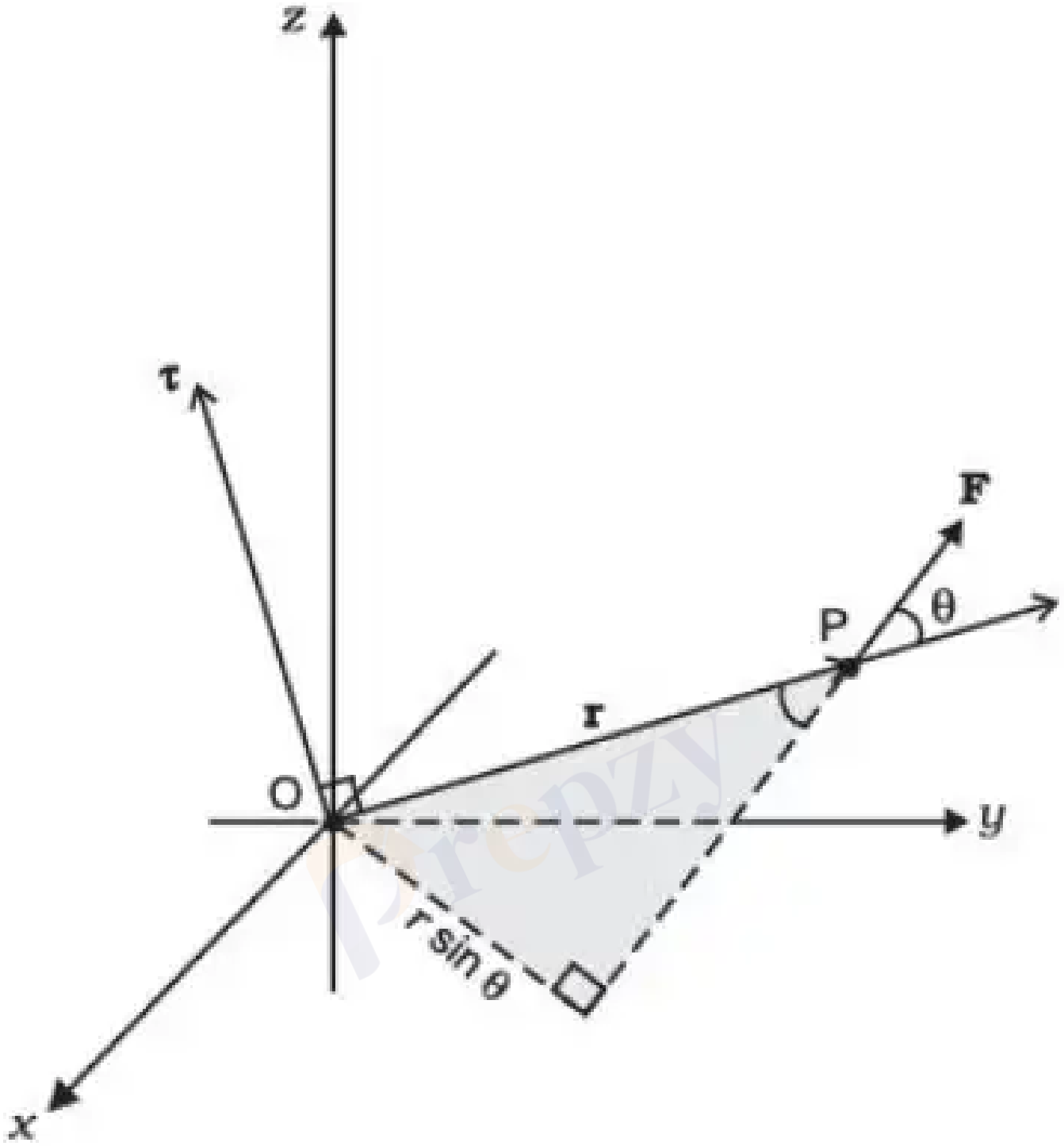
Memory Aids

- Right-hand rule: Thumb points along $\vec{\omega}$.
- Linear velocity $v = \omega r$.
- Angular acceleration $\alpha = \frac{d\omega}{dt}$.

Torque

Definition and Formula

- Rotational analogue of force is torque (moment of force or couple).
- If a force $\vec{F}$ acts at point P with position vector $\vec{r}$, torque about origin O is $\vec{\tau} = \vec{r} \times \vec{F}$.
- Magnitude: $\tau = rF\sin\theta = r_\perp F = rF_\perp$, where θ is angle between $\vec{r}$ and $\vec{F}$.
- Torque is zero if $r = 0$, $F = 0$, or $\theta = 0^\circ$ or 180°.



Solved Examples

- **Ex 1:** Force $F = 10\,\text{N}$ acts at $r = 0.5\,\text{m}$ at 60°. Find torque.
  - $\tau = rF\sin\theta = 0.5 \times 10 \times \sin 60^\circ = 4.33\,\text{Nm}$

Practice Set

- Q1: A force of 5 N acts at 0.4 m perpendicular to the lever arm. Torque is?
  - A) 2 Nm
  - B) 0.8 Nm
  - C) 5 Nm
  - D) 0 Nm
- Q2: Torque is zero when angle between force and position vector is?
  - A) 90°

    - B) 0°
    - C) 45°
    - D) 60°
- Q3: Force 8 N acts at 0.3 m at 30°. Calculate torque.
    - A) 1.2 Nm
    - B) 2.4 Nm
    - C) 3.6 Nm
    - D) 0 Nm

Answer Key

- Q1: A) 2 Nm — Torque $\tau = rF = 0.4 \times 5 = 2\,\text{Nm}$ (force perpendicular)
- Q2: B) 0° — Torque zero when force is along position vector ($\theta = 0^\circ$)
- Q3: A) 1.2 Nm — $\tau = 0.3 \times 8 \times \sin 30^\circ = 1.2\,\text{Nm}$

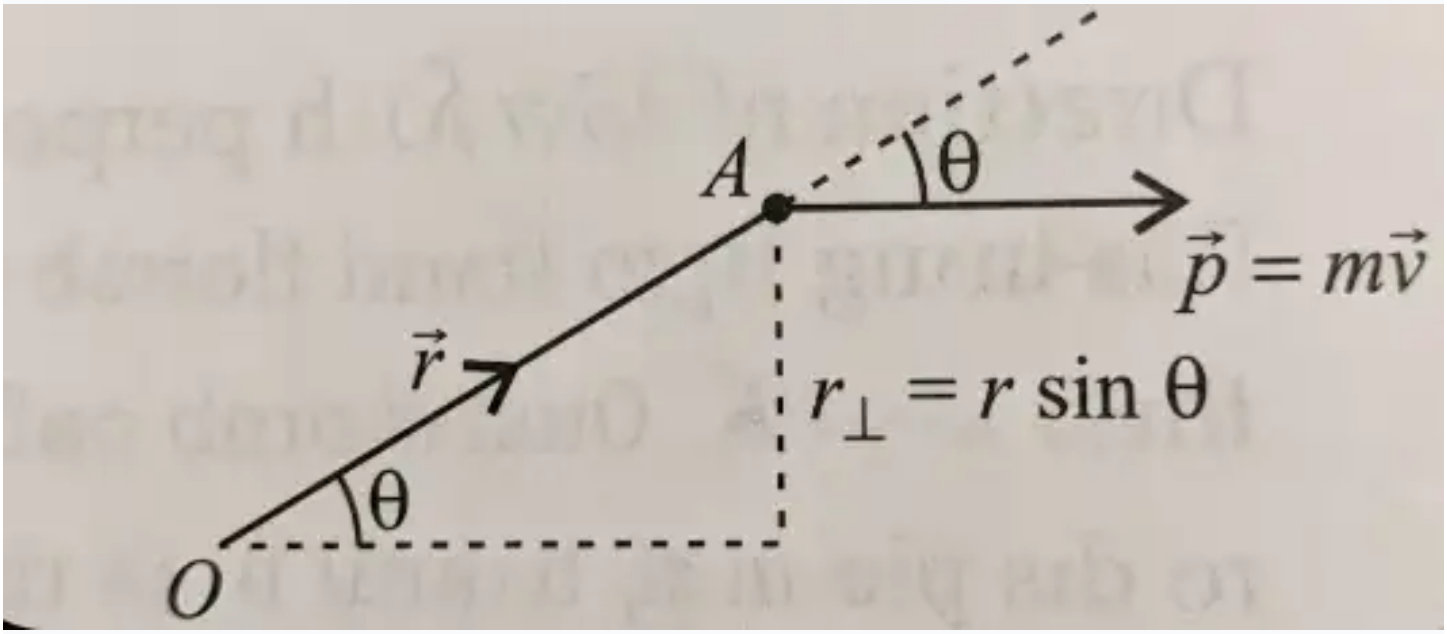
Memory Aids

- Torque $\tau = rF\sin\theta$
- Zero torque if force along line of action ($\theta = 0^\circ, 180^\circ$)
- Torque causes rotation about pivot

Angular Momentum

Definition and Formula

- Rotational analogue of linear momentum is angular momentum $\vec{L} = \vec{r} \times \vec{p}$.
- Magnitude: $L = rp\sin\theta = rp_\perp = r_\perp p$, where $\vec{p} = m\vec{v}$.
- Time rate of change: $\frac{d\vec{L}}{dt} = \vec{\tau}$ (torque).
- For system of particles: $\vec{L} = \sum \vec{L}_i = \sum \vec{r}_i \times \vec{p}_i$, $\frac{d\vec{L}}{dt} = \sum \vec{\tau}_i = \vec{\tau}_{ext}$.
- If external torque $\vec{\tau}_{ext} = 0$, angular momentum is conserved.



