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Introduction

Historical Background

- Nicolaus Copernicus (1473–1543) proposed heliocentric model: planets move in circles around the Sun.
- Galileo supported Copernicus but faced prosecution for his beliefs.
- Tycho Brahe (1546–1601) made precise naked-eye planetary observations.
- Johannes Kepler (1571–1640) derived three laws of planetary motion from Brahe's data.
- Newton used Kepler's laws to formulate the universal law of gravitation.

Solved Examples

Practice Set

- Q1: Who proposed the heliocentric model?
 - A. Galileo
 - B. Tycho Brahe
 - C. Nicolaus Copernicus
 - D. Johannes Kepler
- Q2: What did Tycho Brahe contribute?
 - A. Laws of planetary motion
 - B. Naked-eye planetary observations
 - C. Universal gravitation
 - D. Elliptical orbits
- Q3: Who formulated the universal law of gravitation?
 - A. Copernicus
 - B. Galileo
 - C. Newton
 - D. Kepler

Answer Key

- Q1: C - Copernicus proposed the heliocentric model.
- Q2: B - Brahe made precise naked-eye observations.
- Q3: C - Newton formulated universal gravitation using Kepler's laws.

Memory Aids

- Copernicus: Sun-centered model.
- Brahe: Data collector.
- Kepler: Laws from Brahe's data.
- Newton: Gravity law from Kepler.

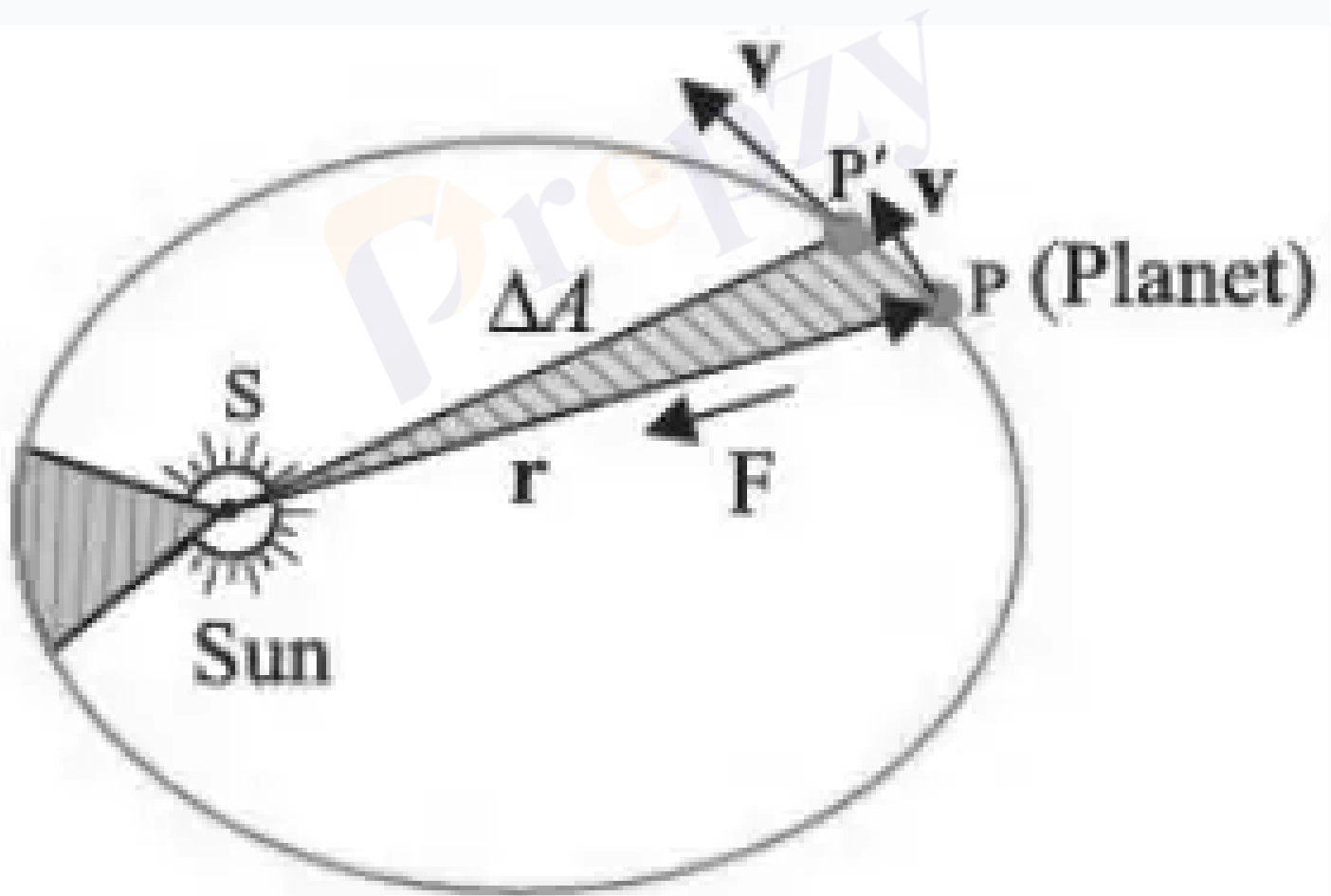
Kepler's Laws

Law of Orbits

- Planets move in elliptical orbits with the Sun at one focus.

Law of Areas

- Line joining planet and Sun sweeps equal areas in equal times.
- Area swept in small time Δt : $\Delta A = \frac{1}{2}rv\Delta t$
- Vector form: $\Delta \vec{A} = \frac{1}{2}(\vec{r} \times \vec{v}\Delta t)$
- Rate of area swept: $\frac{\Delta \vec{A}}{\Delta t} = \frac{1}{2}(\vec{r} \times \vec{v})$
- Using momentum $\vec{p} = m\vec{v}$: $\frac{\Delta \vec{A}}{\Delta t} = \frac{\vec{r} \times \vec{p}}{2m}$
- Angular momentum $\vec{L} = \vec{r} \times \vec{p}$ is constant for central force.
- Therefore, $\frac{\Delta \vec{A}}{\Delta t} = \frac{\vec{L}}{2m} = \text{constant}$



Law of Periods

- Square of orbital period T^2 is proportional to cube of semi-major axis R^3 :
- $T^2 \propto R^3$

Solved Examples

Practice Set

- Q1: What shape is a planet's orbit according to Kepler's first law?
A. Circle
B. Ellipse
C. Parabola
D. Hyperbola
- Q2: According to Kepler's second law, the line joining planet and Sun sweeps out equal areas in equal times. What does this imply about the planet's speed?
A. Constant speed
B. Faster when closer to Sun
C. Slower when closer to Sun
D. No relation to speed
- Q3: If the semi-major axis of a planet's orbit is doubled, how does its orbital period change?
A. Doubles
B. Quadruples
C. Increases by $2^{3/2}$
D. Halves

Answer Key

- Q1: B - Orbits are elliptical with Sun at one focus.
- Q2: B - Planet moves faster when closer to the Sun to sweep equal areas.
- Q3: C - $T^2 \propto R^3$ so $T \propto R^{3/2}$, doubling R increases T by $2^{3/2}$.

Memory Aids

- Orbits: Ellipse with Sun focus.
- Areas: Equal areas in equal times.
- Periods: $T^2 \propto R^3$.

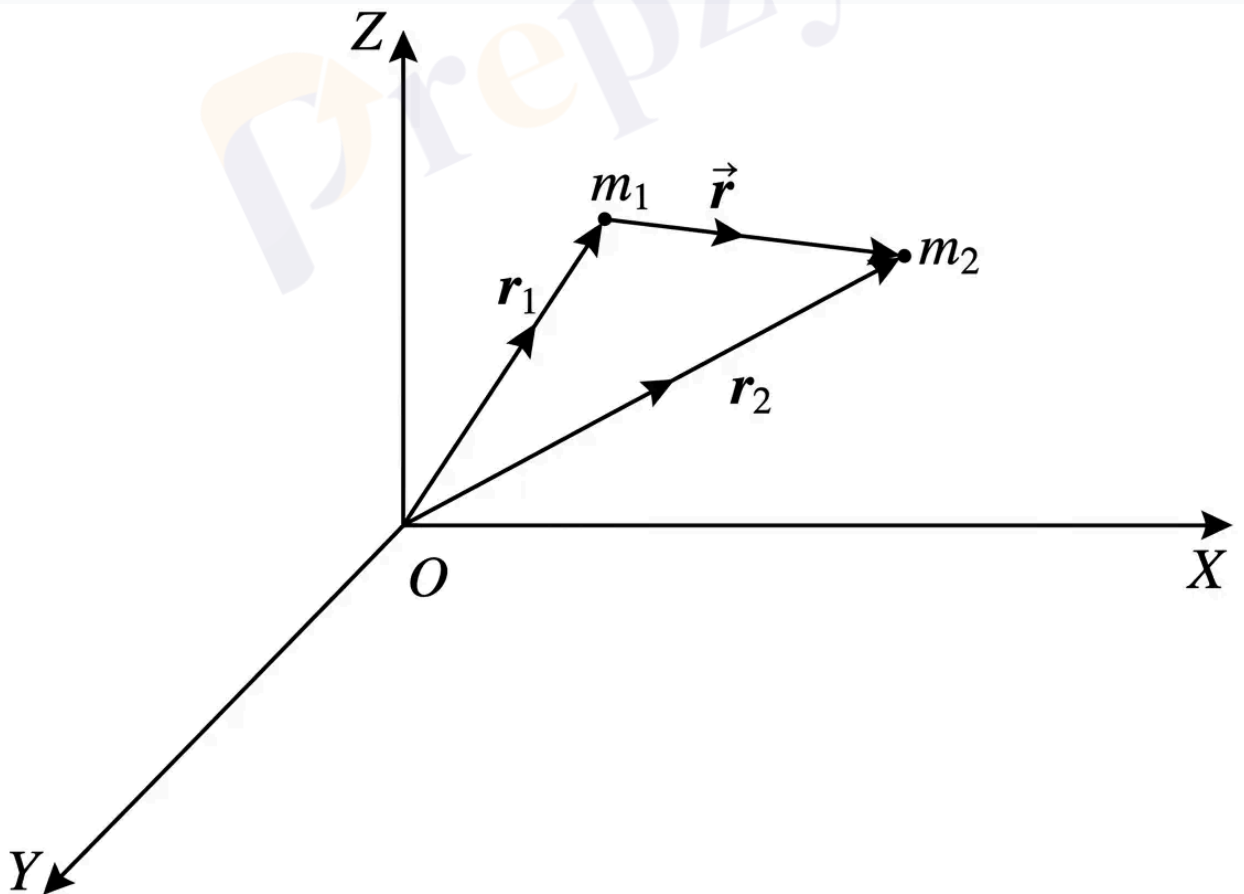
Universal Law of Gravitation

Newton's Law

- Every particle attracts every other particle.
- Force $F \propto$ product of masses $m_1 m_2$.
- Force $F \propto$ inverse square of distance r^2 .
- Mathematical form: $F = \frac{Gm_1 m_2}{r^2}$.
- G is universal gravitational constant, same everywhere.

