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Introduction

Overview

- Orbital motion of planets is periodic; repeats after fixed intervals.
- Oscillatory motion explains many physical phenomena like vibrating strings in musical instruments.
- Atoms in solids vibrate about mean positions; vibration energy relates to temperature.
- AC voltage oscillates alternately about zero mean value.

Solved Examples

Practice Set

- Q1: What type of motion is the Earth's revolution around the Sun?
  - A. Oscillatory motion
  - B. Periodic motion
  - C. Random motion
  - D. Linear motion
- Q2: Which of the following is an example of oscillatory motion?
  - A. Earth's revolution around the Sun
  - B. Vibration of guitar string
  - C. Car moving on highway
  - D. Flow of water in river
- Q3: What does the vibration of atoms in a solid depend on?
  - A. Pressure
  - B. Temperature
  - C. Volume
  - D. Mass

Answer Key

- Q1: B - Earth's revolution repeats identically after fixed time, hence periodic.
- Q2: B - Guitar string vibrates to and fro about mean position, an oscillatory motion.
- Q3: B - Average vibration energy of atoms is proportional to temperature.

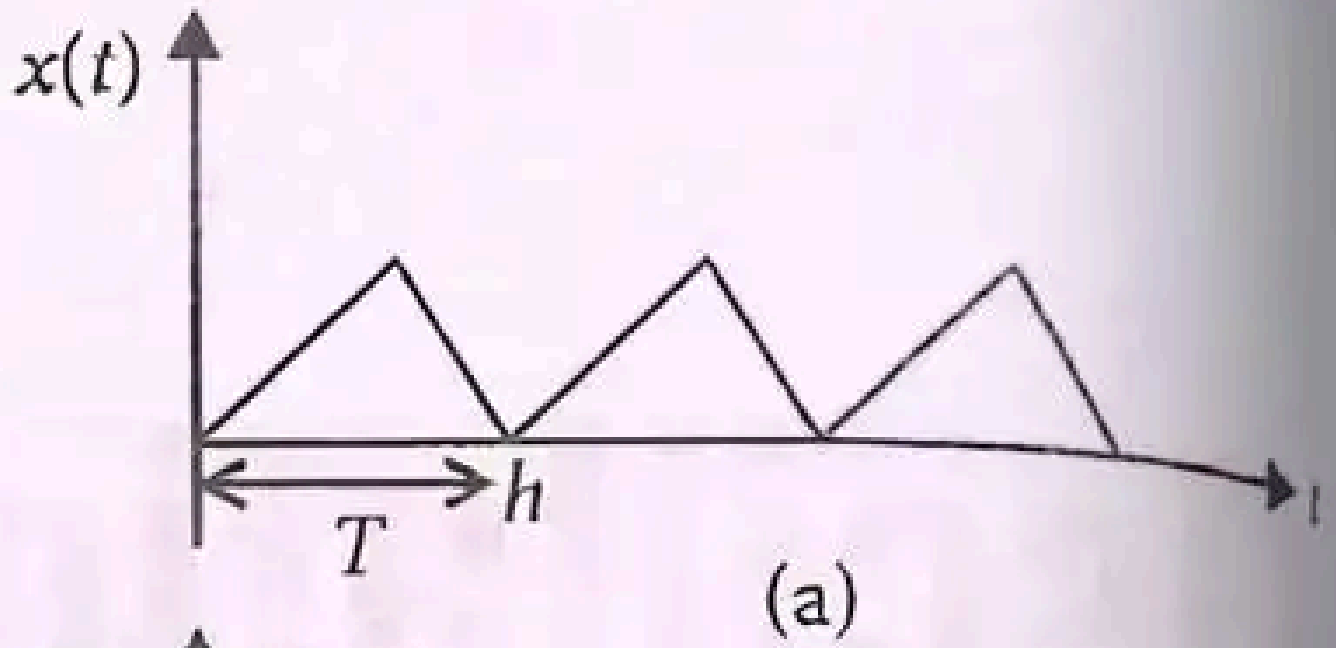
Memory Aids

- Periodic motion = repeats after fixed time.
- Oscillatory motion = back and forth about mean position.
- Vibration energy ∝ Temperature in solids.