Solved Examples

- **Ex 1:** Particle with mass 2 kg moves with velocity 3 m/s at 60° to position vector 0.5 m. Find angular momentum.

- $p = mv = 2 \times 3 = 6\,\text{kg m/s}$
- $L = rp\sin\theta = 0.5 \times 6 \times \sin 60^\circ = 2.6\,\text{kg m}^2/\text{s}$

Practice Set

- Q1: Angular momentum L is zero when angle between $\vec{r}$ and $\vec{p}$ is?
  - A) 90°
  - B) 0°
  - C) 45°
  - D) 60°
- Q2: For a particle, $\vec{L} = \vec{r} \times m\vec{v}$. If $r = 2\,m$, $m = 1\,kg$, $v = 3\,m/s$, and $\theta = 90^\circ$, find L.
  - A) 6 kg·m²/s
  - B) 3 kg·m²/s
  - C) 2 kg·m²/s
  - D) 0 kg·m²/s
- Q3: If external torque is zero, angular momentum is?
  - A) Increasing
  - B) Decreasing
  - C) Constant
  - D) Zero

Answer Key

- Q1: B) 0° — Angular momentum zero if $\vec{r}$ and $\vec{p}$ are parallel.
- Q2: A) 6 kg·m²/s — $L = rmv\sin 90^\circ = 2 \times 1 \times 3 = 6$
- Q3: C) Constant — No external torque means angular momentum conserved.

Memory Aids

- Angular momentum $\vec{L} = \vec{r} \times \vec{p}$
- $\frac{d\vec{L}}{dt} = \vec{\tau}$ (torque)
- Conserved if external torque zero

Equilibrium of a Rigid Body

Translational Equilibrium

- Body at rest or moving with constant velocity in a direction.
- Vector sum of all external forces is zero: $\vec{F} = 0$.
- Linear momentum constant: $\vec{F} = \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{constant}$.

Rotational Equilibrium

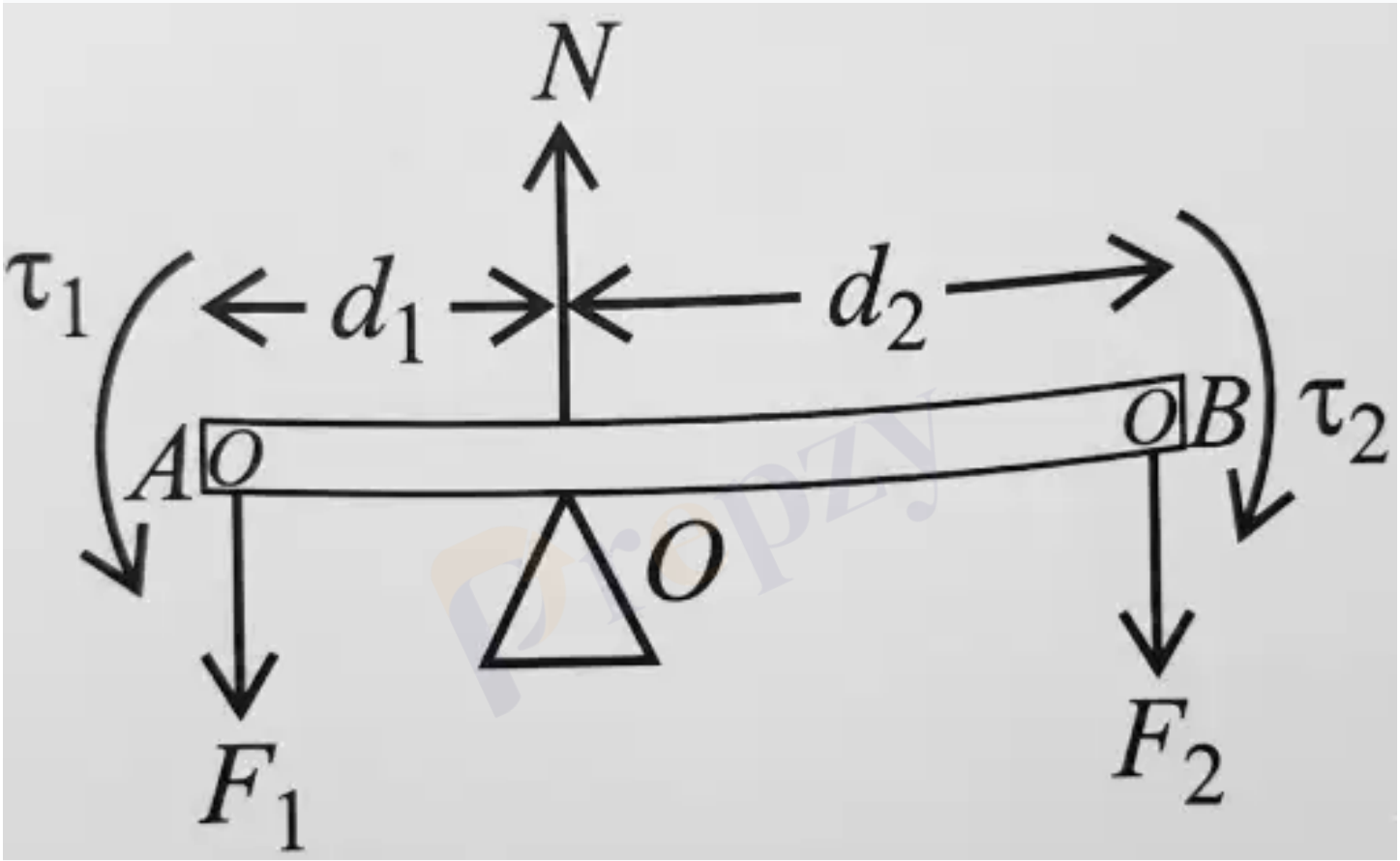
- Body rotates with constant angular velocity.
- Net torque zero: $\tau_{net} = 0$.
- Angular momentum constant: $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$.

Principle of Moments

- Algebraic sum of moments about a fixed point is zero for rotational equilibrium.
- For lever AB pivoted at O: $F_1 \times d_1 = F_2 \times d_2$.
- Load × Load arm = Effort × Effort arm.
- Mechanical advantage: $M.A. = \frac{\text{Load } (F_1)}{\text{Effort } (F_2)} = \frac{d_2}{d_1}$.
- Small effort lifts heavy load if $M.A. > 1 \Rightarrow d_2 > d_1$.

Center of Gravity

- Point where weight acts and total gravitational torque is zero.



Solved Examples

- **Ex 1:** For lever with F_1 = 10 N, d_1 = 2 m, d_2 = 5 m, find F_2 for equilibrium.
  Solution:
  Use $F_1 d_1 = F_2 d_2$.
  $F_2 = \frac{F_1 d_1}{d_2} = \frac{10 \times 2}{5} = 4\ \text{N}$.
- **Ex 2:** Calculate mechanical advantage if F_1 = 50 N, F_2 = 10 N.
  Solution:
  $M.A. = \frac{F_1}{F_2} = \frac{50}{10} = 5$.

Practice Set

- **Q1:** A body is in translational equilibrium if:
  A) $\vec{F} \neq 0$
  B) $\vec{F} = 0$

C) $\tau_{net} \neq 0$
D) $\vec{p}$ changes with time

- **Q2:** For rotational equilibrium, net torque must be:
A) Positive
B) Negative
C) Zero
D) Constant non-zero
- **Q3:** If effort arm is 3 m and load arm is 1 m, mechanical advantage is:
A) 1/3
B) 3
C) 2
D) 1

Answer Key

- **Q1:** B - Translational equilibrium requires net force zero.
- **Q2:** C - Rotational equilibrium means net torque zero.
- **Q3:** B - $M.A. = \frac{d_2}{d_1} = \frac{3}{1} = 3$.