Characteristics of Gravitational Force

- Always attractive, along line joining particles.
- Action-reaction pair: $\vec{F}_1 = -\vec{F}_2$.
- Independent of medium.
- Valid for all distances (interplanetary to atomic).
- Conservative force: work done independent of path.
- No work done in closed path.
- Vector form: $\vec{F} = -G \frac{m_1 m_2}{r^2} \hat{r} = -G \frac{m_1 m_2}{|\vec{r}|^3} \vec{r}$.



Solved Examples

- **Ex 1:** Calculate force between masses $m_1 = 2 \text{ kg}$, $m_2 = 3 \text{ kg}$, distance $r = 4 \text{ m}$, $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$.
 - Use $F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 2 \times 3}{4^2} = 2.5 \times 10^{-11} \text{ N}$.

Practice Set

- Q1: The gravitational force between two masses is doubled if the distance between them is changed from r to:
 - A) $r/2$
 - B) $2r$
 - C) $r/4$
 - D) $4r$
- Q2: Two masses m and $2m$ are separated by distance d . The force between them is F . What is the force if distance is doubled?
 - A) $F/2$
 - B) $F/4$
 - C) $F/8$
 - D) $F/16$
- Q3: If the gravitational force between two masses is $3 \times 10^{-10} \text{ N}$ at distance 1 m , what is the force at 3 m ?
 - A) $3.33 \times 10^{-11} \text{ N}$
 - B) $3.33 \times 10^{-10} \text{ N}$
 - C) $3 \times 10^{-9} \text{ N}$
 - D) $1 \times 10^{-10} \text{ N}$

Answer Key

- Q1: A) $r/2$ - Force $\propto 1/r^2$, halving distance quadruples force.
- Q2: B) $F/4$ - Doubling distance reduces force by 4.
- Q3: A) $3.33 \times 10^{-11} \text{ N}$ - Force decreases by $(1/3)^2 = 1/9$.

Memory Aids

- Force $\propto m_1m_2/r^2$.
- Always attractive, along line joining masses.
- Action = Reaction (equal & opposite).
- Conservative force: path independent work.

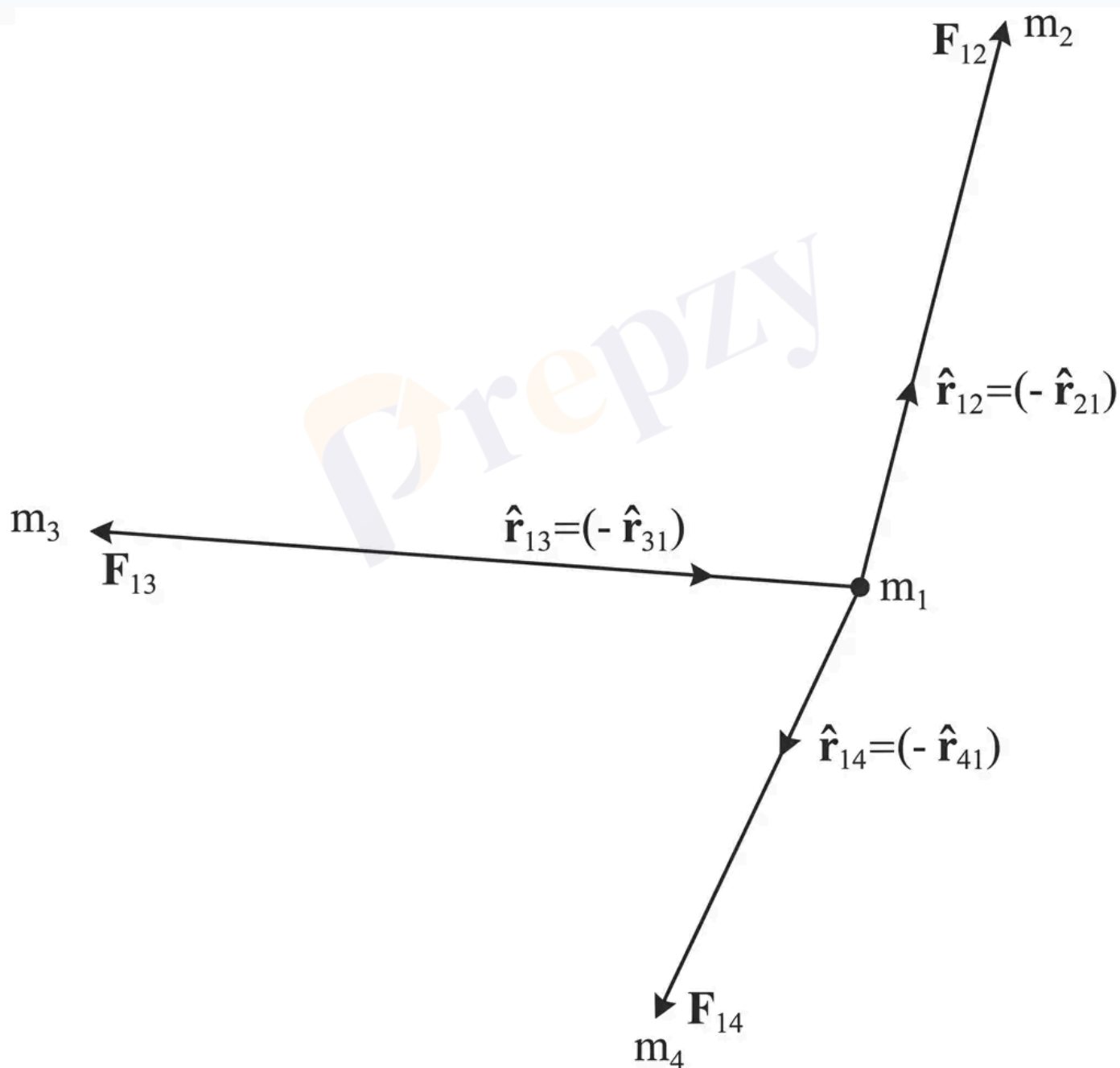
Gravitational Force on Multiple Masses

Vector Sum of Forces

- Force on a mass due to multiple masses is vector sum of individual forces.
- For mass m_1 due to m_2, m_3, m_4 :

$$\vec{F}_1 = \frac{Gm_1m_2}{r_{21}^2}\hat{r}_{21} + \frac{Gm_1m_3}{r_{31}^2}\hat{r}_{31} + \frac{Gm_1m_4}{r_{41}^2}\hat{r}_{41}$$

- Unit vectors $\hat{r}_{ij}$ point from mass m_i to m_j .
- $\hat{r}_{ij} = -\hat{r}_{ji}$ (opposite directions).



Solved Examples

- **Ex 1:** Calculate net force on $m_1 = 2 \text{ kg}$ due to $m_2 = 3 \text{ kg}$ at 4 m and $m_3 = 4 \text{ kg}$ at 3 m along different directions.
 - Calculate each force using $F = \frac{Gm_1m_i}{r^2}$ and add vectorially.

Practice Set

- Q1: If $m_1 = 1 \text{ kg}$, $m_2 = 2 \text{ kg}$, $m_3 = 3 \text{ kg}$, distances 1 m and 2 m respectively, net force on m_1 is:
 - A) Sum of magnitudes
 - B) Vector sum of forces
 - C) Zero
 - D) Product of forces
- Q2: Unit vector $\hat{r}_{ij}$ points from:
 - A) m_i to m_j
 - B) m_j to m_i
 - C) Origin to m_i
 - D) Origin to m_j
- Q3: If $\vec{F}_{12} = -\vec{F}_{21}$, this means:
 - A) Forces are equal and opposite
 - B) Forces are zero
 - C) Forces are perpendicular
 - D) Forces are unrelated

Answer Key

- Q1: B) Vector sum of forces - Forces add vectorially.
- Q2: A) m_i to m_j - Unit vector direction.
- Q3: A) Forces are equal and opposite - Action-reaction pair.

Memory Aids

- Net force = vector sum of individual forces.
- Unit vector points from source mass to target mass.
- Action = Reaction forces are equal & opposite.