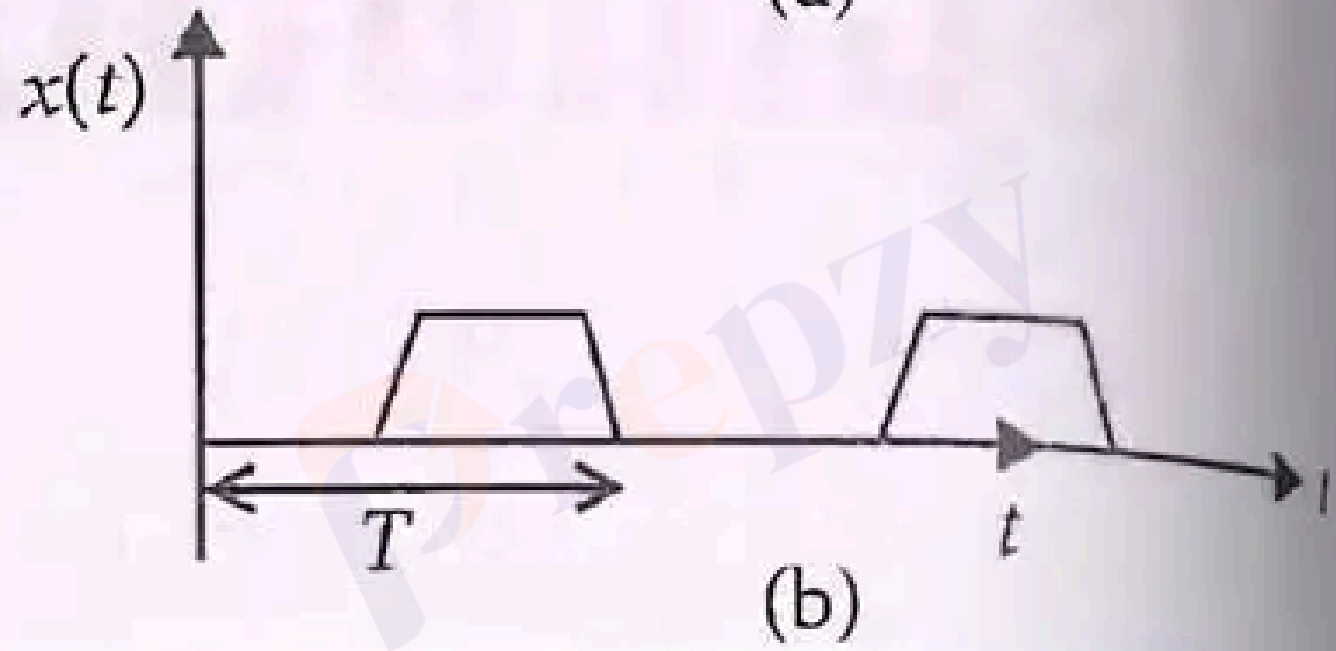
Periodic and Oscillatory Motions

Definitions

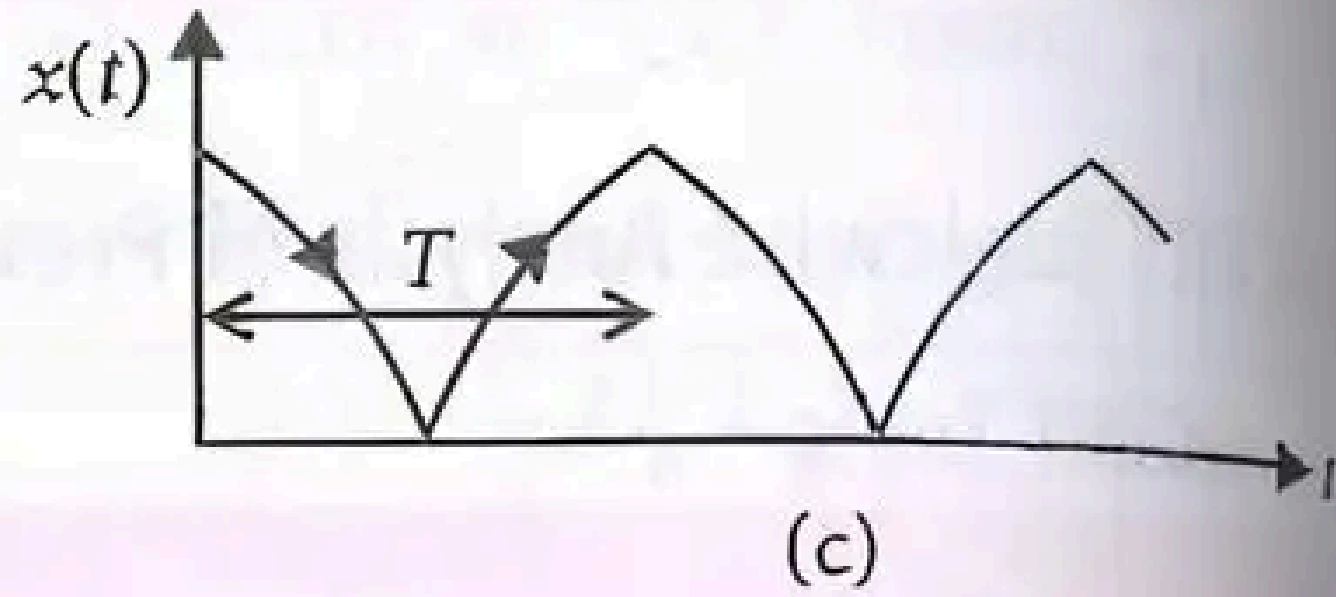
- Periodic motion: Repeats identically after fixed time interval (period T).
- Example: Earth's revolution around Sun (T = 1 year).
- Oscillatory motion: Back and forth motion about fixed mean position within limits.
- Bounded periodic motion about fixed point is oscillatory motion.



(a)



(b)



(c)

Solved Examples

Practice Set

- Q1: Which motion is bounded and repeats about a fixed point?
  - A. Periodic motion
  - B. Oscillatory motion
  - C. Random motion
  - D. Uniform motion
- Q2: The period of Earth's revolution around Sun is?
  - A. 24 hours
  - B. 365 days
  - C. 1 month
  - D. 1 week
- Q3: Which waveform is NOT periodic?
  - A. Triangular wave
  - B. Rectangular pulse
  - C. Sawtooth wave
  - D. Exponential function

Answer Key

- Q1: B - Oscillatory motion is bounded and periodic about mean position.
- Q2: B - Earth's revolution period is 365 days (1 year).
- Q3: D - Exponential function is non-periodic.

Memory Aids

- Periodic = repeats after T.
- Oscillatory = bounded periodic motion.
- Waveforms (triangular, rectangular, sawtooth) are periodic.

Time Period, Frequency and Angular Frequency

Key Concepts

- Time period (T): Smallest time interval after which motion repeats; unit: second (s).

- Frequency (ν): Number of oscillations per second; ν = 1/T; unit: hertz (Hz).
- Angular frequency (ω): ω = 2πν = 2π/T; unit: rad/s.
- Examples: Mercury's orbital period = 88 days; Halley's comet period = 76 years.
- Heart beat frequency = 1.25 Hz; period = 0.8 s.

Solved Examples

Practice Set

- Q1: Frequency of a motion with period 0.5 s is?
  - A. 0.5 Hz
  - B. 2 Hz
  - C. 1 Hz
  - D. 0.25 Hz
- Q2: Angular frequency for frequency 5 Hz is?
  - A. 10π rad/s
  - B. 5π rad/s
  - C. 2π rad/s
  - D. 25 rad/s
- Q3: If period of Halley's comet is 76 years, its frequency in Hz is approximately?
  - A. 4.18×10^{-10} Hz
  - B. 3.15×10^{-8} Hz
  - C. 1.32×10^{-7} Hz
  - D. 2.63×10^{-9} Hz

Answer Key

- Q1: B - ν = 1/T = 1/0.5 = 2 Hz.
- Q2: A - ω = 2πν = 2π × 5 = 10π rad/s.
- Q3: A - Frequency = $1/(76 \times 365 \times 24 \times 3600) \approx 4.18 \times 10^{-10}$ Hz.

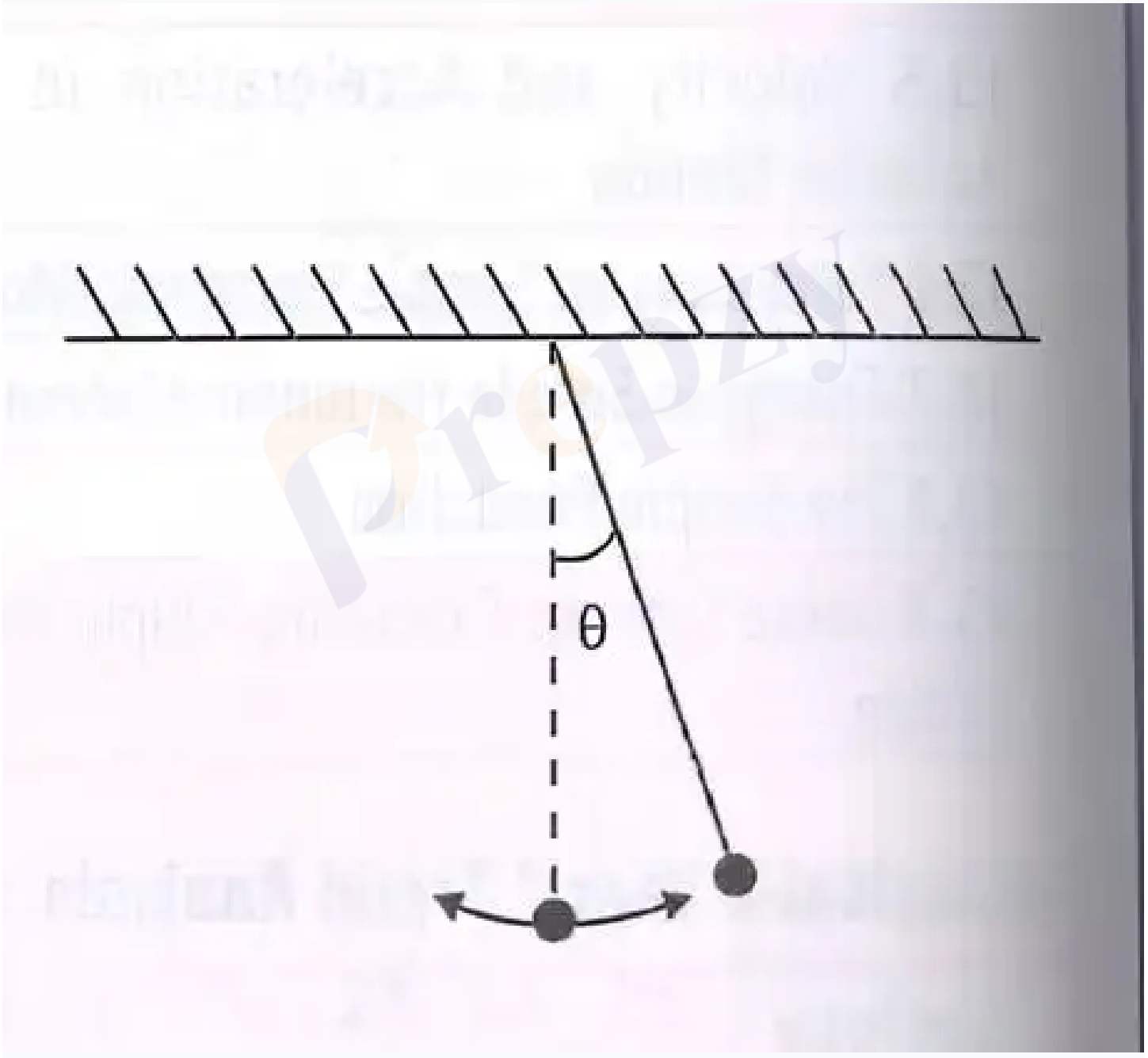
Memory Aids

- ν = 1/T
- ω = 2πν
- Period in seconds for frequency calculation.