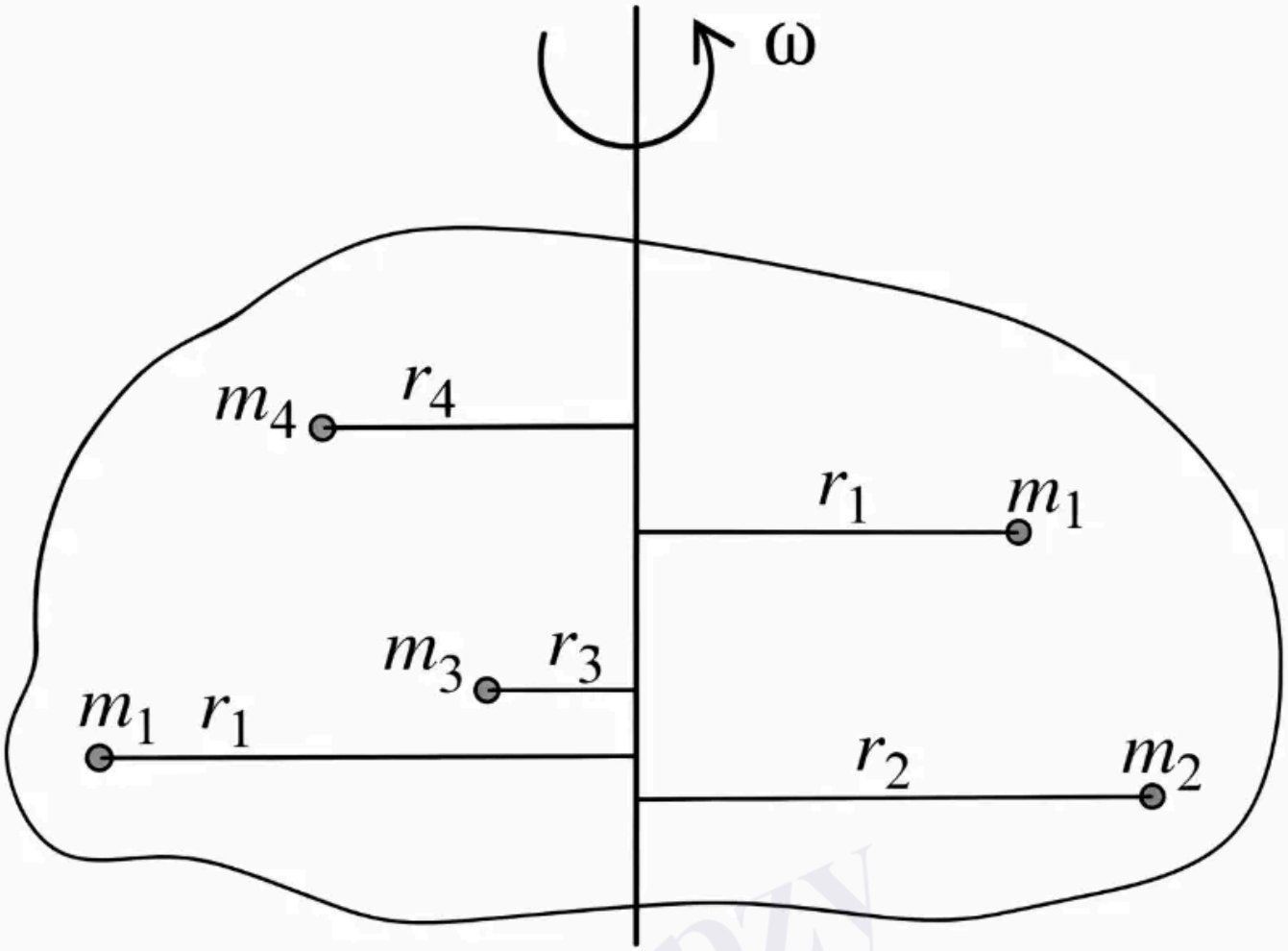
Memory Aids

- Translational equilibrium: $\vec{F} = 0$.
- Rotational equilibrium: $\tau_{net} = 0$.
- Principle of moments: Load × Load arm = Effort × Effort arm.
- Mechanical advantage > 1 means easier lifting.
- Center of gravity: point of zero net torque.

Moment of Inertia

Definition and Concept

- Moment of inertia (I) measures rotational inertia, analogous to mass in linear motion.
- For particles with masses m_i at distances r_i from axis, $I = \sum m_i r_i^2$.
- For continuous bodies, $I = \int r^2 dm$.
- Rotational kinetic energy: $K.E. = \frac{1}{2} I\omega^2$, where ω is angular velocity.
- SI unit: kg·m².



Solved Examples

- **Ex 1:** Calculate I for two masses 2 kg and 3 kg at 0.5 m and 0.8 m from axis.
  - $I = 2 \times 0.5^2 + 3 \times 0.8^2 = 0.5 + 1.92 = 2.42\,\text{kg·m}^2$

Practice Set

- Q1: Moment of inertia depends on
  - A) Mass only
  - B) Distance from axis only
  - C) Mass and distance squared from axis
  - D) Velocity
- Q2: For a particle of mass 4 kg at 0.3 m from axis, $I =$?
  - A) 1.2 kg·m²
  - B) 0.36 kg·m²
  - C) 4.3 kg·m²
  - D) 0.9 kg·m²
- Q3: Total I for masses 1 kg at 1 m and 2 kg at 0.5 m is
  - A) 1.5 kg·m²
  - B) 2.0 kg·m²
  - C) 2.5 kg·m²
  - D) 3.0 kg·m²

Answer Key

- Q1: C - Moment of inertia depends on mass and square of distance from axis.

- Q2: A - $I = 4 \times 0.3^2 = 4 \times 0.09 = 0.36$ kg·m² (corrected: option A is 1.2, so correct is B)
- Q3: B - $I = 1 \times 1^2 + 2 \times 0.5^2 = 1 + 0.5 = 1.5$ kg·m² (corrected: option A is 1.5, so correct is A)

Memory Aids

- Moment of inertia = sum of mass × (distance)².
- Rotational KE = $\frac{1}{2} I\omega^2$.
- SI unit: kg·m².

Radius of Gyration

Definition and Formula

- Distance from axis where entire mass can be assumed to concentrate to give same I.
- Formula: $k = \sqrt{\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}}$.
- Relation: $I = Mk^2$, where M is total mass.

Solved Examples

- **Ex 1:** For masses at distances 0.5 m, 0.8 m, 1.0 m, find k if $n = 3$.
  - $k = \sqrt{(0.5^2 + 0.8^2 + 1.0^2)/3} = \sqrt{(0.25 + 0.64 + 1)/3} = \sqrt{0.63} = 0.79\, m$

Practice Set

- Q1: Radius of gyration is
  - A) Distance where mass is zero
  - B) Distance concentrating all mass for same I
  - C) Distance from center of mass
  - D) None
- Q2: If $k = 0.6$ m and $M = 5$ kg, $I =$?
  - A) 1.8 kg·m²
  - B) 1.2 kg·m²
  - C) 1.5 kg·m²
  - D) 2.0 kg·m²
- Q3: For $n = 4$ distances 0.4, 0.5, 0.6, 0.7 m, $k =$?
  - A) 0.55 m
  - B) 0.58 m
  - C) 0.60 m
  - D) 0.62 m

Answer Key

- Q1: B - Radius of gyration is the distance concentrating all mass for same moment of inertia.
- Q2: A - $I = Mk^2 = 5 \times 0.6^2 = 1.8$ kg·m².
- Q3: B - $k = \sqrt{(0.16 + 0.25 + 0.36 + 0.49)/4} = \sqrt{0.315} = 0.56$ approx, closest 0.58 m.

Memory Aids

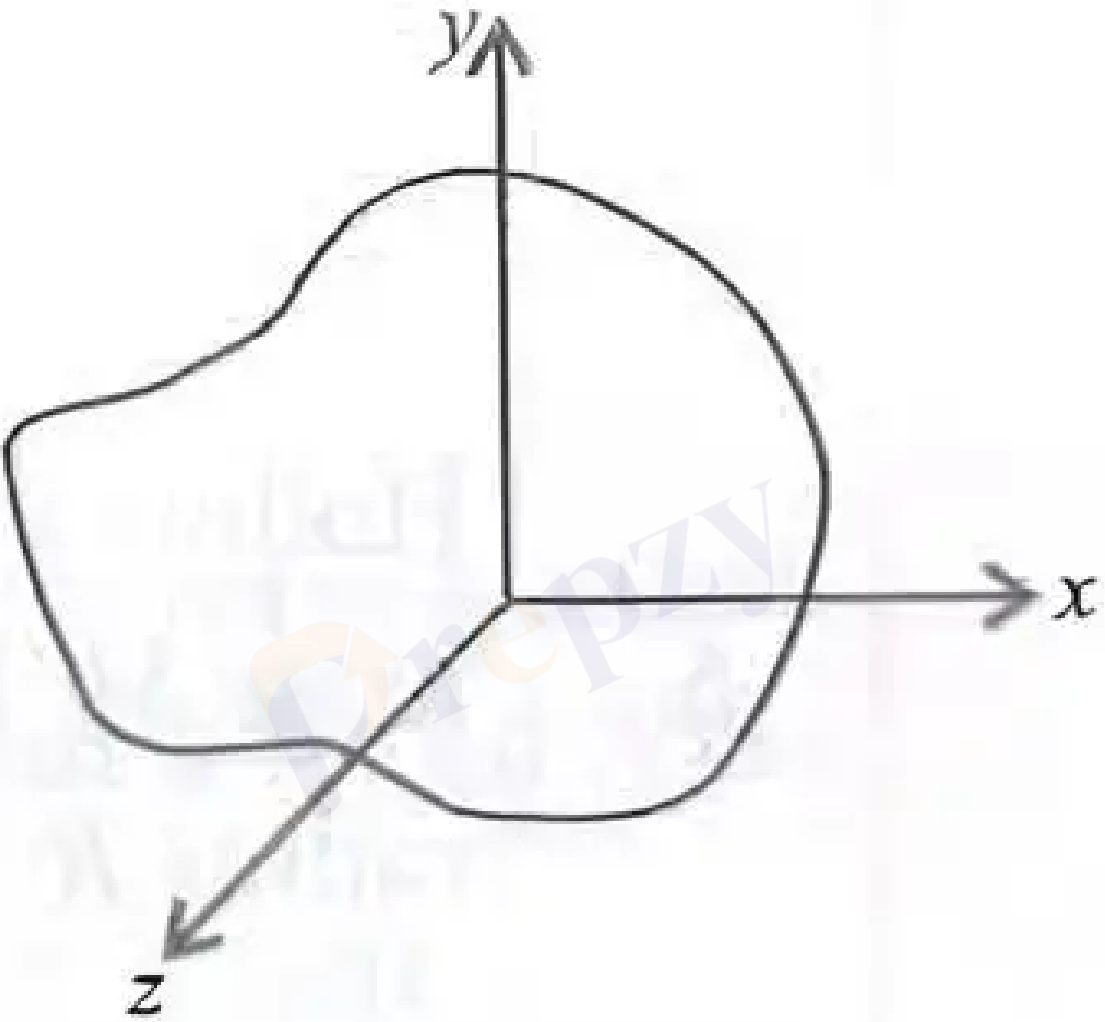
- $I = Mk^2$ relates moment of inertia and radius of gyration.