The Gravitational Constant

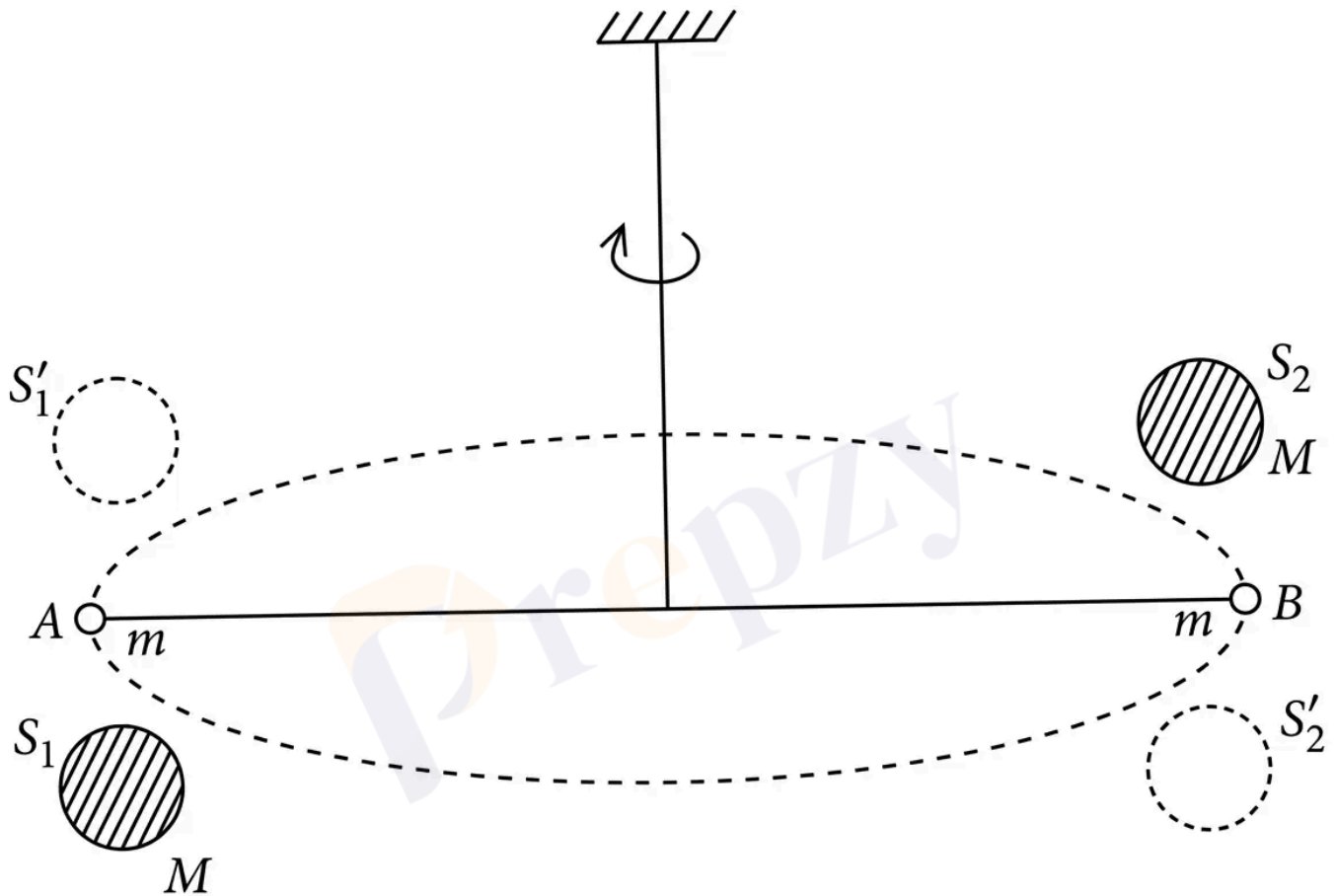
Definition and Historical Context

- The gravitational constant G quantifies the strength of gravity between two masses.
- First measured by Henry Cavendish in 1798 using a torsion balance experiment.

- Current accepted value: $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

Schematic of Cavendish's Experiment

- Two small masses m fixed at points A and B on a light rod, rotating about a vertical axis.
- Two large spheres S_1 and S_2 placed near the small masses exert gravitational force.
- When large spheres are moved to opposite sides (positions S'_1 and S'_2), the rod rotates due to torque reversal.
- Rotation angle θ is measured to calculate G .



Equilibrium Condition and Formula

- At equilibrium, gravitational torque equals restoring torque from the wire.
- Mathematically: $G \frac{Mm}{r^2} L = k\theta$, where:
 - M = mass of large sphere
 - m = mass of small sphere
 - r = distance between centers of large and small spheres
 - L = length of the rod
 - k = torsion constant of the wire
 - θ = angle of twist
- Rearranged formula to find G :

$$G = \frac{k\theta r^2}{MmL}$$

Solved Examples

- **Ex 1:** Given $k = 2 \times 10^{-6} \text{ Nm/rad}$, $\theta = 0.01 \text{ rad}$, $r = 0.05 \text{ m}$, $M = 1 \text{ kg}$, $m = 0.01 \text{ kg}$, $L = 0.4 \text{ m}$, find G .
- Solution: $G = \frac{2 \times 10^{-6} \times 0.01 \times (0.05)^2}{1 \times 0.01 \times 0.4} = 1.25 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

Practice Set

- Q1: Who first determined the value of G ?
A. Isaac Newton
B. Albert Einstein
C. Henry Cavendish
D. Galileo Galilei
- Q2: In Cavendish's experiment, the torque due to gravitational force is balanced by:
A. Magnetic force
B. Restoring torque of the wire
C. Centrifugal force
D. Electrostatic force
- Q3: Calculate G if $k = 3 \times 10^{-6} \text{ Nm/rad}$, $\theta = 0.02 \text{ rad}$, $r = 0.04 \text{ m}$, $M = 2 \text{ kg}$, $m = 0.02 \text{ kg}$, $L = 0.5 \text{ m}$.
A. 1.92×10^{-11}
B. 2.4×10^{-11}
C. 3.84×10^{-11}
D. 4.8×10^{-11}

Answer Key

- Q1: C - Henry Cavendish first measured G in 1798 using a torsion balance.
- Q2: B - The restoring torque of the wire balances the gravitational torque at equilibrium.
- Q3: A - Using formula $G = \frac{k\theta r^2}{MmL} = \frac{3 \times 10^{-6} \times 0.02 \times (0.04)^2}{2 \times 0.02 \times 0.5} = 1.92 \times 10^{-11}$.

Memory Aids

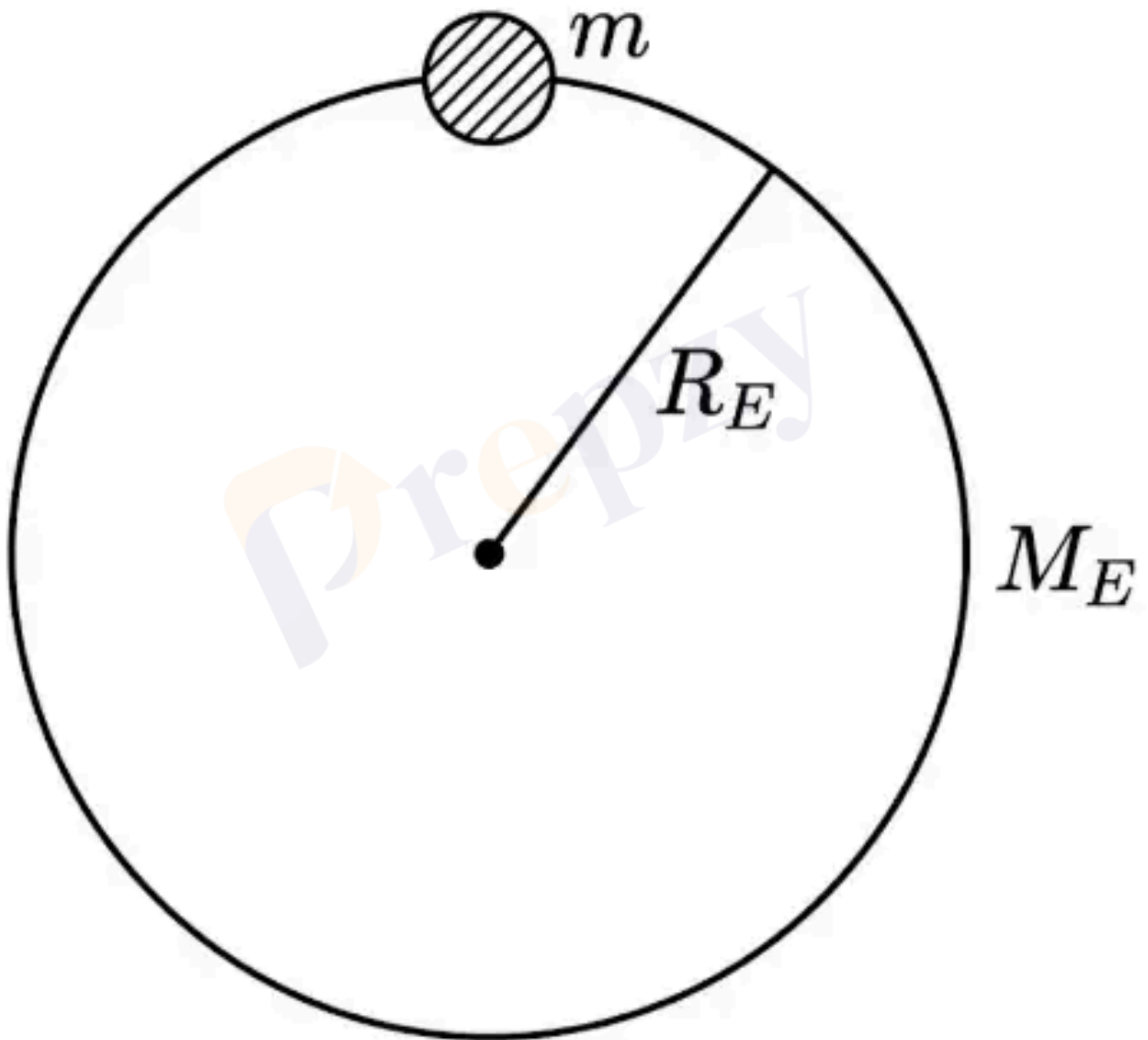
- Remember G relates gravitational force to masses and distance squared.
- Formula: $G = \frac{k\theta r^2}{MmL}$ from Cavendish's torsion balance.
- Value: $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

Acceleration due to Gravity

Concept and Definition

- Acceleration produced in a body falling freely towards Earth due to gravity.
- Standard value on Earth's surface: $g = 9.8 \text{ m/s}^2$.
- Depends on Earth's mass M_E and radius R_E .

Explanation with Diagram



- Earth represented as a large circle with center point.
- Mass of Earth: M_E .
- Small object mass: m , orbiting at radius R_E from Earth's center.
- Object moves due to gravitational attraction.