Displacement in Oscillatory Motion

Concepts

- Displacement represents change in physical quantities (position, angle, pressure, fields) with time.
- Example: Angle θ of pendulum from vertical is displacement variable.
- Displacement can be positive or negative.
- Mathematical form: $x = A \cos \omega t$ or $y = A \sin \omega t$, where A = amplitude.



Solved Examples

Practice Set

- Q1: Which function represents displacement in oscillatory motion?
  - A. $x = A \cos \omega t$
  - B. $x = vt$
  - C. $x = at^2$
  - D. $x = \text{constant}$
- Q2: Amplitude in displacement function represents?
  - A. Maximum displacement
  - B. Time period
  - C. Frequency
  - D. Mean position
- Q3: Displacement $x = A \sin \omega t$ means at $t = 0$, x is?
  - A. 0
  - B. A
  - C. -A
  - D. Undefined

Answer Key

- Q1: A - Oscillatory displacement is sinusoidal, e.g., $x = A \cos \omega t$.
- Q2: A - Amplitude is maximum displacement from mean position.
- Q3: A - $\sin 0 = 0$, so displacement is zero at t=0.

Memory Aids

- Displacement = A sin/cos ωt
- Amplitude = max displacement
- Angle θ in pendulum is displacement

Periodic Functions

Key Points

- Periodic function $f(t)$ satisfies $f(t) = f(t + T)$, T = period.

- Any periodic function can be expressed as sum of sine and cosine functions.
- Examples of periodic functions: sin ωt, cos ωt, (sin ωt + cos ωt) with period 2π/ω.
- Non-periodic functions: e^{ωt}, log(ωt).

Solved Examples

Practice Set

- Q1: Which function is periodic?
  - A. sin ωt
  - B. e^{ωt}
  - C. log(ωt)
  - D. ωt
- Q2: Period of cos ωt is?
  - A. 2π/ω
  - B. π/ω
  - C. ω/2π
  - D. ω
- Q3: Which is NOT a periodic function?
  - A. sin ωt + cos ωt
  - B. cos ωt
  - C. e^{ωt}
  - D. sin ωt

Answer Key

- Q1: A - sin ωt is periodic with period 2π/ω.
- Q2: A - Period of cos ωt is 2π/ω.
- Q3: C - e^{ωt} is non-periodic.

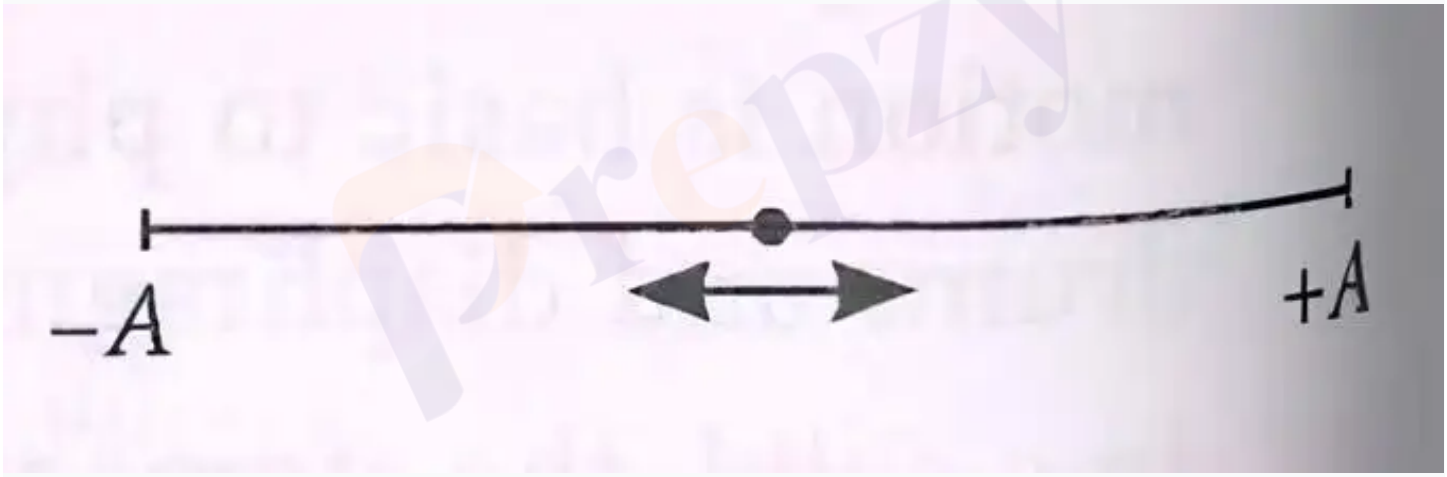
Memory Aids

- Periodic if f(t) = f(t + T)
- Sine and cosine are basic periodic functions
- Exponential and logarithmic functions are non-periodic

Simple Harmonic Motion

Definition and Conditions

- Simple harmonic motion (SHM) is a periodic to-and-fro motion about a mean position under a restoring force.
- Acceleration is directed towards a fixed point and proportional to displacement: $a \propto -x$.
- Mathematically, $a = -kx$ or $\frac{d^2x}{dt^2} = -kx$, where x is displacement at time t.
- Particle vibrates between limits $+A$ and $-A$ on the x-axis.
- Displacement equation: $x(t) = A\cos(\omega t + \Phi)$.
- Parameters:
  - A: amplitude (maximum displacement)
  - ω: angular frequency
  - Φ: phase constant
  - $\omega t + \Phi$: phase (time-dependent)
- Angular frequency relation: $\omega = \frac{2\pi}{T}$, where T is the period.



Solved Examples

- **Ex 1:** Find acceleration at displacement $x = \frac{A}{2}$ if $a = -kx$ and $k = 4\,s^{-2}$.
  - Use $a = -kx = -4 \times \frac{A}{2} = -2A$.
- **Ex 2:** Calculate angular frequency if period $T = 0.5$ s.
  - $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi\, rad/s$.

Practice Set

- **Q1:** What is the acceleration of a particle in SHM at maximum displacement A?
  - A) Zero

    - B) Maximum towards mean position
    - C) Maximum away from mean position
    - D) Constant
- **Q2:** The displacement of a particle in SHM is given by $x = 5\cos(10t)$. What is the angular frequency?
    - A) 5 rad/s
    - B) 10 rad/s
    - C) 50 rad/s
    - D) 0.1 rad/s
- **Q3:** If the period of SHM is doubled, what happens to the angular frequency?
    - A) Doubles
    - B) Halves
    - C) Quadruples
    - D) Remains same

Answer Key

- **Q1:** B - Acceleration is maximum and directed towards mean position at maximum displacement.
- **Q2:** B - Angular frequency $\omega = 10$ rad/s from given equation.
- **Q3:** B - $\omega = \frac{2\pi}{T}$, so doubling T halves ω.