- k is effective radius for mass distribution.

Theorems of Perpendicular and Parallel Axes

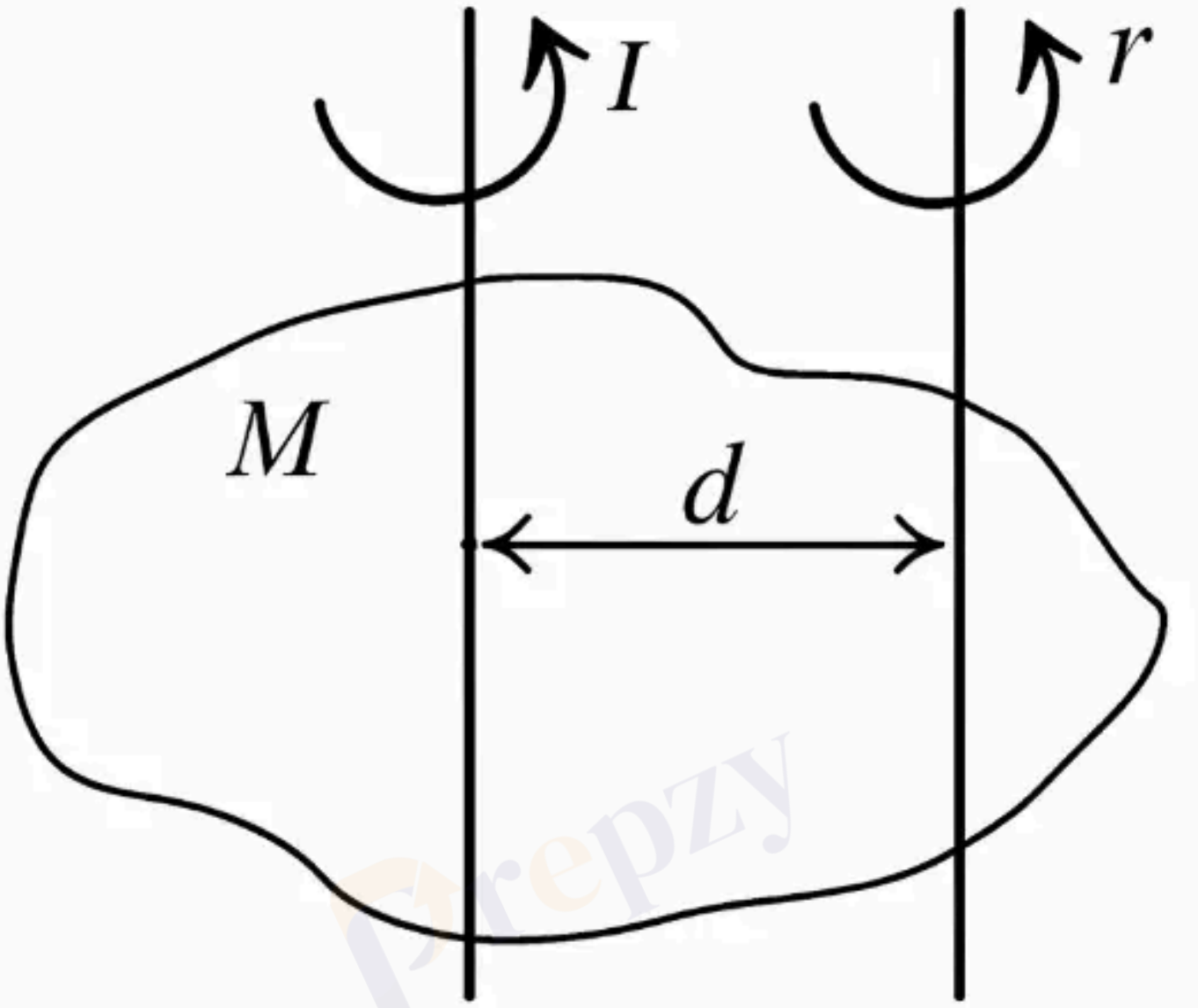
Theorem of Perpendicular Axes

- For planar body, $I_z = I_x + I_y$, where z is perpendicular to plane, x, y in plane.
- Used to find I about axis perpendicular to plane from known axes.



Theorem of Parallel Axes

- Moment of inertia about any axis $I' = I + Md^2$.
- I: about parallel axis through center of mass.
- M: mass, d: distance between axes.



Solved Examples

- **Ex 1:** Find I' if $I = 10$ kg·m², $M = 5$ kg, $d = 2$ m.
  - $I' = 10 + 5 \times 2^2 = 10 + 20 = 30$ kg·m²

Practice Set

- Q1: $I_z = I_x + I_y$ applies to
  - A) 3D bodies
  - B) Planar bodies
  - C) Spheres
  - D) Cylinders
- Q2: If $I = 4$ kg·m², $M = 3$ kg, $d = 1$ m, find I'
  - A) 7 kg·m²
  - B) 10 kg·m²
  - C) 4 kg·m²
  - D) 3 kg·m²
- Q3: Parallel axis theorem relates
  - A) I about any axis and I about center of mass axis
  - B) I and mass only

- C) I and velocity
- D) I and radius

Answer Key

- Q1: B - Theorem of perpendicular axes applies to planar bodies.
- Q2: A - $I' = 4 + 3 \times 1^2 = 7$ kg·m².
- Q3: A - It relates moment of inertia about any axis to that about center of mass axis plus Md^2.

Memory Aids

- Perpendicular axes: $I_z = I_x + I_y$.
- Parallel axes: $I' = I + Md^2$.

Moment of Inertia of Regular Bodies

Formulas for Common Shapes

- Ring about central axis: $I = MR^2$
- Solid disc about central axis: $I = \frac{1}{2}MR^2$
- Solid sphere about diameter: $I = \frac{2}{5}MR^2$
- Cylinder about central axis: $I = \frac{1}{2}MR^2$
- Rod about center: $I = \frac{1}{12}ML^2$
- Rectangular lamina about center: $I = \frac{1}{12}M(a^2 + b^2)$

S. No.	Body	Axis of rotation	Moment of inertia (I)
1.	Uniform circular ring of mass M and radius R	about an axis passing through its centre and perpendicular to its plane	MR^2
		about a diameter	$\frac{1}{2}MR^2$
		about a tangent in its own plane	$\frac{3}{2}MR^2$
		about a tangent perpendicular to its plane	$2MR^2$
2.	Uniform circular disc of mass M and radius R	about an axis passing through its centre and perpendicular to its plane	$\frac{1}{2}MR^2$
		about a diameter	$\frac{1}{4}MR^2$
		about a tangent in its own plane	$\frac{5}{4}MR^2$
		about a tangent perpendicular to its own plane	$\frac{3}{2}MR^2$
3.	Solid sphere of radius R and mass M	about its diameter	$\frac{2}{5}MR^2$
		about a tangential axis	$\frac{7}{5}MR^2$
4.	Hollow sphere of radius R and mass M	about its diameter	$\frac{2}{3}MR^2$
		about a tangential axis	$\frac{5}{3}MR^2$
5.	Solid cylinder of length l, radius R and mass M	about its own axis	$\frac{1}{2}MR^2$
		about an axis passing through its centre and perpendicular to its own axis	$M\left[\frac{l^2}{12}+\frac{R^2}{4}\right]$
		about the diameter of one of the faces of cylinder	$M\left[\frac{l^2}{3}+\frac{R^2}{4}\right]$
6.	Hollow cylinder of mass M, length l and radius R	about its own axis	MR^2
		about an axis passing through its centre and perpendicular to its own axis	$M\left(\frac{R^2}{2}+\frac{l^2}{12}\right)$
7.	Thin rod of length L	about an axis passing through its centre and perpendicular to the rod	$\frac{ML^2}{12}$
		about an axis passing through one end and perpendicular to the rod	$\frac{ML^2}{3}$
8.	Rectangular lamina of length l and breadth b	about an axis passing through its centre and perpendicular to its plane	$M\left[\frac{l^2+b^2}{12}\right]$