Gravitational Force Formula

- Force between Earth and object: $F = \frac{GM_E m}{R_E^2}$.
- Weight of object: $W = mg$.
- Equating weight and force: $mg = \frac{GM_E m}{R_E^2} \Rightarrow g = \frac{GM_E}{R_E^2}$.
- Numerical: $g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6400 \times 10^3)^2} = 9.8 \text{ m/s}^2$.

Solved Examples

- **Ex 1:** Calculate g using $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$, $M_E = 6 \times 10^{24} \text{ kg}$, $R_E = 6400 \text{ km}$.
 - Use formula: $g = \frac{GM_E}{R_E^2}$.
 - Convert R_E to meters: $6400 \times 10^3 \text{ m}$.
 - Calculate $g = 9.8 \text{ m/s}^2$.

Practice Set

- **Q1:** What is the acceleration due to gravity on Earth's surface?
 - A) 9.8 m/s^2
 - B) 10 m/s^2
 - C) 9.2 m/s^2
 - D) 8.9 m/s^2
- **Q2:** If Earth's radius doubles, what happens to g ?
 - A) Doubles
 - B) Halves
 - C) Quartered
 - D) Remains same
- **Q3:** Given $G = 6.67 \times 10^{-11}$, $M_E = 6 \times 10^{24}$, $R_E = 6400 \times 10^3$, calculate g .
 - A) 9.8 m/s^2
 - B) 10.5 m/s^2
 - C) 8.9 m/s^2
 - D) 9.2 m/s^2

Answer Key

- **Q1:** A - Standard acceleration due to gravity on Earth's surface is 9.8 m/s^2 .
- **Q2:** C - g is inversely proportional to square of radius, so doubling radius quarters g .
- **Q3:** A - Direct calculation using formula gives 9.8 m/s^2 .

Memory Aids

- Formula: $g = \frac{GM_E}{R_E^2}$.

- Remember: g depends on Earth's mass and radius only.
- Value on Earth's surface: 9.8 m/s^2 .

Relationship between g and G

Formula and Explanation

- $g = \frac{GM_E}{R_E^2} = G \frac{4}{3}\pi R_E^3 \rho / R_E^2 = \frac{4}{3}\pi G R_E \rho$.
- ρ is uniform density of Earth.
- Shows g depends on G , Earth's radius, and density.

Solved Examples

- **Ex 1:** Calculate g if $\rho = 5500 \text{ kg/m}^3$, $R_E = 6400 \times 10^3 \text{ m}$, $G = 6.67 \times 10^{-11}$.
 - Use formula: $g = \frac{4}{3}\pi G R_E \rho$.
 - Calculate $g \approx 9.8 \text{ m/s}^2$.

Practice Set

- **Q1:** Which quantity does g depend on?
 - A) Mass of object
 - B) Earth's radius and density
 - C) Velocity of object
 - D) Temperature
- **Q2:** If Earth's density doubles, what happens to g ?
 - A) Doubles
 - B) Halves
 - C) No change
 - D) Quadruples
- **Q3:** Given $G = 6.67 \times 10^{-11}$, $R_E = 6400 \times 10^3$, $\rho = 5500$, calculate g .
 - A) 9.8 m/s^2
 - B) 10.5 m/s^2
 - C) 8.9 m/s^2
 - D) 9.2 m/s^2

Answer Key

- **Q1:** B - g depends on Earth's radius and density, not object mass.
- **Q2:** A - g is directly proportional to density, so doubling density doubles g .
- **Q3:** A - Calculation using formula yields 9.8 m/s^2 .

Memory Aids

- Formula: $g = \frac{4}{3}\pi GR_E\rho$.
- g depends on Earth's radius and density.
- G is universal gravitational constant.

Gravitational Field

Definition and Intensity

- Space around a body where its gravitational pull is felt is called gravitational field.
- Intensity E at a point: force per unit mass placed at that point.
- $E = -\frac{GM}{r^2}$, negative sign indicates attraction.

Solved Examples

- **Ex 1:** Calculate gravitational field intensity at $r = 10$ m from mass $M = 1000$ kg, $G = 6.67 \times 10^{-11}$.
 - Use formula: $E = -\frac{GM}{r^2}$.
 - Calculate $E = -\frac{6.67 \times 10^{-11} \times 1000}{10^2} = -6.67 \times 10^{-11}$ N/kg.

Practice Set

- **Q1:** What does gravitational field represent?
 - A) Space where gravitational pull is felt
 - B) Magnetic field
 - C) Electric field
 - D) Space with no forces
- **Q2:** Intensity of gravitational field at distance r is?
 - A) $\frac{GM}{r}$
 - B) $-\frac{GM}{r^2}$
 - C) $\frac{GM}{r^3}$
 - D) $-\frac{GM}{r}$
- **Q3:** Calculate gravitational field intensity at $r = 5$ m from mass 500 kg, $G = 6.67 \times 10^{-11}$.
 - A) -1.33×10^{-9} N/kg
 - B) -2.67×10^{-9} N/kg
 - C) -6.67×10^{-10} N/kg
 - D) -1.33×10^{-10} N/kg

Answer Key

- **Q1:** A - Gravitational field is the space where gravitational pull is experienced.

- **Q2:** B - Intensity formula is $E = -\frac{GM}{r^2}$.
- **Q3:** A - Calculation: $E = -\frac{6.67 \times 10^{-11} \times 500}{5^2} = -1.33 \times 10^{-9} \text{ N/kg}$.

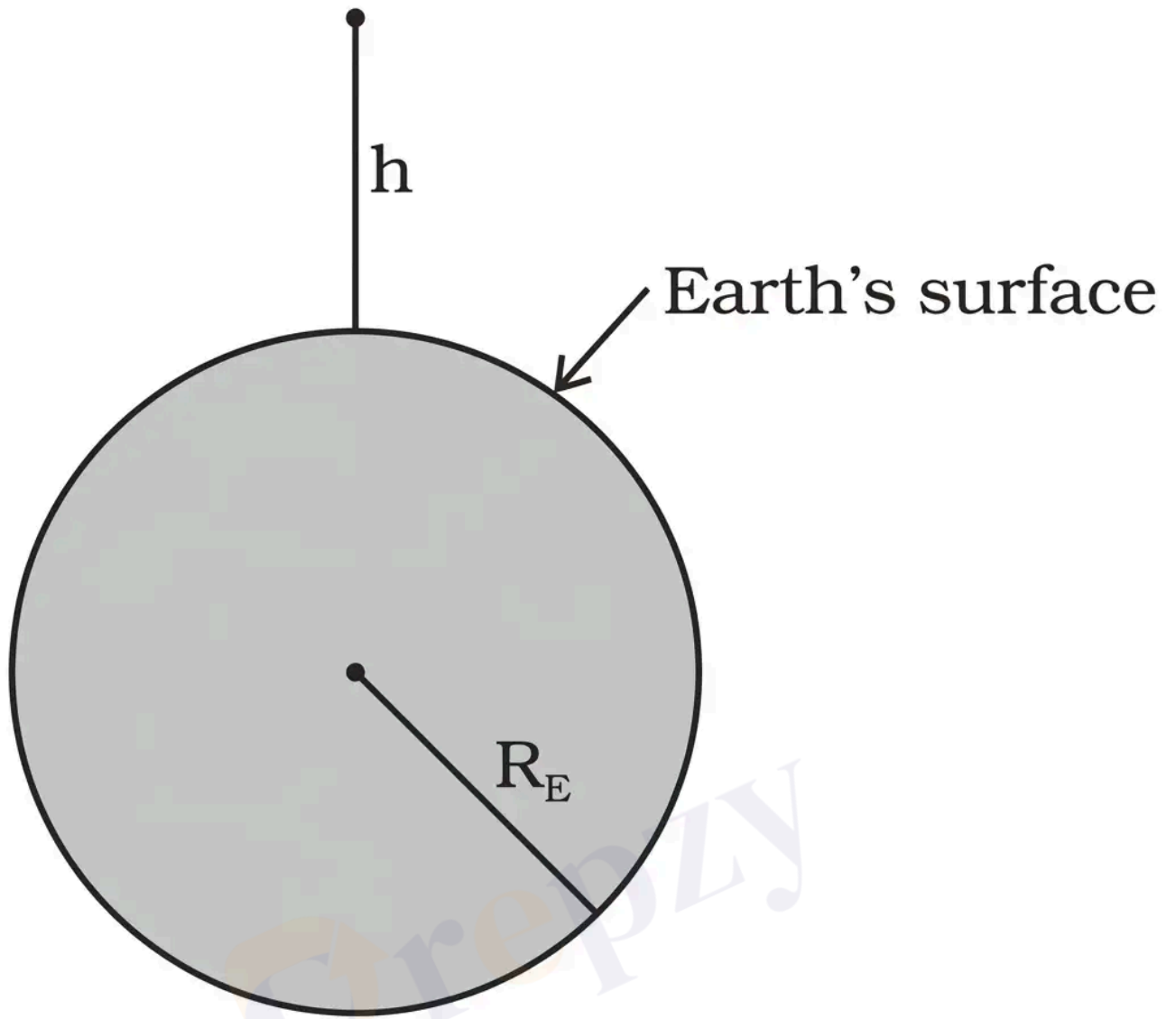
Memory Aids

- Formula: $E = -\frac{GM}{r^2}$.
- Negative sign means attraction.
- Field intensity = force per unit mass.