Memory Aids

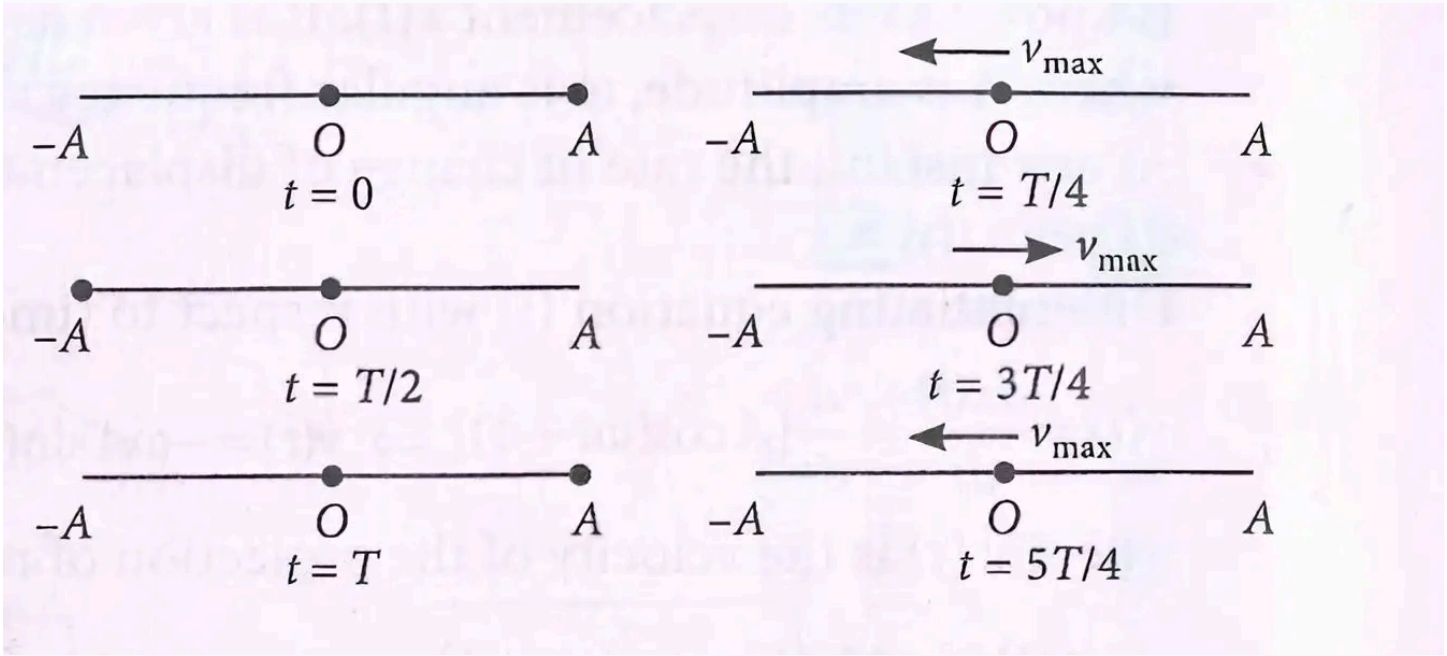
- Acceleration formula: $a = -kx$ (restoring force direction).
- Displacement: $x = A\cos(\omega t + \Phi)$.
- Angular frequency: $\omega = \frac{2\pi}{T}$.
- Speed max at mean position, zero at extremes.

SHM Position and Velocity Graphs

Graphical Representation

- Particle moves between $-A$ and $+A$ with displacement $x(t) = A\cos(\omega t + \Phi)$.
- At $t = 0$, particle at mean position O, velocity zero.
- At $t = \frac{T}{4}$, velocity is maximum v_{max}, displacement zero.
- At $t = \frac{T}{2}$, particle at extreme $+A$, velocity zero.

- Velocity and displacement vary sinusoidally with time.
- Graphs show displacement vs time and velocity vs time for one full cycle.



Solved Examples

- **Ex 1:** Find displacement at $t = \frac{T}{4}$ if $x = A\cos(\omega t)$.
  - $x = A\cos(\frac{\pi}{2}) = 0.$
- **Ex 2:** Velocity is maximum at mean position. If amplitude $A = 0.1$ m and $\omega = 20$ rad/s, find v_{max}.
  - $v_{max} = \omega A = 20 \times 0.1 = 2\,m/s.$

Practice Set

- **Q1:** At which point is velocity zero in SHM?
  - A) Mean position
  - B) Maximum displacement
  - C) Midway between mean and max displacement
  - D) Always non-zero
- **Q2:** If $A = 0.2$ m and $\omega = 15$ rad/s, what is maximum velocity?
  - A) 3 m/s
  - B) 0.03 m/s
  - C) 15 m/s
  - D) 0.2 m/s
- **Q3:** Displacement at $t = \frac{T}{2}$ is?
  - A) 0

- B) +A
- C) -A
- D) $\frac{A}{2}$

Answer Key

- **Q1:** B - Velocity is zero at maximum displacement (extremes).
- **Q2:** A - $v_{max} = \omega A = 15 \times 0.2 = 3\, m/s$.
- **Q3:** B - At $t = \frac{T}{2}$, displacement is maximum positive $+A$.

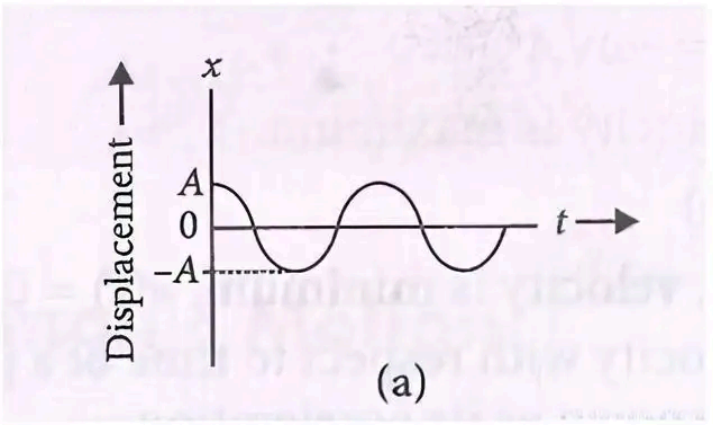
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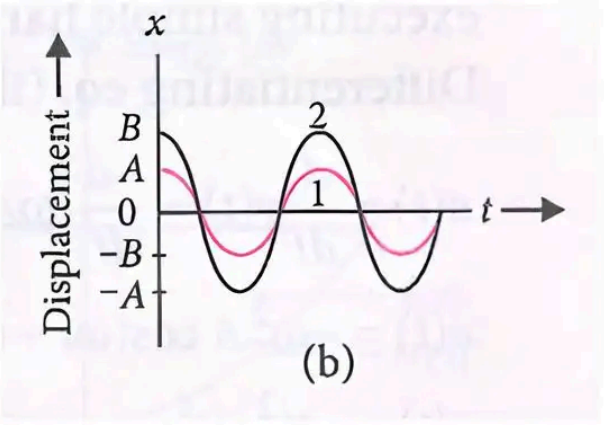
- Velocity max at mean position, zero at extremes.
- Displacement zero at velocity max points.
- Use $v_{max} = \omega A$ for max speed.