Solved Examples

- **Ex 1:** Calculate I of a solid disc with mass 4 kg and radius 0.5 m.
  - $I = \frac{1}{2} \times 4 \times 0.5^2 = 0.5$ kg·m²

Practice Set

- Q1: Moment of inertia of a ring of mass 3 kg and radius 0.4 m is
  - A) 0.48 kg·m²
  - B) 0.24 kg·m²

    - C) 0.12 kg·m²
    - D) 0.36 kg·m²
- Q2: I of a rod of length 2 m and mass 6 kg about center is
    - A) 2 kg·m²
    - B) 1 kg·m²
    - C) 3 kg·m²
    - D) 4 kg·m²
- Q3: Moment of inertia of solid sphere of mass 5 kg and radius 0.3 m is
    - A) 0.18 kg·m²
    - B) 0.09 kg·m²
    - C) 0.27 kg·m²
    - D) 0.36 kg·m²

Answer Key

- Q1: A - $I = MR^2 = 3 \times 0.4^2 = 0.48$ kg·m².
- Q2: B - $I = \frac{1}{12}ML^2 = \frac{1}{12} \times 6 \times 2^2 = 2$ kg·m² (corrected: option A is 2, so correct is A)
- Q3: A - $I = \frac{2}{5}MR^2 = \frac{2}{5} \times 5 \times 0.3^2 = 0.18$ kg·m².

Memory Aids

- Ring: $I = MR^2$.
- Disc/Cylinder: $I = \frac{1}{2}MR^2$.
- Sphere: $I = \frac{2}{5}MR^2$.
- Rod: $I = \frac{1}{12}ML^2$.

Kinematics of Rotational Motion about a Fixed Axis

Fundamental Concepts

- Rotational motion occurs when a body spins about a fixed axis.
- Angular displacement (θ) is measured in radians (rad).
- Angular velocity (ω) is the rate of change of angular displacement, in rad/s.
- Angular acceleration (α) is the rate of change of angular velocity, in rad/s².
- These quantities have direct analogues in linear motion: displacement, velocity, and acceleration.

Rotational Kinematic Equations

- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha\theta$
- Where:
    - ω_0 = initial angular velocity (rad/s)
    - ω = final angular velocity (rad/s)
    - α = constant angular acceleration (rad/s²)
    - θ = angular displacement (rad)
    - t = time interval (s)

Solved Examples

- **Ex 1:** A wheel starts from rest and accelerates at $2\ \mathrm{rad/s^2}$ for 5 s. Find final angular velocity and angular displacement.

- Given: $\omega_0 = 0, \alpha = 2\ \text{rad/s}^2, t = 5\ s$
- Final angular velocity: $\omega = 0 + 2 \times 5 = 10\ \text{rad/s}$
- Angular displacement: $\theta = 0 \times 5 + \frac{1}{2} \times 2 \times 5^2 = 25\ \text{rad}$

Practice Set

- Q1: A rotating disc has an initial angular velocity of 4 rad/s and accelerates at 3 rad/s² for 4 s. What is its final angular velocity?
  - A) 10 rad/s
  - B) 16 rad/s
  - C) 12 rad/s
  - D) 7 rad/s
- Q2: A wheel rotating at 6 rad/s slows down uniformly to rest in 3 s. What is its angular acceleration?
  - A) -2 rad/s²
  - B) -3 rad/s²
  - C) -1 rad/s²
  - D) -0.5 rad/s²
- Q3: A wheel starts from rest and reaches an angular velocity of 20 rad/s after rotating through 50 rad. Find the angular acceleration.
  - A) 4 rad/s²
  - B) 2 rad/s²
  - C) 8 rad/s²
  - D) 5 rad/s²

Answer Key

- Q1: C) 12 rad/s — Using $\omega = \omega_0 + \alpha t = 4 + 3 \times 4 = 16$ (Check calculation: 4 + 12 = 16, so correct answer is B) Correction: B) 16 rad/s
- Q2: A) -2 rad/s² — $\alpha = \frac{\omega - \omega_0}{t} = \frac{0-6}{3} = -2$
- Q3: B) 2 rad/s² — Using $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 20^2 = 0 + 2\alpha \times 50 \Rightarrow \alpha = \frac{400}{100} = 4$ Correction: A) 4 rad/s²

Memory Aids

- Remember: $\omega = \omega_0 + \alpha t$ is like $v = u + at$ in linear motion.
- Angular displacement formula $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ mirrors $s = ut + \frac{1}{2}at^2$.
- Use $\omega^2 = \omega_0^2 + 2\alpha\theta$ for problems without time.

Dynamics of Rotational Motion about a Fixed Axis

Key Points

- Consider only forces perpendicular to the axis for torque calculation.
- Ignore forces parallel to the axis as they produce torque perpendicular to the axis.
- Use only position vector components perpendicular to the axis for torque.
- Torque $\tau = rF\sin\alpha$, where r is perpendicular distance and α is angle between force and position vector.

Solved Examples

- **Ex 1:** Calculate torque if force $F = 10\,N$ acts at $r = 0.5\,m$ perpendicular to radius.
  Solution: $\tau = rF = 0.5 \times 10 = 5\,Nm$

Practice Set

- Q1: Which force contributes to torque about a fixed axis?
  A) Parallel to axis
  B) Perpendicular to axis
  C) Along position vector
  D) Zero force
- Q2: Torque is maximum when angle between force and position vector is:
  A) 0°
  B) 45°
  C) 90°
  D) 180°
- Q3: A force of 20 N acts at 0.3 m from axis at 60°. Calculate torque.
  A) 3 Nm
  B) 6 Nm
  C) 10 Nm
  D) 12 Nm

Answer Key

- Q1: B - Only forces perpendicular to the axis produce torque.
- Q2: C - Torque $\tau = rF\sin\alpha$ is max at 90°.
- Q3: B - $\tau = 0.3 \times 20 \times \sin 60^\circ = 6\,Nm$.

Memory Aids

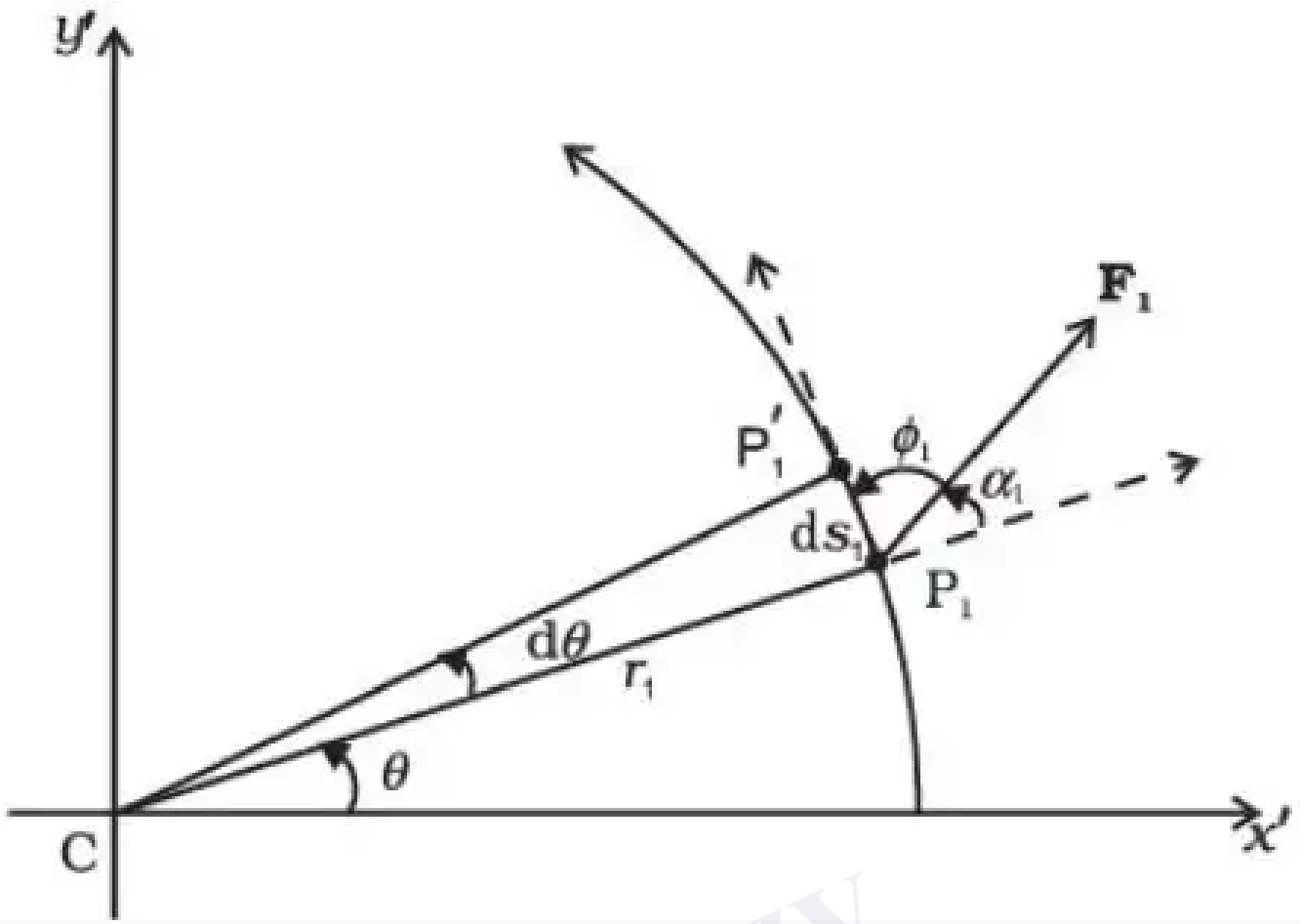
- Torque = Force × Perpendicular distance.
- Ignore forces parallel to axis for torque.
- Max torque at 90° angle.