Acceleration due to Gravity Below and Above Earth Surface

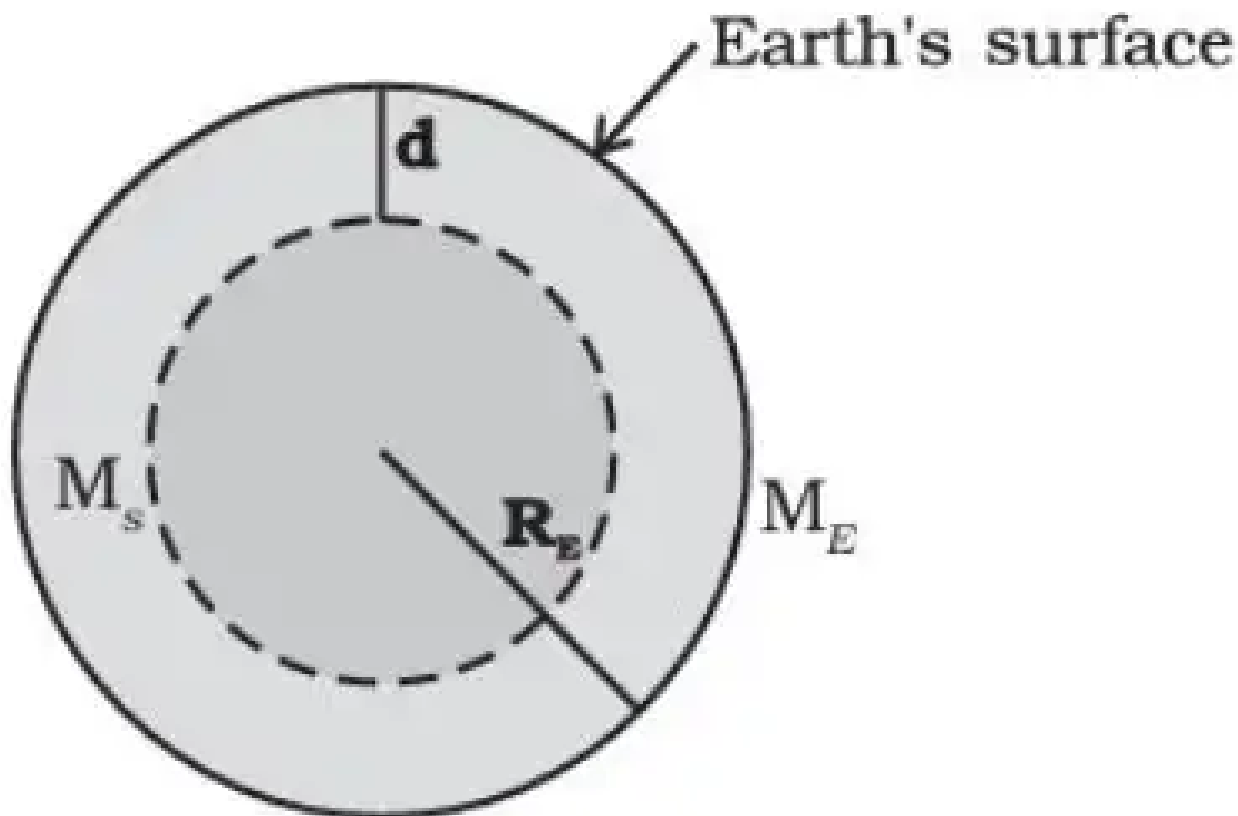
Effect of Altitude

- Force on mass m at height h : $F(h) = \frac{GM_E m}{(R_E + h)^2}$
- Acceleration due to gravity at height h : $g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E + h)^2} = g \left(1 + \frac{h}{R_E}\right)^{-2}$
- For small height h ($h \ll R_E$), using binomial approximation: $g(h) \approx g \left(1 - \frac{2h}{R_E}\right)$
- Gravity decreases with altitude by factor $(1 - 2h/R_E)$



Effect of Depth

- Mass of smaller sphere at depth d : $\frac{M_s}{M_E} = \left(\frac{R_E - d}{R_E}\right)^3$
- Force on mass m at depth d : $F(d) = \frac{GM_s m}{(R_E - d)^2} = \frac{GM_E m (R_E - d)}{R_E^3}$
- Acceleration due to gravity at depth d : $g(d) = \frac{F(d)}{m} = g \left(1 - \frac{d}{R_E}\right)$
- Gravity decreases linearly with depth by factor $(1 - d/R_E)$



Solved Examples

- **Ex 1:** Calculate $g(h)$ at height $h = 1000$ m above Earth surface. Given $g = 9.8 \text{ m/s}^2$, $R_E = 6.4 \times 10^6$ m.
 - Use $g(h) \approx g \left(1 - \frac{2h}{R_E}\right) = 9.8 \left(1 - \frac{2 \times 1000}{6.4 \times 10^6}\right) = 9.8 \times 0.9996875 = 9.796 \text{ m/s}^2$
- **Ex 2:** Find $g(d)$ at depth $d = 5000$ m below Earth surface.
 - Use $g(d) = g \left(1 - \frac{d}{R_E}\right) = 9.8 \left(1 - \frac{5000}{6.4 \times 10^6}\right) = 9.8 \times 0.99922 = 9.792 \text{ m/s}^2$

Practice Set

- Q1: At a height h above Earth surface, acceleration due to gravity $g(h)$ is given by:
 - A) $g \left(1 + \frac{h}{R_E}\right)^2$
 - B) $g \left(1 + \frac{h}{R_E}\right)^{-2}$
 - C) $g \left(1 - \frac{h}{R_E}\right)^2$
 - D) $g \left(1 - \frac{2h}{R_E}\right)^{-1}$
- Q2: At depth d below Earth surface, acceleration due to gravity $g(d)$ is:
 - A) $g \left(1 + \frac{d}{R_E}\right)$
 - B) $g \left(1 - \frac{d}{R_E}\right)$
 - C) $g \left(1 - \frac{2d}{R_E}\right)$
 - D) $g \left(1 + \frac{2d}{R_E}\right)$
- Q3: If $g = 9.8 \text{ m/s}^2$, $R_E = 6.4 \times 10^6$ m, find $g(h)$ at $h = 6400$ m.

- A) 9.78 m/s^2
- B) 9.76 m/s^2
- C) 9.74 m/s^2
- D) 9.72 m/s^2

Answer Key

- Q1: B - Formula for gravity at height is $g(h) = g\left(1 + \frac{h}{R_E}\right)^{-2}$
- Q2: B - Gravity decreases linearly with depth as $g(d) = g\left(1 - \frac{d}{R_E}\right)$
- Q3: A - Using $g(h) \approx g\left(1 - \frac{2h}{R_E}\right) = 9.8 \times \left(1 - \frac{2 \times 6400}{6.4 \times 10^6}\right) = 9.78$

Memory Aids

- Gravity decreases with altitude: $g(h) \approx g(1 - 2h/R_E)$
- Gravity decreases with depth: $g(d) = g(1 - d/R_E)$
- Use binomial approximation for small heights

Variation in Acceleration due to Gravity

Factors Affecting Gravity

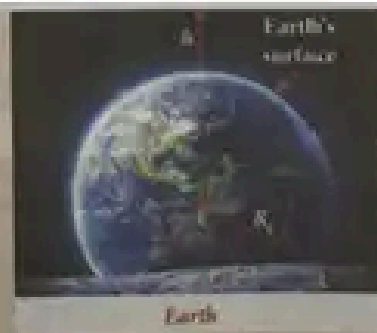
- Gravity decreases above Earth's surface with height.
- Gravity decreases below Earth's surface with depth, zero at center.
- Earth's rotation reduces gravity at equator due to centrifugal force.
- Gravity varies with latitude λ : $g_\lambda = g - R_E \omega^2 \cos^2 \lambda$
- Minimum gravity at equator ($\lambda = 0^\circ$), maximum at poles ($\lambda = 90^\circ$).
- Earth's shape: equatorial bulge reduces gravity at equator.

Due to altitude (h) : The acceleration due to gravity at height h above

the Earth's surface is given by $g_h = \frac{GM_E}{(R_E + h)^2} = g \left(1 + \frac{h}{R_E}\right)^{-2}$ ($\because g = \frac{GM_E}{R_E^2}$)

for $h \ll R_E \therefore g_h = g \left(1 - \frac{2h}{R_E}\right)$

When we move above the Earth's surface, the value of acceleration due to gravity goes on decreasing.



Due to depth (d) : The acceleration due to gravity at a depth d below the Earth's surface is given by

$$g_d = \frac{GM_E}{R_E^3} (R_E - d)$$

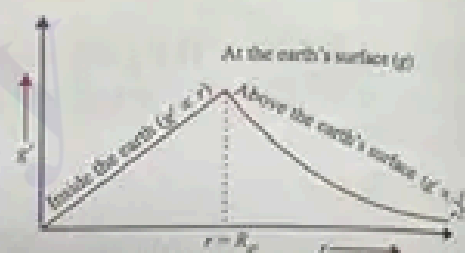
$$= g \left(\frac{R_E - d}{R_E}\right) = g \left(1 - \frac{d}{R_E}\right)$$

At the centre of the earth, $d = R_E \therefore g_d = 0$.



When we move below the earth's surface, the value of acceleration due to gravity also decreases. The value of acceleration due to gravity is maximum at the earth's surface and becomes zero at the centre of the earth.

The variation of acceleration due to gravity (g) with distance from the centre of the earth (r) is as shown in the figure.



Due to rotation of the earth about its axis : The acceleration due to gravity at latitude λ is given by $g_\lambda = g - R_E \omega^2 \cos^2 \lambda$

where ω is the angular speed of rotation of the earth about its axis and its value is $7.3 \times 10^{-5} \text{ rad s}^{-1}$.

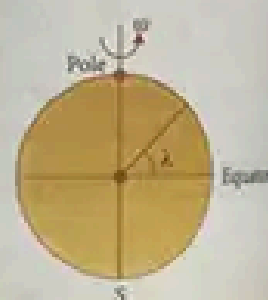
* At the equator, $\lambda = 0^\circ$

$$g_\lambda = g_e = g - R_E \omega^2 \cos^2 0^\circ = g - R_E \omega^2$$

* At the poles, $\lambda = 90^\circ$; $g_\lambda = g_p = g - R_E \omega^2 \cos^2 90^\circ = g$

The value of acceleration due to gravity increases from equator to the pole due to rotation of the earth.

$$g_p - g_e = g - (g - R_E \omega^2) = R_E \omega^2$$



If the earth stops rotating about its axis ($\omega = 0$), the value of g will increase everywhere, except at the poles. On the contrary, if there is increase in the angular speed of the earth, then except at the poles the value of g will decrease at all places.