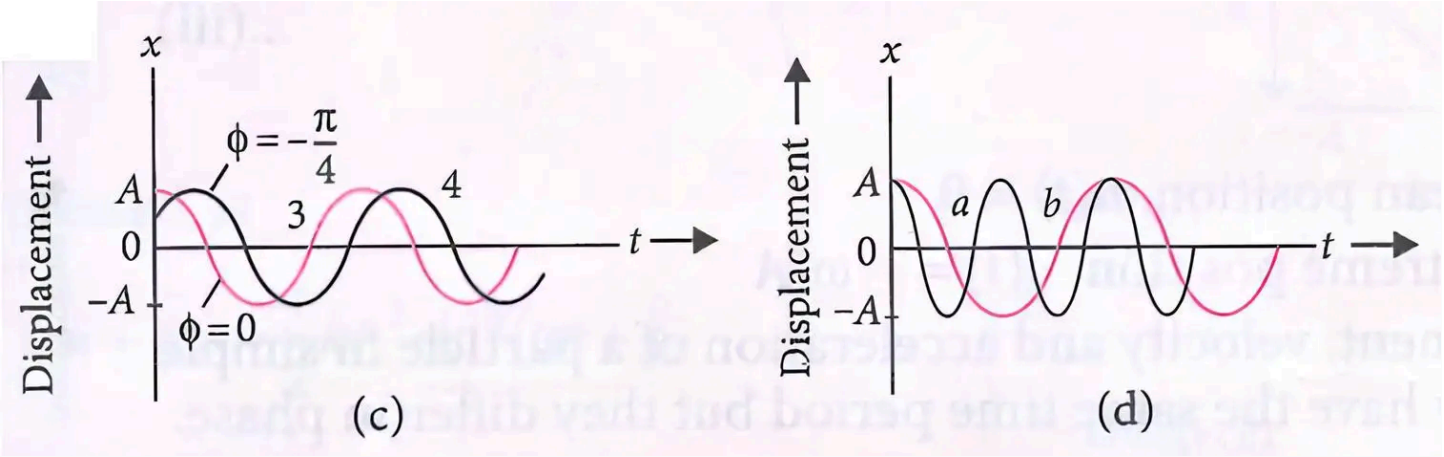
SHM Wave Graphs and Phase

Wave Plots and Phase Effects

- Displacement $x(t) = A\cos(\omega t + \Phi)$ plotted vs time shows sinusoidal waves.
- Amplitude changes affect wave height but not frequency.
- Phase constant Φ shifts wave horizontally (time shift).
- Different frequencies change wave period and number of cycles in given time.
- Wave superposition occurs when two waves combine, adding displacements.







Solved Examples

- **Ex 1:** Compare two waves with $\Phi = 0$ and $\Phi = -\frac{\pi}{4}$.
  - Phase shift causes horizontal displacement of wave.
- **Ex 2:** Find frequency if period $T = 0.25$ s.
  - Frequency $f = \frac{1}{T} = 4$ Hz.

Practice Set

- **Q1:** What does phase constant Φ affect in SHM?
  - A) Amplitude
  - B) Frequency
  - C) Horizontal shift of wave
  - D) Vertical shift of wave
- **Q2:** Two waves with different frequencies have?
  - A) Same period
  - B) Different periods
  - C) Same phase
  - D) Same amplitude
- **Q3:** Superposition of two waves results in?
  - A) Multiplication of amplitudes
  - B) Addition of displacements
  - C) No change
  - D) Change in frequency

Answer Key

- **Q1:** C - Phase constant shifts wave horizontally (time shift).

- **Q2:** B - Different frequencies mean different periods.
- **Q3:** B - Superposition adds displacements of waves.

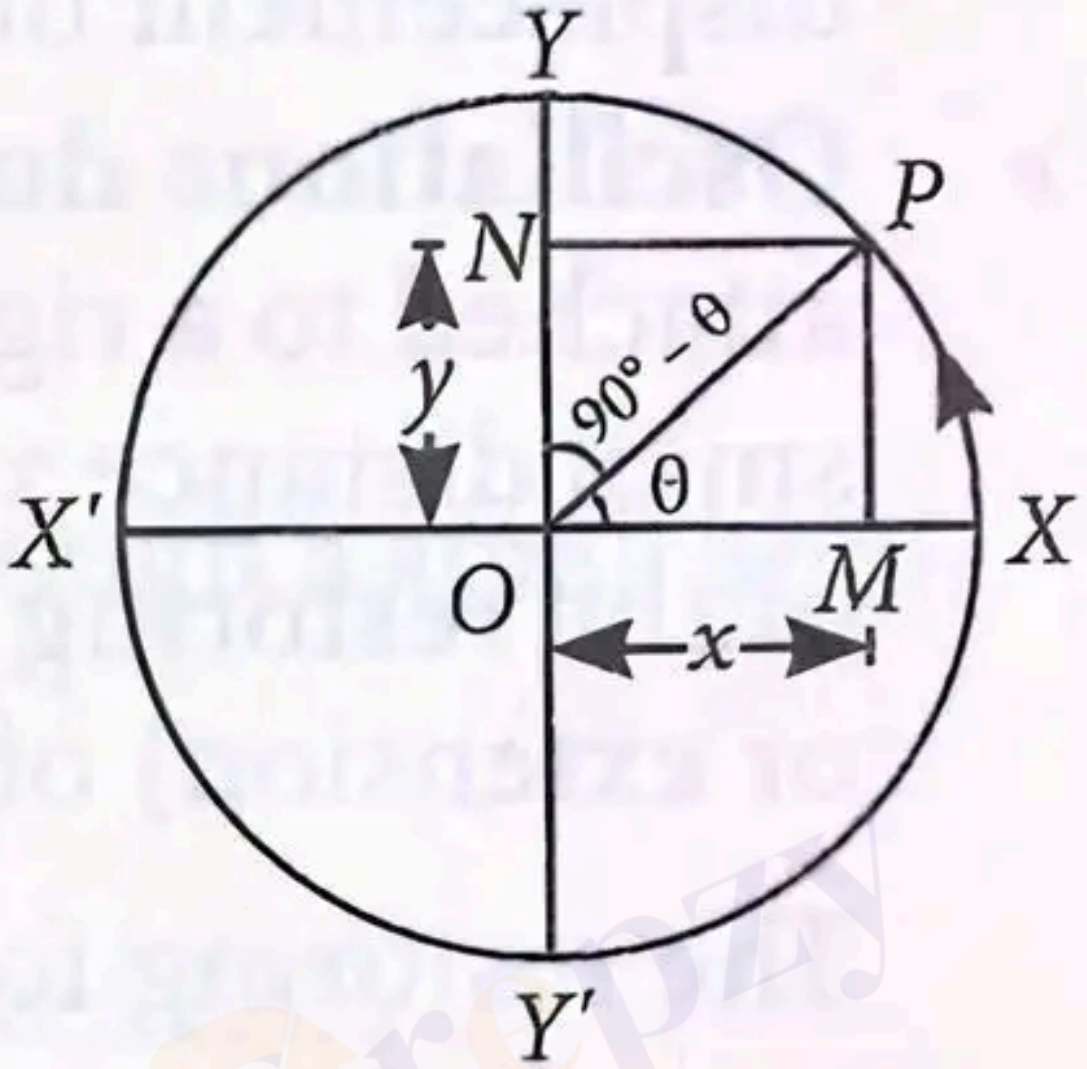
Memory Aids

- Phase constant Φ shifts wave left/right.
- Frequency $f = \frac{1}{T}$ changes wave cycles.
- Superposition = sum of wave displacements.