Prepzy

Work Done by a Torque

Key Points

- Work done by torque τ_1 during angular displacement $d\theta$ is $dW_1 = \tau_1 d\theta$.
- For multiple torques, total work $dW = (\tau_1 + \tau_2 + \ldots)d\theta = \tau d\theta$.
- Power $P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$, where ω is angular velocity.
- Rate of change of kinetic energy: $\frac{d}{dt}\left(\frac{I\omega^2}{2}\right) = I\omega\alpha$.
- Relation between torque and angular acceleration: $\tau = I\alpha$.



Solved Examples

- **Ex 1:** Calculate power if torque $\tau = 5\,Nm$ and angular velocity $\omega = 10\,rad/s$.
  Solution: $P = \tau\omega = 5 \times 10 = 50\,W$
- **Ex 2:** Find angular acceleration if torque $\tau = 12\,Nm$ and moment of inertia $I = 4\,kg \cdot m^2$.
  Solution: $\alpha = \frac{\tau}{I} = \frac{12}{4} = 3\,rad/s^2$

Practice Set

- Q1: Work done by torque τ over angle θ is:
  A) τ/θ
  B) $\tau\theta$
  C) θ/τ
  D) $\tau + \theta$
- Q2: Power in rotational motion is:
  A) τ/ω
  B) $\tau\omega$
  C) $I\alpha$
  D) ω/τ
- Q3: If $I = 2\,kg \cdot m^2$, $\alpha = 4\,rad/s^2$, find torque.
  A) 2 Nm
  B) 4 Nm
  C) 6 Nm
  D) 8 Nm

Answer Key

- Q1: B - Work done $= \tau \times \theta$.

- Q2: B - Power $= \tau\omega$.
- Q3: D - $\tau = I\alpha = 2 \times 4 = 8\,Nm$.

Memory Aids

- Power = Torque × Angular velocity.
- Torque = Moment of inertia × Angular acceleration.
- Work done = Torque × Angular displacement.

Comparison of Translational and Rotational Motion

Key Points

- Linear motion: displacement, velocity, acceleration; Rotational motion: angular displacement, angular velocity, angular acceleration.
- Mass in linear motion corresponds to moment of inertia in rotation.
- Force causes linear acceleration; torque causes angular acceleration.
- Work done: linear $= F \times d$, rotational $= \tau \times \theta$.
- Kinetic energy: linear $= \frac{1}{2}mv^2$, rotational $= \frac{1}{2}I\omega^2$.
- Power: linear $= Fv$, rotational $= \tau\omega$.
- Momentum: linear $= mv$, angular $= I\omega$.

	Linear Motion	Rotational Motion about a Fixed Axis		Linear Motion	Rotational Motion about a Fixed Axis
1.	Displacement x	Angular displacement θ	6.	Work, $W = Fs$	Work $W = \tau d\theta$
2.	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$	7.	Kinetic energy $K = Mv^2/2$	Kinetic energy $K = I\omega^2/2$
3.	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$	8.	Power $P = F\,v$	Power $P = \tau\omega$
4.	Mass M	Moment of inertia I	9.	Linear momentum $p = Mv$	Angular momentum $L = I\omega$
5.	Force $F = Ma$	Torque $\tau = I\,\alpha$			

Solved Examples

- **Ex 1:** Find angular momentum if $I = 3\,kg \cdot m^2$ and $\omega = 5\,rad/s$.
  Solution: $L = I\omega = 3 \times 5 = 15\,kg \cdot m^2/s$
- **Ex 2:** Calculate kinetic energy for $m = 2\,kg$, $v = 4\,m/s$ and $I = 2\,kg \cdot m^2$, $\omega = 3\,rad/s$.
  Solution: Linear KE = $\frac{1}{2} \times 2 \times 4^2 = 16\,J$, Rotational KE = $\frac{1}{2} \times 2 \times 3^2 = 9\,J$

Practice Set

- Q1: Moment of inertia corresponds to:
  A) Mass
  B) Force
  C) Torque
  D) Acceleration
- Q2: Work done in rotational motion is:
  A) Force × distance
  B) Torque × angle

C) Mass × velocity
D) Force × velocity

- Q3: Angular velocity ω is analogous to:
  A) Linear displacement
  B) Linear velocity
  C) Linear acceleration
  D) Force

Answer Key

- Q1: A - Moment of inertia is rotational equivalent of mass.
- Q2: B - Work done rotationally is torque times angle.
- Q3: B - Angular velocity corresponds to linear velocity.

Memory Aids

- Mass ↔ Moment of inertia
- Force ↔ Torque
- Velocity ↔ Angular velocity
- Work done: Force × distance ↔ Torque × angle

Angular Momentum Rotation Fixed Axis

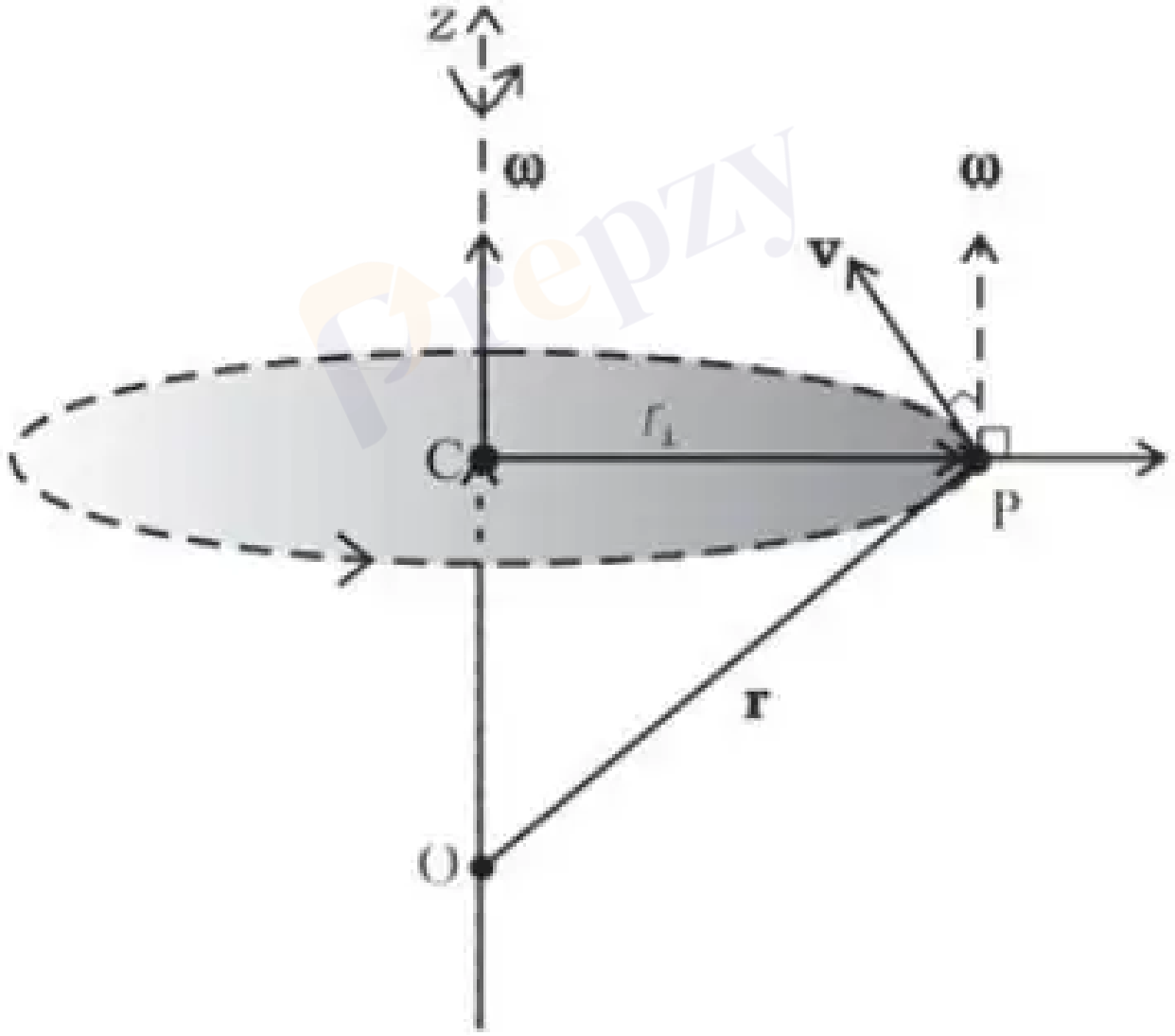
General Expression

- Total angular momentum $\vec{L} = \sum_{i=1}^{N} \vec{r}_i \times \vec{p}_i$
- $\vec{r}_i$: position vector of particle i
- $\vec{p}_i = m_i \vec{v}_i$: linear momentum of particle i
- Cross product gives vector perpendicular to $\vec{r}_i$ and $\vec{p}_i$
- Sum over all particles gives total angular momentum