Due to shape of the earth : Earth is not a perfect sphere but it is an ellipsoid. The earth's radius is 21 km larger at the equator than at the poles. Thus, the earth has an equatorial bulge and is flattened at the poles. Both, rotation and equatorial bulge, contribute additively to keep the g smaller at the equator than at the poles.

Solved Examples

- **Ex 1:** Calculate gravity at equator given $g = 9.8 \text{ m/s}^2$, $R_E = 6.4 \times 10^6 \text{ m}$, $\omega = 7.27 \times 10^{-5} \text{ rad/s}$.
 - $g_e = g - R_E \omega^2 = 9.8 - 6.4 \times 10^6 \times (7.27 \times 10^{-5})^2 = 9.8 - 0.0339 = 9.766 \text{ m/s}^2$
- **Ex 2:** Find difference in gravity between poles and equator.
 - $g_p - g_e = R_E \omega^2 = 0.0339 \text{ m/s}^2$

Practice Set

- Q1: Gravity at latitude λ is given by:
 - A) $g + R_E \omega^2 \cos^2 \lambda$
 - B) $g - R_E \omega^2 \cos^2 \lambda$
 - C) $g - R_E \omega^2 \sin^2 \lambda$
 - D) $g + R_E \omega^2 \sin^2 \lambda$
- Q2: Gravity is minimum at:
 - A) Poles
 - B) Equator
 - C) Mid-latitudes
 - D) Center of Earth
- Q3: If Earth did not rotate, gravity at equator would be:
 - A) Greater than current
 - B) Less than current
 - C) Same as poles
 - D) Zero

Answer Key

- Q1: B - Gravity reduces by centrifugal force term $R_E \omega^2 \cos^2 \lambda$
- Q2: B - Minimum gravity at equator due to rotation and bulge
- Q3: A - Without rotation, no centrifugal force, gravity stronger at equator

Memory Aids

- Gravity $g_\lambda = g - R_E \omega^2 \cos^2 \lambda$
- Minimum gravity at equator, maximum at poles
- Earth's bulge reduces gravity at equator

Gravitational Field and Potential of Continuous Mass Distributions

Key Formulas

- Gravitational potential V : work done per unit mass from infinity to point

- Relation: $E = -\frac{dV}{dr}$

- Uniform ring (mass M , radius R):**

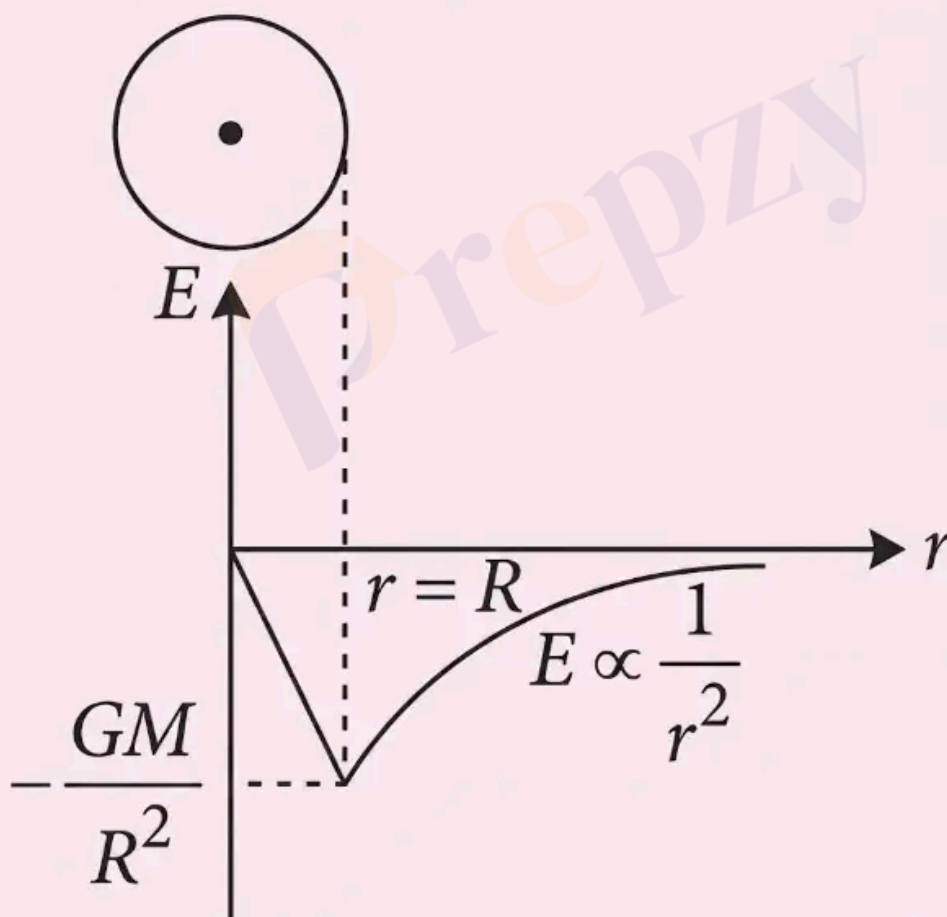
- Field on axis: $E = -\frac{GMx}{(R^2+x^2)^{3/2}}$
- Potential on axis: $V = -\frac{GM}{\sqrt{R^2+x^2}}$

- Uniform disc (mass M , radius R):**

- Field on axis: $E = -\frac{2GM}{R^2} \left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$
- Potential on axis: $V = -\frac{2GM}{R^2} \left(x - \sqrt{R^2+x^2}\right)$

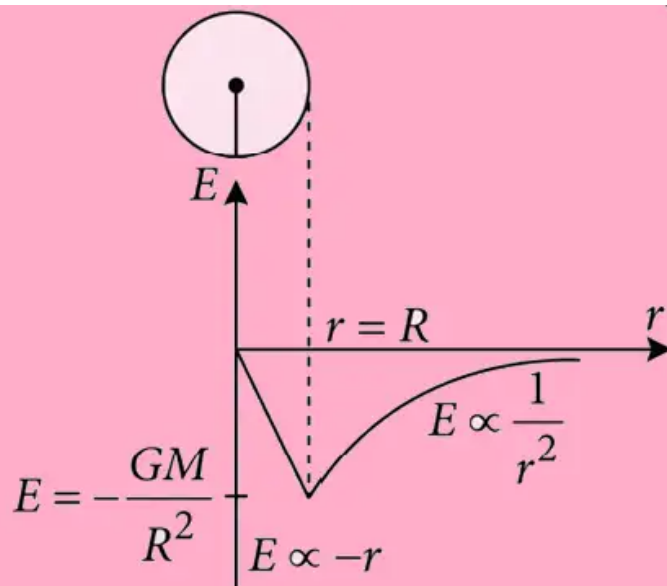
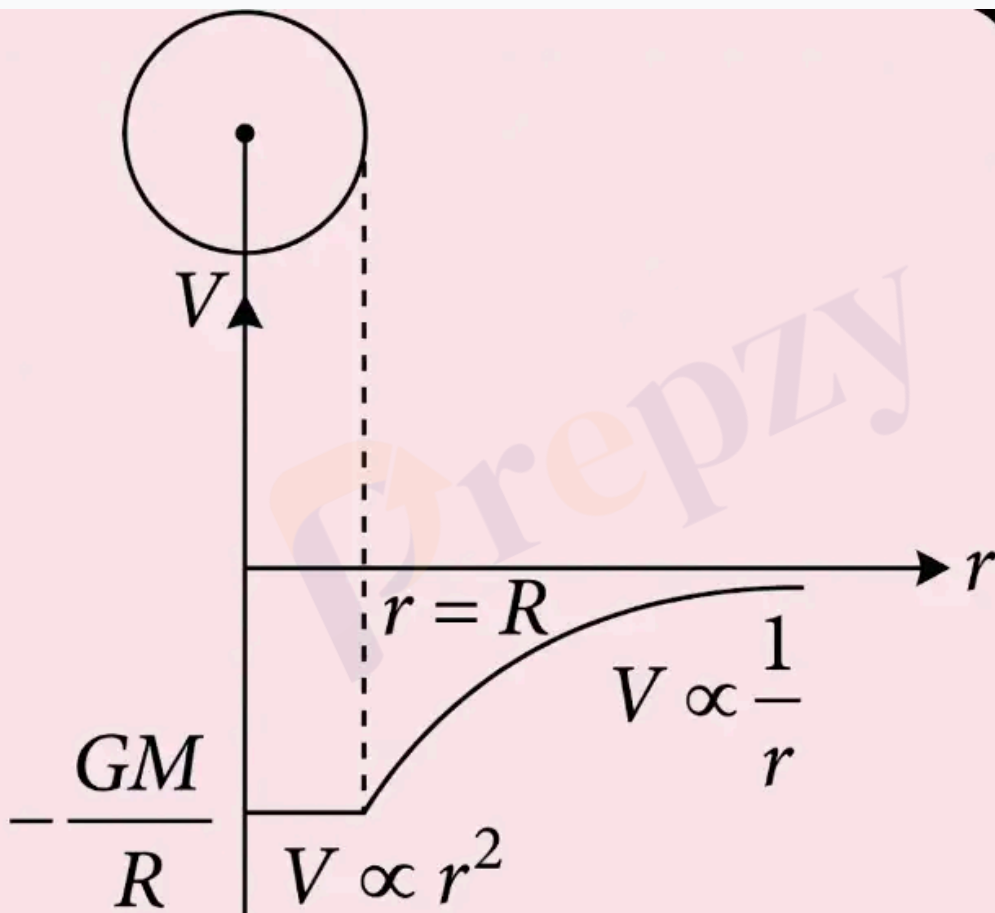
- Thin spherical shell (mass M , radius R):**