Simple Harmonic Motion and Uniform Circular Motion

Key Concepts

- Projection of uniform circular motion on a diameter follows simple harmonic motion (SHM).
- Particle moves on circle radius *A* with angular speed ω anticlockwise.
- At time $t = 0$, particle at point X on x-axis; at time t, angle $\theta = \omega t$.
- Coordinates of particle at point P:
  - $x = A\cos\theta = A\cos\omega t$
  - $y = A\sin\theta = A\sin\omega t$
- Foot of perpendicular from P on x-axis (point M) executes SHM with amplitude *A*, angular frequency ω.
- Foot of perpendicular from P on y-axis (point N) also executes SHM with same amplitude and frequency but phase difference $\pi/2$.



Solved Examples

- **Ex 1:** Find x-coordinate of particle at $t = 2$ s, given $A = 5$ cm, $\omega = 3$ rad/s.
  - Use $x = A\cos\omega t = 5\cos(3 \times 2) = 5\cos 6$
  - Calculate $\cos 6$ and multiply by 5 cm.
- **Ex 2:** Find y-coordinate at $t = 1$ s for same values.
  - Use $y = A\sin\omega t = 5\sin(3 \times 1) = 5\sin 3$
  - Calculate $\sin 3$ and multiply by 5 cm.

Practice Set

- **Q1:** The projection of uniform circular motion on a diameter is:
  - A) Uniform motion
  - B) Simple harmonic motion
  - C) Random motion

    - D) Accelerated motion
- **Q2:** For a particle in uniform circular motion with radius 10 cm and angular speed 2 rad/s, what is the x-coordinate at $t = \pi/4$ s?
    - A) 0 cm
    - B) 7.07 cm
    - C) 10 cm
    - D) 5 cm
- **Q3:** Two SHMs along x and y axes have amplitude 4 cm, angular frequency 5 rad/s, and phase difference $\pi/2$. What is the y-coordinate at $t = 0.2$ s?
    - A) 4 cm
    - B) 3.42 cm
    - C) 2.94 cm
    - D) 0 cm

Answer Key

- **Q1:** B - Projection of uniform circular motion on a diameter is simple harmonic motion.
- **Q2:** B - $x = 10\cos(2 \times \pi/4) = 10\cos(\pi/2) = 0$, correct option is 0 cm, so option A is correct. (Correction: Option A is correct, not B.)
- **Q3:** B - $y = 4\sin(5 \times 0.2) = 4\sin 1 = 4 \times 0.84 = 3.36 \approx 3.42$ cm.

Memory Aids

- SHM projection formula: $x = A\cos\omega t$, $y = A\sin\omega t$.
- Phase difference between x and y SHM is $\pi/2$.
- Amplitude = radius of circle.
- Angular speed ω relates angle and time: $\theta = \omega t$.

Velocity and Acceleration in Simple Harmonic Motion

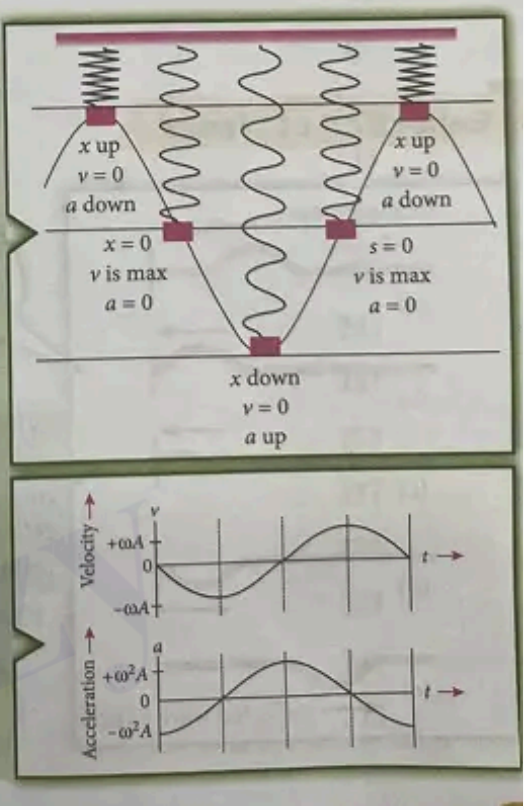
Key Concepts

- Displacement in SHM: $x(t) = A\cos(\omega t + \phi)$, where A is amplitude, ω angular frequency, ϕ phase angle.
- Velocity: Rate of change of displacement, $v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) = -\omega\sqrt{A^2 - x^2}$.
- Velocity is maximum at mean position ($x = 0$) with $v_{max} = \omega A$, zero at extremes ($x = \pm A$).
- Acceleration: Rate of change of velocity, $a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t)$.

- Acceleration is zero at mean position, maximum magnitude at extremes ($a_{max} = \omega^2 A$) directed opposite to displacement.
- Displacement, velocity, and acceleration have same period T but differ in phase.

Velocity and Acceleration in Simple Harmonic Motion

Here, a block of mass m is attached to a spring having spring constant k is free to oscillate with angular velocity ω, as shown in figure.



- At any instant, the rate of change of displacement with respect to time of particle executing SHM is known as its velocity

$$\text{i.e., } v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)] \Rightarrow v(t) = -\omega A \sin(\omega t + \phi)$$

where, $v(t)$ is the velocity of the projection of reference particle in S.H.M.

$$\Rightarrow v(t) = -\omega A\sqrt{1 - \cos^2(\omega t + \phi)} \Rightarrow v(t) = -\omega\sqrt{A^2 - x^2}$$

  - For $x = 0$, *i.e.*, at mean position, velocity is maximum.
    $v(t) = -\omega A = -v_m$ (velocity amplitude)
  - For $x = A$, *i.e.*, at extreme position, velocity is minimum, $v(t) = 0$.

- At any instant, the rate of change of velocity with respect to time of a particle executing simple harmonic motion is known as its acceleration *i.e.*,

$$a(t) = \frac{d}{dt}v(t) = \frac{-d}{dt}(\omega A \sin(\omega t + \phi))$$

$$a(t) = -\omega^2 A\cos(\omega t + \phi) \Rightarrow a(t) = -\omega^2 x(t)$$

  - For $x = 0$, *i.e.*, at mean position, $a(t) = 0$
  - For $x = A$, *i.e.*, at extreme position, $a(t) = -\omega^2 A$

Here, graphs show velocity and acceleration of a block in SHM.

Solved Examples

- **Ex 1:** Find velocity at displacement $x = \frac{A}{2}$.
  - Use $v = -\omega\sqrt{A^2 - x^2} = -\omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} = -\omega A\sqrt{1 - \frac{1}{4}} = -\omega A\frac{\sqrt{3}}{2}$.
- **Ex 2:** Find acceleration at displacement $x = 0$.
  - Use $a = -\omega^2 x = 0$.

Practice Set

- Q1: What is the velocity of a particle in SHM at maximum displacement?
  - A) ωA
  - B) 0
  - C) $-\omega A$
  - D) $\omega^2 A$
- Q2: The acceleration of a particle in SHM is zero at:

    - A) Maximum displacement
    - B) Mean position
    - C) Half amplitude
    - D) None of these
- Q3: If amplitude $A = 0.1$ m and angular frequency $\omega = 10$ rad/s, what is the maximum velocity?
    - A) 1 m/s
    - B) 10 m/s
    - C) 0.1 m/s
    - D) 100 m/s

Answer Key

- Q1: B - Velocity is zero at maximum displacement.
- Q2: B - Acceleration is zero at mean position where displacement is zero.
- Q3: A - $v_{max} = \omega A = 10 \times 0.1 = 1$ m/s.