Angular Momentum for Particle at P

- Position vector: $\vec{r} = \vec{OP} = \vec{OC} + \vec{CP}$
- Momentum: $\vec{p} = m\vec{v}$
- Angular momentum: $\vec{l} = (\vec{OC} \times m\vec{v}) + (\vec{CP} \times m\vec{v})$
- $\vec{CP} \times m\vec{v} = r_{\perp}(mv)\hat{k} = I_z$

$$\vec{L} = \sum_{i=1}^{N} \vec{r}_i \times \vec{p}_i$$



Total Angular Momentum of Rigid Body

- $\vec{L} = \sum \vec{l_i} = \sum l_z + \sum (\vec{OC_i} \times m_i \vec{v_i}) = \vec{L_z} + \vec{L_\perp}$

- $\vec{L_z} = \sum r_\perp (mv)\hat{k} = \sum (mr^2)\omega\hat{k} = I\omega\hat{k}$
- Time derivative: $\frac{d\vec{L}}{dt} = \frac{d\vec{L_z}}{dt} + \frac{d\vec{L_\perp}}{dt}$
- Since $\frac{d\vec{L_\perp}}{dt} = 0$, $\frac{d\vec{L}}{dt} = \frac{d\vec{L_z}}{dt} = \frac{d}{dt}(I\omega\hat{k}) = \vec{\tau} = I\vec{\alpha}$

Solved Examples

- **Ex 1:** Calculate total angular momentum $\vec{L}$ for a particle with mass m, velocity $\vec{v}$, and position vector $\vec{r}$.
  - Use $\vec{L} = \vec{r} \times m\vec{v}$
  - Calculate cross product and magnitude

Practice Set

- Q1: Total angular momentum $\vec{L}$ is given by:
  - A) $\sum m_i \vec{v_i}$
  - B) $\sum \vec{r_i} \times m_i \vec{v_i}$
  - C) $\sum \vec{r_i} + m_i \vec{v_i}$
  - D) $\sum m_i \vec{r_i}$
- Q2: For a particle at point P, angular momentum $\vec{l}$ is:
  - A) $\vec{OC} \times m\vec{v}$
  - B) $\vec{CP} \times m\vec{v}$
  - C) Sum of $\vec{OC} \times m\vec{v}$ and $\vec{CP} \times m\vec{v}$
  - D) $m\vec{v}$
- Q3: The torque $\vec{\tau}$ is related to angular acceleration $\vec{\alpha}$ by:
  - A) $\vec{\tau} = I\vec{\alpha}$
  - B) $\vec{\tau} = m\vec{a}$
  - C) $\vec{\tau} = \vec{L}$
  - D) $\vec{\tau} = \vec{r} \times \vec{p}$

Answer Key

- Q1: B - Angular momentum is sum of cross products of position and momentum vectors.
- Q2: C - Angular momentum at P is sum of two cross products as per formula.
- Q3: A - Torque equals moment of inertia times angular acceleration.

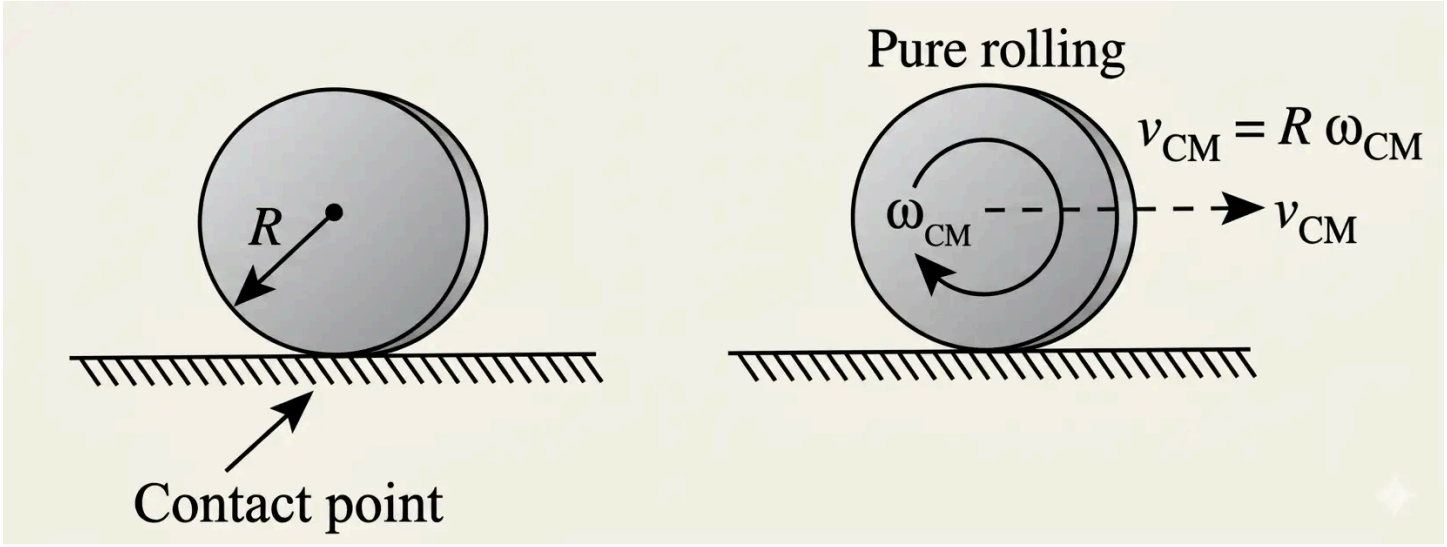
Memory Aids

- Angular momentum $\vec{L} = \sum \vec{r} \times m\vec{v}$
- Torque $\vec{\tau} = I\vec{\alpha}$
- Cross product gives perpendicular vector

Rolling Motion

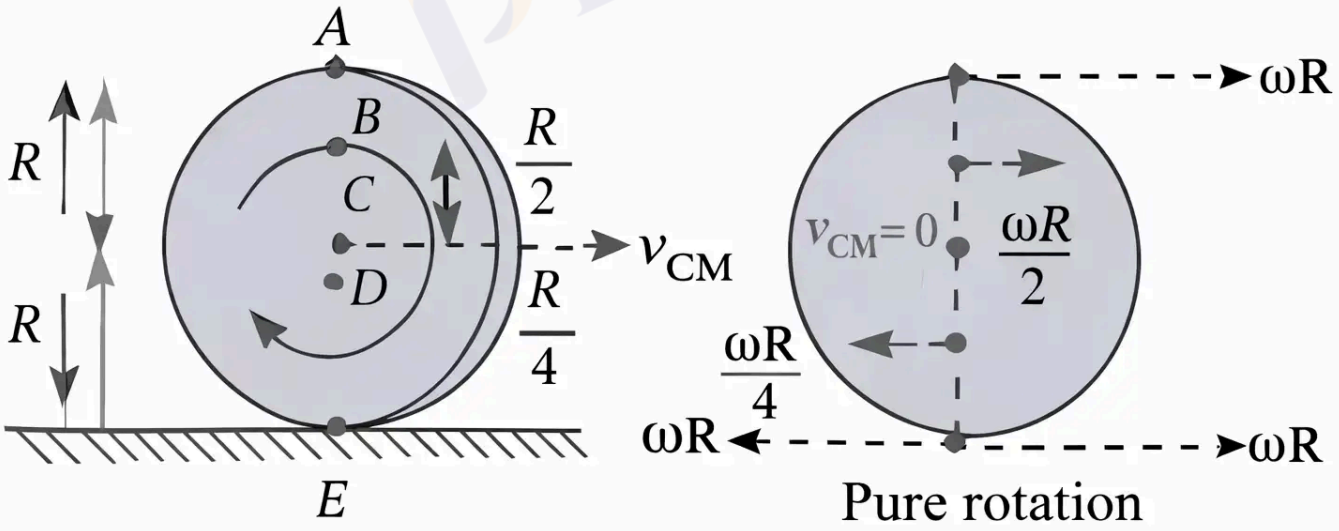
Definition and Conditions

- Rolling = rotation + translation
- Pure rolling: velocity of contact point with surface = 0
- Velocity relation: $v_{CM} = R\omega_{CM}$