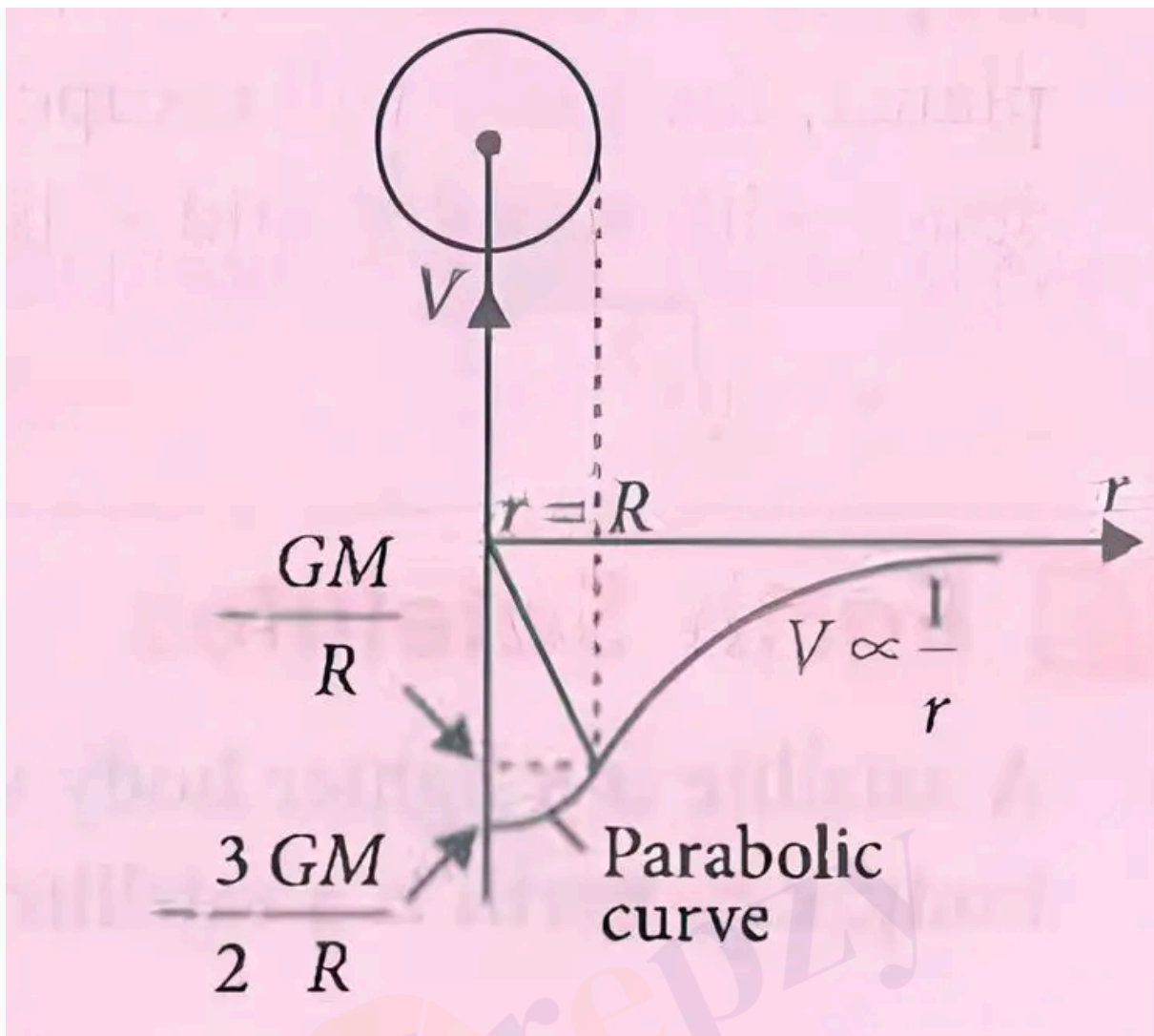
- Inside ($r < R$): $E = 0, V = -\frac{GM}{R}$
- On surface ($r = R$): $E = -\frac{GM}{R^2}, V = -\frac{GM}{R}$
- Outside ($r > R$): $E = -\frac{GM}{r^2}, V = -\frac{GM}{r}$



- Solid sphere (mass M , radius R , uniform density):**

- Inside ($r < R$): $E = -\frac{GMr}{R^3}, V = -\frac{GM}{2R^3} (3R^2 - r^2)$
- On surface ($r = R$): $E = -\frac{GM}{R^2}, V = -\frac{GM}{R}$
- Outside ($r > R$): $E = -\frac{GM}{r^2}, V = -\frac{GM}{r}$
- Center ($r = 0$): $V = -\frac{3GM}{2R}$





Solved Examples

- **Ex 1:** Find gravitational field E on axis of ring at distance $x = R$.
 - $E = -\frac{GMx}{(R^2+x^2)^{3/2}} = -\frac{GMR}{(R^2+R^2)^{3/2}} = -\frac{GMR}{(2R^2)^{3/2}} = -\frac{GM}{2\sqrt{2}R^2}$
- **Ex 2:** Calculate gravitational potential V on axis of uniform disc at $x = 0$.
 - $V = -\frac{2GM}{R^2}(0 - R) = \frac{2GM}{R}$

Practice Set

- Q1: Gravitational field on axis of ring at distance x is:
 - A) $-\frac{GMx}{(R^2+x^2)^{3/2}}$
 - B) $-\frac{GM}{(R^2+x^2)^{1/2}}$
 - C) $-\frac{2GM}{R^2}\left(1 - \frac{x}{\sqrt{R^2+x^2}}\right)$
 - D) 0
- Q2: Gravitational potential on axis of uniform disc at distance x is:
 - A) $-\frac{GM}{\sqrt{R^2+x^2}}$
 - B) $-\frac{2GM}{R^2}\left(x - \sqrt{R^2+x^2}\right)$
 - C) $-\frac{GMx}{(R^2+x^2)^{3/2}}$
 - D) 0
- Q3: Inside thin spherical shell, gravitational field $E(r < R)$ is:
 - A) $-\frac{GM}{R^2}$
 - B) 0

- C) $-\frac{GM}{r^2}$
- D) $-\frac{GM}{r}$

Answer Key

- Q1: A - Formula for gravitational field on ring axis
- Q2: B - Formula for gravitational potential on disc axis
- Q3: B - Field inside shell is zero

Memory Aids

- Field and potential formulas depend on shape and position
- Inside shell: field zero, potential constant
- Outside shell: treat mass as point at center

Gravitational Potential Energy

Definition and Formula

- Work done to bring a body from infinity to a point in a gravitational field.
- Between two masses M and m separated by distance r: $U = -\frac{GMm}{r}$
- Potential energy at infinity is zero: $U \rightarrow 0 \text{ as } r \rightarrow \infty$
- Scalar quantity, SI unit: Joule (J), Dimensional formula: $[ML^2T^{-2}]$

Potential Energy Near Earth

- At height h above Earth surface: $U_h = -\frac{GM_E m}{R_E + h}$
- On Earth surface: $U_s = -\frac{GM_E m}{R_E}$
- Change in potential energy when moved up by h:

$$\Delta U = U_h - U_s = GM_E m \left(\frac{1}{R_E} - \frac{1}{R_E + h} \right) = \frac{GM_E m h}{R_E^2 \left(1 + \frac{h}{R_E} \right)} = \frac{mgh}{1 + \frac{h}{R_E}}$$
- Since $\frac{GM_E}{R_E^2} = g$, for small heights ($h \ll R_E$), $\Delta U \approx mgh$

Solved Examples

- **Ex 1:** Calculate gravitational potential energy of 2 kg mass at 10 m above Earth surface.
 Given: $m=2 \text{ kg}$, $h=10 \text{ m}$, $g=9.8 \text{ m/s}^2$
 Use: $\Delta U = mgh$
 $\Delta U = 2 \times 9.8 \times 10 = 196 \text{ J}$

Practice Set

- Q1: Gravitational potential energy between two masses M and m separated by distance r is:
 A) $\frac{GMm}{r}$ B) $-\frac{GMm}{r}$ C) $\frac{GMm}{r^2}$ D) $-\frac{GMm}{r^2}$
- Q2: The change in gravitational potential energy when a body is moved from surface to height h ($h \ll R_E$) is:
 A) mgh B) mg C) $\frac{mgh}{2}$ D) $2mgh$
- Q3: If the radius of Earth is doubled, the gravitational potential energy on surface for mass m becomes:
 A) $-\frac{GM_E m}{2R_E}$ B) $-\frac{GM_E m}{4R_E}$ C) $-\frac{GM_E m}{R_E}$ D) $-\frac{2GM_E m}{R_E}$

Answer Key

- Q1: B - Formula for gravitational potential energy is negative: $U = -\frac{GMm}{r}$.
- Q2: A - For small heights, change in potential energy is mgh .
- Q3: B - Doubling radius increases denominator, so energy becomes $-\frac{GM_E m}{2R_E \times 2} = -\frac{GM_E m}{4R_E}$.

Memory Aids

- Potential energy is negative and zero at infinity.
- Near Earth, $\Delta U \approx mgh$ for small heights.
- Formula: $U = -\frac{GMm}{r}$ always negative.

Escape Speed

Definition and Formula

- Minimum speed to escape a planet's gravitational field without returning.
- Escape speed, $v_e = \sqrt{\frac{2GM}{R}}$
- Where, M = mass of planet, R = radius of planet, G = gravitational constant.

Alternate Formula Using Density

- Using volume and density: $v_e = \sqrt{\frac{2G \times \text{volume} \times \rho}{R}} = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8\pi G \rho R^2}{3}}$
- ρ = density of the planet.

Earth's Escape Speed

- For Earth, $v_e = 11.2 \text{ km/s}$

Speed Comparison Effects

- If projection speed $v < v_e$, body may orbit or fall back.
- If $v > v_e$, body escapes gravitational field with speed $v' = \sqrt{v^2 - v_e^2}$.