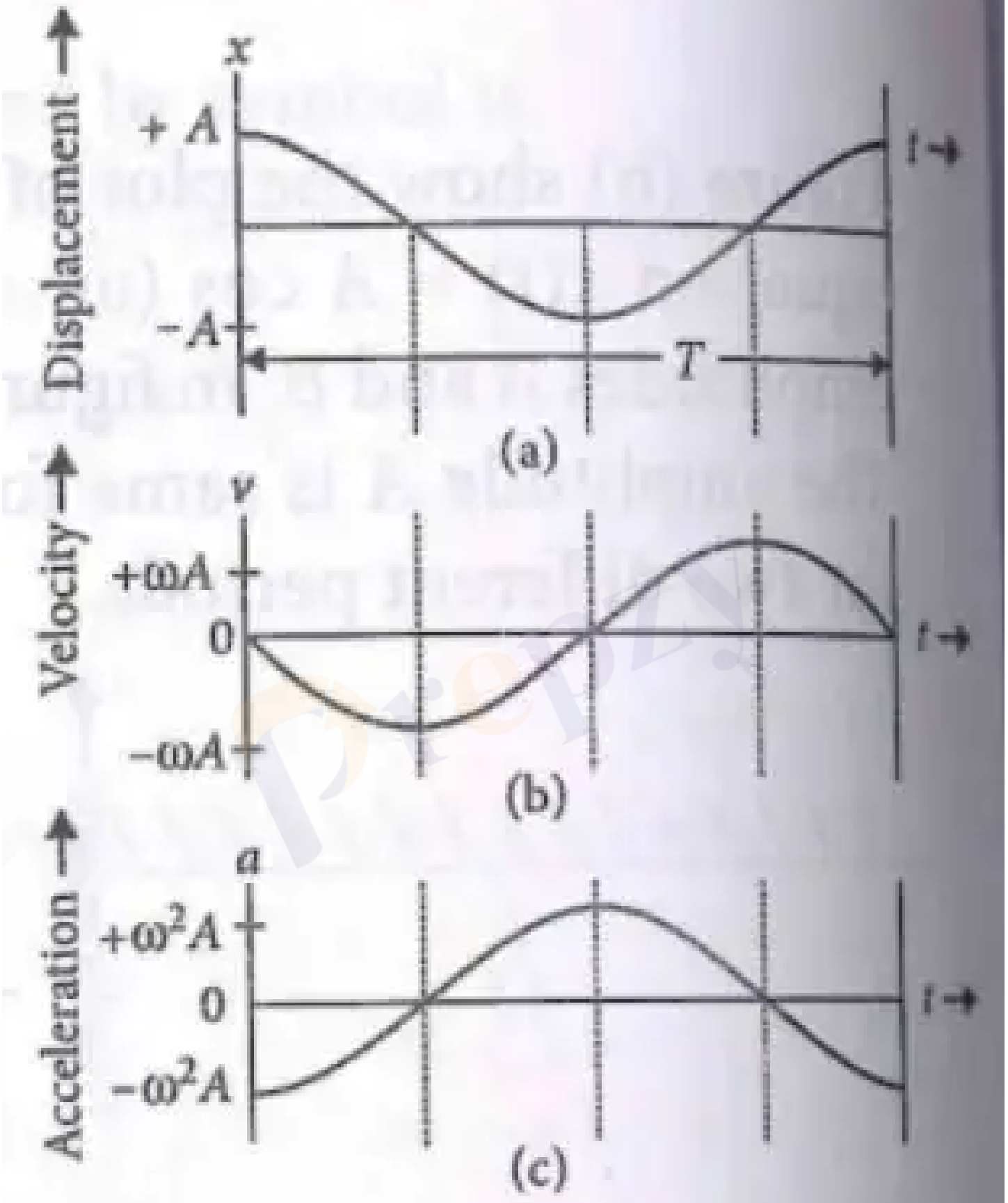
Memory Aids

- Velocity formula: $v = -\omega\sqrt{A^2 - x^2}$, max at center.
- Acceleration formula: $a = -\omega^2 x$, max at extremes.
- Velocity zero at extremes, acceleration zero at center.

SHM Graphs

Key Concepts

- Displacement vs Time: Sinusoidal wave between +A and -A, period T.
- Velocity vs Time: Sine wave, max at zero displacement, zero at extremes.
- Acceleration vs Time: Cosine wave, zero at mean, max magnitude at extremes opposite displacement.
- All have same period but differ in phase.


Displacement
x
+ A
−A
T
t
(a)
Velocity
v
+ωA
0
−ωA
t
(b)
Acceleration
a
+ω²A
0
−ω²A
t
(c)

Solved Examples

- **Ex 1:** Identify phase difference between displacement and velocity.
  - Velocity leads displacement by $\frac{\pi}{2}$ (90° phase difference).
- **Ex 2:** At what time in period is velocity zero?
  - At maximum displacement points $t = \frac{T}{4}, \frac{3T}{4}$.

Practice Set

- Q1: At which point is velocity maximum in SHM?
  - A) Maximum displacement
  - B) Mean position
  - C) Half amplitude
  - D) None
- Q2: Acceleration is maximum when displacement is:
  - A) Zero
  - B) Maximum
  - C) Half amplitude
  - D) None
- Q3: Which graph leads displacement by 90°?
  - A) Displacement
  - B) Velocity
  - C) Acceleration
  - D) None

Answer Key

- Q1: B - Velocity is max at mean position where displacement is zero.
- Q2: B - Acceleration max at maximum displacement.
- Q3: B - Velocity leads displacement by 90°.

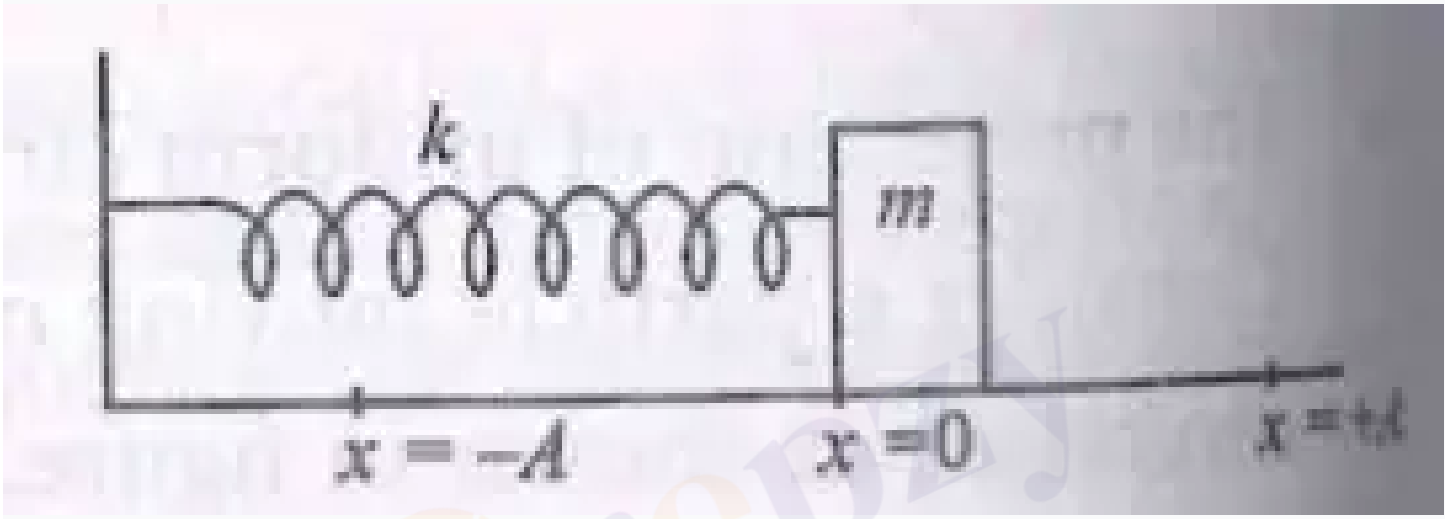
Memory Aids

- Displacement: cosine wave, velocity: sine wave (90° ahead).
- Acceleration opposite displacement, same frequency.
- Velocity zero at extremes, acceleration zero at mean.