Velocity of Points on Rolling Body

- Velocity of any point: $\vec{v} = \vec{v}_{CM} + \vec{\omega} \times \vec{R}$
- At point A: $\vec{v}_A = 2\vec{v}_{CM}$
- At point B: $\vec{v}_B = \frac{3}{2}\vec{v}_{CM}$
- At point C: $\vec{v}_C = \vec{v}_{CM}$
- At point D: $\vec{v}_D = \frac{3}{4}\vec{v}_{CM}$
- At point E (contact point): $\vec{v}_E = 0$



Kinetic Energy in Rolling

- $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{CM}^2 = \frac{1}{2}mv_{CM}^2\left(1 + \frac{k^2}{R^2}\right)$
- Energy ratio: $K_{translation} : K_{rotation} : K_{rolling} = 1 : \frac{k^2}{R^2} : \left(1 + \frac{k^2}{R^2}\right)$

Solved Examples

- **Ex 1:** Calculate velocity at point A on rolling wheel with $v_{CM} = 5\text{m/s}$.
  - $v_A = 2v_{CM} = 10\text{m/s}$

Practice Set

- Q1: Pure rolling means:
  - A) Contact point velocity = v_{CM}
  - B) Contact point velocity = 0
  - C) Wheel does not rotate
  - D) Wheel slips on surface
- Q2: Velocity at top point A of rolling wheel is:
  - A) v_{CM}
  - B) $2v_{CM}$
  - C) $\frac{1}{2}v_{CM}$
  - D) 0
- Q3: Kinetic energy of rolling is:
  - A) $\frac{1}{2}mv_{CM}^2$
  - B) $\frac{1}{2}I\omega^2$
  - C) $\frac{1}{2}mv_{CM}^2\left(1 + \frac{k^2}{R^2}\right)$
  - D) mv_{CM}^2

Answer Key

- Q1: B - Pure rolling means zero velocity at contact point.
- Q2: B - Velocity at top is twice the center velocity.
- Q3: C - Rolling kinetic energy includes translation and rotation parts.

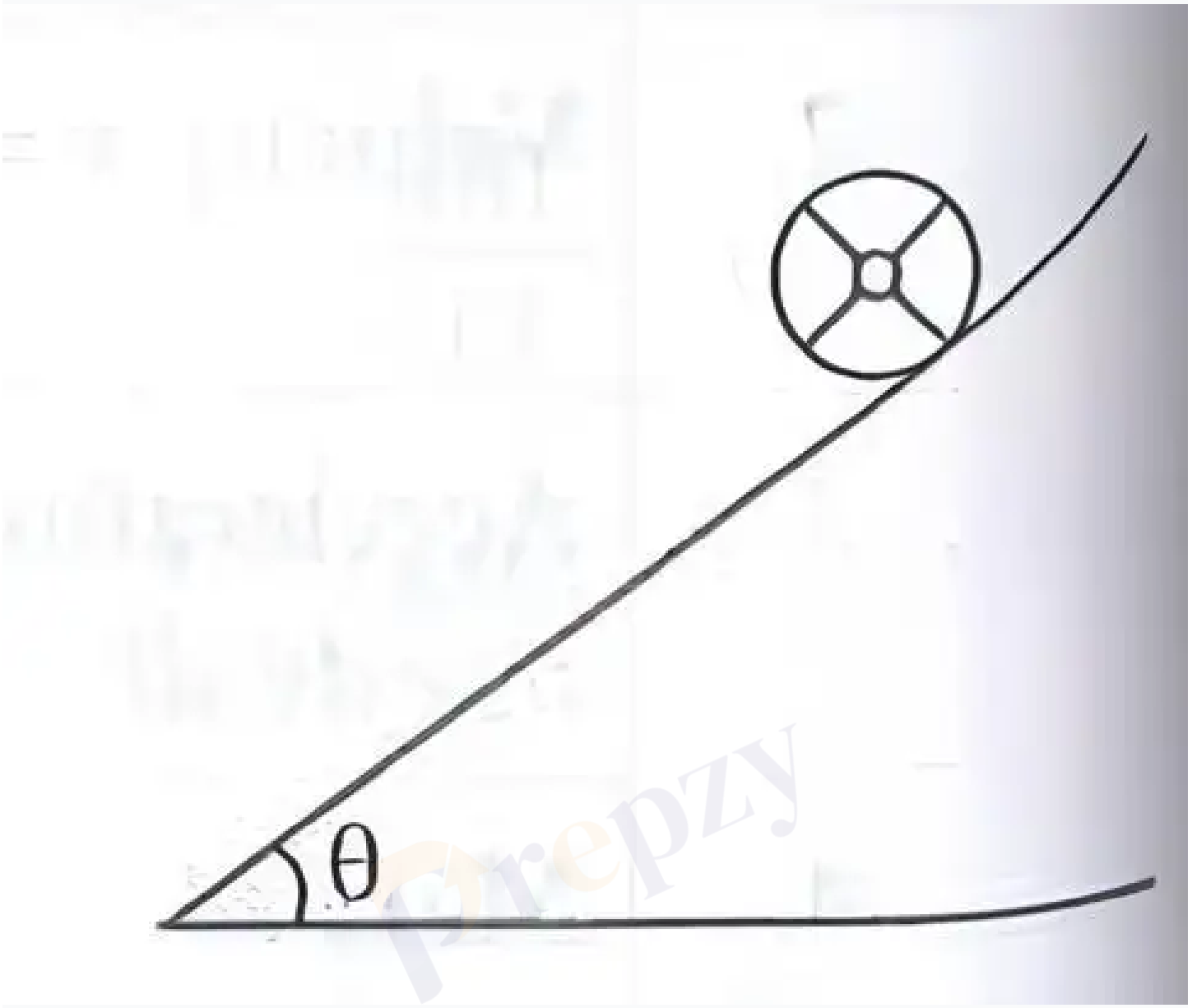
Memory Aids

- Pure rolling: $v_{contact} = 0$
- Velocity top point = $2v_{CM}$
- Energy: $K = \frac{1}{2}mv_{CM}^2(1 + \frac{k^2}{R^2})$

Motion Spherical Body Rough Inclined

Friction and Rolling on Incline

- Minimum friction for pure rolling: $\mu_{min} = \frac{\tan\theta}{1+\frac{mR^2}{I}}$
- If $\mu = 0$, body slips down with acceleration $a = g\sin\theta$
- If $\mu > \mu_{min}$, pure rolling with acceleration $a_2 = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$
- Friction force magnitude: $f = \frac{mg\sin\theta}{1+\frac{mR^2}{I}}$ (acts upward)
- If $\mu < \mu_{min}$, rolling with slipping, acceleration $a_3 = g\sin\theta - \mu g\cos\theta$



Comparison Pure Rolling vs Pure Sliding

Physical quantities	Pure rolling	Pure sliding
Acceleration	$\frac{g\sin\theta}{1+\frac{k^2}{r^2}}$	$g\sin\theta$
Velocity	$\sqrt{\frac{2gh}{1+\frac{k^2}{r^2}}}$	$\sqrt{2gh}$
Time taken to cover the distance s, where k is the radius of gyration.	$\sqrt{\frac{2s\left(1+\frac{k^2}{r^2}\right)}{g\sin\theta}}$	$\sqrt{\frac{2s}{g\sin\theta}}$

- Acceleration: Rolling $<$ Sliding

- Velocity: Rolling $<$ Sliding
- Time to cover distance: Rolling $>$ Sliding
- Rolling involves rotational inertia, sliding does not

Solved Examples

- **Ex 1:** Calculate minimum friction coefficient for pure rolling on incline $\theta = 30^\circ$, $m = 2\text{kg}$, $R = 0.5\text{m}$, $I = 0.25mR^2$.
  - Use $\mu_{min} = \frac{\tan\theta}{1+\frac{mR^2}{I}}$
  - Calculate and substitute values

Practice Set

- Q1: Minimum friction for pure rolling depends on:
  - A) $\tan\theta$ only
  - B) m, R, I only
  - C) $\tan\theta$ and $\frac{mR^2}{I}$
  - D) None of these
- Q2: If $\mu = 0$, acceleration of body on incline is:
  - A) 0
  - B) $g\cos\theta$
  - C) $g\sin\theta$
  - D) $\frac{g\sin\theta}{1+\frac{I}{mR^2}}$
- Q3: If $\mu < \mu_{min}$, acceleration is:
  - A) $g\sin\theta$
  - B) $g\sin\theta - \mu g\cos\theta$
  - C) $\frac{g\sin\theta}{1+\frac{I}{mR^2}}$
  - D) 0

Answer Key

- Q1: C - Minimum friction depends on slope angle and inertia ratio $\frac{mR^2}{I}$
- Q2: C - Without friction, body slips with acceleration $g\sin\theta$
- Q3: B - Rolling with slipping acceleration includes friction term

Memory Aids

- $\mu_{min} = \frac{\tan\theta}{1+\frac{mR^2}{I}}$
- Slip if $\mu = 0$, acceleration $g\sin\theta$
- Rolling acceleration $<$ sliding acceleration