Solved Examples

- **Ex 1:** Calculate escape speed from a planet of mass $5 \times 10^{24} \text{ kg}$ and radius $6 \times 10^6 \text{ m}$.
 - Use $v_e = \sqrt{\frac{2GM}{R}}$
 - Substitute $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
 - Calculate v_e
- **Ex 2:** Find speed v' if a body is projected at 15 km/s from Earth.
 - Given $v = 15 \text{ km/s}$, $v_e = 11.2 \text{ km/s}$
 - Calculate $v' = \sqrt{v^2 - v_e^2} = \sqrt{15^2 - 11.2^2}$
 - Result v' in km/s

Practice Set

- **Q1:** Escape speed depends on
 - A) Mass and radius of planet
 - B) Only mass
 - C) Only radius
 - D) None
- **Q2:** Escape speed from Earth is approximately
 - A) 9.8 m/s
 - B) 11.2 km/s
 - C) 30 km/s
 - D) 3 km/s
- **Q3:** A body projected at 14 km/s from a planet with escape speed 12 km/s will
 - A) Fall back
 - B) Orbit planet

- C) Escape with speed $\sqrt{14^2 - 12^2}$ km/s
- D) Stop at max height

Answer Key

- **Q1:** A - Escape speed depends on both mass and radius as $v_e = \sqrt{\frac{2GM}{R}}$.
- **Q2:** B - Earth's escape speed is 11.2 km/s.
- **Q3:** C - Speed after escape is $\sqrt{v^2 - v_e^2}$.

Memory Aids

- Escape speed formula: $v_e = \sqrt{\frac{2GM}{R}}$
- For Earth, remember 11.2 km/s
- If $v < v_e$, body returns or orbits; if $v > v_e$, body escapes

Earth Satellites

Definition

- A satellite is a lighter body revolving continuously in an orbit around a much larger body.
- Example: Earth is a satellite of the Sun; Moon is a satellite of Earth.

Solved Examples

- **Ex 1:** Identify the satellite in the Earth-Sun system.
Answer: Earth is the satellite revolving around the Sun.

Practice Set

- Q1: Which of the following is a natural satellite?
 - A. Sun
 - B. Earth
 - C. Moon
 - D. Mars
- Q2: The Earth is a satellite of which body?
 - A. Moon
 - B. Sun
 - C. Mars
 - D. Jupiter
- Q3: Which of the following is NOT a satellite?
 - A. Moon
 - B. Earth
 - C. Sun
 - D. Artificial satellite

Answer Key

- Q1: C - Moon is a natural satellite of Earth.
- Q2: B - Earth revolves around the Sun, making it a satellite of the Sun.
- Q3: C - Sun is a star, not a satellite.

Memory Aids

- Satellite = smaller body orbiting a larger one.
- Earth → Sun, Moon → Earth.

Motion of a Satellite

Key Points

- Satellites move in circular orbits around Earth.
- Velocity direction is tangent to the orbit.
- Acceleration is directed towards the center of the orbit (centripetal acceleration).

Solved Examples

- **Ex 1:** Direction of velocity and acceleration of a satellite in orbit.
Velocity is tangent; acceleration points to orbit center.

Practice Set

- Q1: The acceleration of a satellite in circular orbit is directed:
A. Tangent to orbit
B. Away from Earth
C. Towards the center of orbit
D. Opposite to velocity
- Q2: The velocity of a satellite is always:
A. Radial
B. Tangential
C. Zero
D. Constant in direction
- Q3: What type of path does a satellite follow?
A. Elliptical
B. Circular
C. Parabolic
D. Straight line

Answer Key

- Q1: C - Acceleration is centripetal, towards orbit center.
- Q2: B - Velocity is tangent to the orbit path.
- Q3: B - Satellite moves in a circular path.

Memory Aids

- Velocity tangent, acceleration center-directed.
- Circular orbit = constant radius.

Orbital Speed of a Satellite

Definition & Formula

- Minimum speed to keep satellite in orbit around Earth.
- Orbital speed at height h :
$$v_0 = \sqrt{\frac{GM_E}{R_E+h}} = R_E \sqrt{\frac{g}{R_E+h}}$$
- Near Earth's surface ($h \ll R_E$):
$$v_0 = \sqrt{gR_E}$$
- Orbital speed is independent of satellite's mass.

Solved Examples

- **Ex 1:** Calculate orbital speed near Earth's surface.

Use $v_0 = \sqrt{gR_E}$.

Practice Set

- Q1: Orbital speed depends on:
 - A. Mass of satellite
 - B. Height of orbit
 - C. Both A and B
 - D. None
- Q2: Orbital speed near Earth's surface is approximately:
 - A. $\sqrt{gR_E}$
 - B. gR_E
 - C. $\frac{g}{R_E}$
 - D. $\sqrt{\frac{g}{R_E}}$
- Q3: If height h increases, orbital speed:
 - A. Increases
 - B. Decreases
 - C. Remains same
 - D. Becomes zero

Answer Key

- Q1: B - Orbital speed depends on orbit height, not satellite mass.
- Q2: A - Near surface, $v_0 = \sqrt{gR_E}$.
- Q3: B - Orbital speed decreases with increasing height.

Memory Aids

- $v_0 = \sqrt{\frac{GM_E}{R_E+h}}$
- Near surface: $v_0 = \sqrt{gR_E}$

Time Period of a Satellite

Definition & Formula

- Time for one complete revolution around Earth.
- General formula:

$$T = 2\pi\sqrt{\frac{(R_E+h)^3}{GM_E}} = \frac{2\pi}{\sqrt{g}}\sqrt{\frac{(R_E+h)^3}{R_E^2}}$$

- Near Earth's surface ($h \ll R_E$):

$$T = 2\pi\sqrt{\frac{R_E}{g}} = 84.6 \text{ min}$$

Solved Examples

- **Ex 1:** Calculate time period near Earth's surface.

Use $T = 2\pi\sqrt{\frac{R_E}{g}}$.

Practice Set

- Q1: Time period depends on:
 - A. Mass of satellite

- B. Height of orbit
- C. Both A and B
- D. None
- Q2: Time period near Earth's surface is approximately:
 - A. 60 min
 - B. 84.6 min
 - C. 120 min
 - D. 30 min
- Q3: If height h increases, time period:
 - A. Increases
 - B. Decreases
 - C. Remains same
 - D. Becomes zero

Answer Key

- Q1: B - Time period depends on orbit height, not satellite mass.
- Q2: B - Near surface, time period is about 84.6 minutes.
- Q3: A - Time period increases with height.

Memory Aids

- $T = 2\pi\sqrt{\frac{(R_E+h)^3}{GM_E}}$
- Near surface: $T = 2\pi\sqrt{\frac{R_E}{g}}$

Height of Satellite Above Earth's Surface

Formula

- Height h from Earth's surface related to time period T :

$$h = \left(\frac{T^2 R_E^2 g}{4\pi^2} \right)^{1/3} - R_E$$

Solved Examples

- **Ex 1:** Calculate height for given time period T using formula.

Practice Set

- Q1: Height of satellite depends on:
 - A. Time period
 - B. Mass of satellite
 - C. Both A and B
 - D. None
- Q2: If time period T increases, height h :
 - A. Increases
 - B. Decreases
 - C. Remains same
 - D. Becomes zero
- Q3: Formula for height involves:
 - A. T^2
 - B. R_E^2
 - C. g
 - D. All of the above

Answer Key

- Q1: A - Height depends on time period, not satellite mass.
- Q2: A - Height increases with increasing time period.
- Q3: D - Formula includes T^2 , R_E^2 , g .

Memory Aids

- $h = \left(\frac{T^2 R_E^2 g}{4\pi^2} \right)^{1/3} - R_E$
- Height linked to time period cube root.

Energy of an Orbiting Satellite

Key Concepts

- Satellite energy consists of potential energy (U), kinetic energy (K), and total mechanical energy (E).
- Potential energy: $U = -\frac{GM_E m}{R_E + h}$
- Kinetic energy: $K = \frac{1}{2} m v_0^2 = \frac{1}{2} \frac{GM_E m}{R_E + h} = \frac{|U|}{2}$
- Total energy: $E = K + U = -\frac{GM_E m}{2(R_E + h)} = \frac{U}{2}$
- For orbits close to Earth surface ($h \ll R_E$), $E = -\frac{GM_E m}{2R_E}$
- Kinetic energy equals negative total energy: $K = -E$
- Potential energy equals twice total energy: $U = 2E$
- Binding energy (energy to remove satellite to infinity): $E_B = -E = \frac{GM_E m}{2(R_E + h)}$
- In elliptical orbits, K.E. and P.E. vary, but total energy remains constant and negative.

Solved Examples

- **Ex 1:** Calculate total energy of a satellite at height h from Earth surface.
 - Use $E = -\frac{GM_E m}{2(R_E + h)}$
 - Substitute known values of G , M_E , m , R_E , h
 - Calculate E in joules (J)
- **Ex 2:** Find kinetic energy if potential energy is $U = -2 \times 10^7$ J.
 - Use $K = \frac{|U|}{2} = 1 \times 10^7$ J

Practice Set

- **Q1:** What is the formula for potential energy of a satellite at height h ?
 - A) $-\frac{GM_E m}{R_E + h}$
 - B) $\frac{GM_E m}{R_E + h}$
 - C) $-\frac{GM_E m}{2(R_E + h)}$
 - D) $\frac{GM_E m}{2(R_E + h)}$
- **Q2:** If total energy of a satellite is -5×10^6 J, what is its kinetic energy?
 - A) 5×10^6 J
 - B) -5×10^6 J
 - C) 2.5×10^6 J
 - D) -2.5×10^6 J
- **Q3:** Binding energy required to remove a satellite at height h is given by?
 - A) $\frac{GM_E m}{R_E + h}$
 - B) $-\frac{GM_E m}{2(R_E + h)}$
 - C) $\frac{GM_E m}{2(R_E + h)}$
 - D) $-\frac{GM_E m}{R_E + h}$

Answer Key

- **Q1:** A; Potential energy formula is $U = -\frac{GM_E m}{R_E + h}$.
- **Q2:** A; Kinetic energy $K = -E = 5 \times 10^6 \text{ J}$.
- **Q3:** C; Binding energy $E_B = -E = \frac{GM_E m}{2(R_E + h)}$.

Memory Aids

- Potential energy is negative and inversely proportional to orbit radius.
- Kinetic energy is half the magnitude of potential energy.
- Total energy is half of potential energy and negative.
- Binding energy equals the magnitude of total energy.

Prepzy