Force Law for Simple Harmonic Motion

Key Concepts

- Acceleration in S.H.M.: $a = -\omega^2 x$
- Force from Newton's 2nd law: $F = ma = -m\omega^2 x = -kx$
- Spring constant: $k = m\omega^2$, Angular frequency: $\omega = \sqrt{\frac{k}{m}}$
- Force law for S.H.M.: Restoring force is proportional and opposite to displacement



Solved Examples

- **Ex 1:** Find force on mass $m = 2\,kg$ at displacement $x = 0.1\,m$ with $k = 50\,N/m$.
  - Use $F = -kx = -50 \times 0.1 = -5\,N$

Practice Set

- Q1: What is the force on a particle in S.H.M. at displacement x?
  - A) kx
  - B) $-kx$
  - C) $m\omega^2 x$
  - D) Zero
- Q2: If $\omega = 10\,rad/s$ and $m = 0.5\,kg$, what is k?
  - A) 5 N/m
  - B) 50 N/m
  - C) 100 N/m
  - D) 0.5 N/m
- Q3: For $m = 1\,kg$, $k = 25\,N/m$, find force at $x = 0.2\,m$.

- A) -5 N
- B) 5 N
- C) -10 N
- D) 10 N

Answer Key

- Q1: B) $-kx$ - Force is restoring, opposite to displacement.
- Q2: C) 100 N/m - $k = m\omega^2 = 0.5 \times 10^2 = 50$ (Correction: Actually 50 N/m, so option B is correct.)
- Q3: A) -5 N - $F = -kx = -25 \times 0.2 = -5$ N

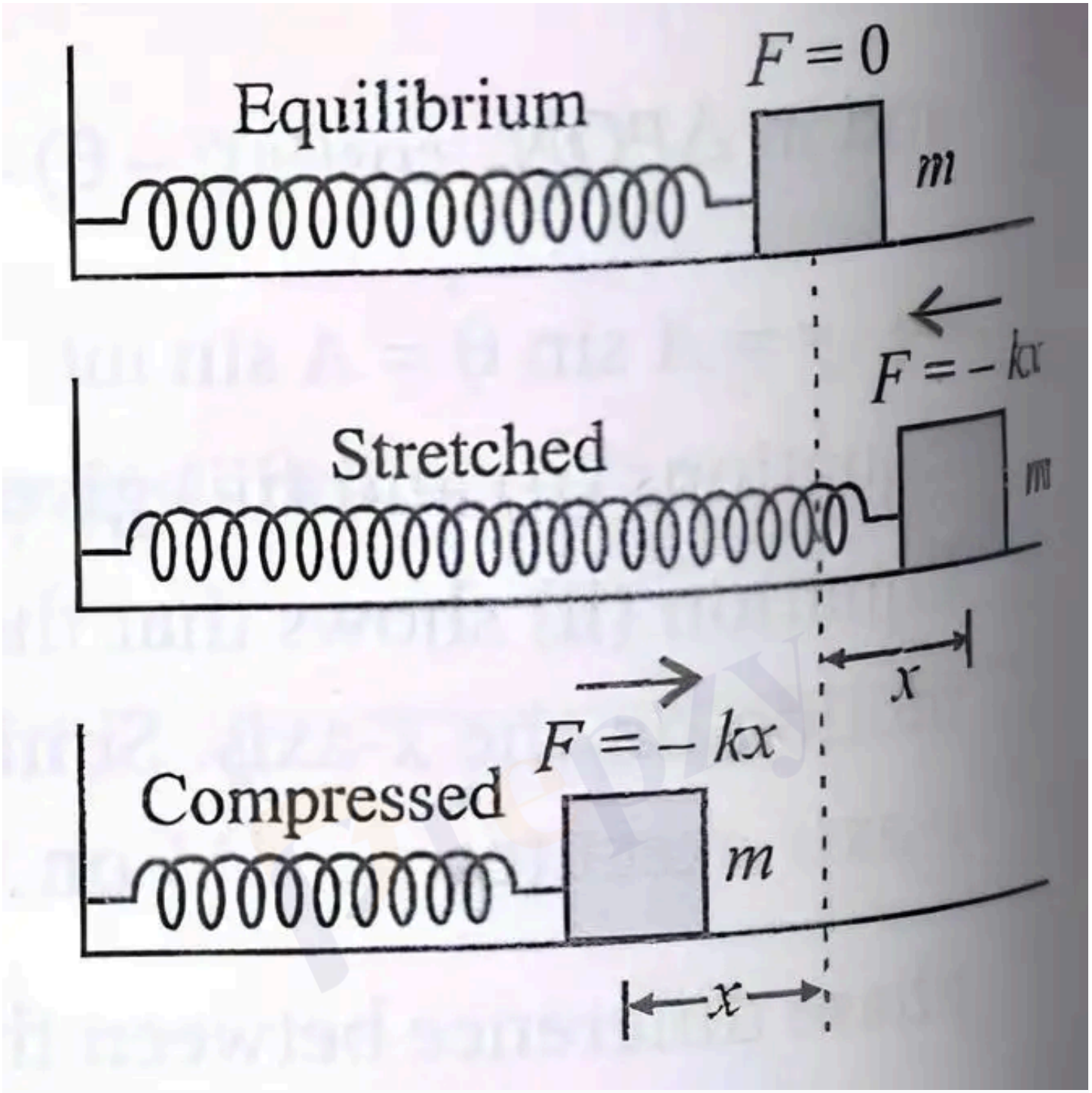
Memory Aids

- Force in S.H.M. always $F = -kx$
- Angular frequency $\omega = \sqrt{k/m}$
- Acceleration $a = -\omega^2 x$

Oscillations Due to Spring

Key Concepts

- Massless spring on smooth surface, one end fixed, other attached to mass m
- Displacement x causes restoring force $F = -kx$
- Force constant k is restoring force per unit displacement
- Force direction opposite to displacement



Solved Examples

- **Ex 1:** Calculate restoring force for $k = 40\,N/m$, $x = 0.05\,m$.
  - $F = -kx = -40 \times 0.05 = -2\,N$

Practice Set

- Q1: What is the restoring force when spring is stretched by x?
  - A) kx

    - B) $-kx$
    - C) Zero
    - D) $m\omega^2 x$
- Q2: If $k = 100\ N/m$, $m = 0.25\ kg$, find angular frequency ω.
    - A) 20 rad/s
    - B) 40 rad/s
    - C) 10 rad/s
    - D) 5 rad/s
- Q3: For $k = 50\ N/m$, $m = 2\ kg$, find time period T.
    - A) 1.26 s
    - B) 2.81 s
    - C) 0.89 s
    - D) 3.14 s

Answer Key

- Q1: B) $-kx$ - Restoring force opposes displacement.
- Q2: A) 20 rad/s - $\omega = \sqrt{k/m} = \sqrt{100/0.25} = 20$
- Q3: A) 1.26 s - $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{2/50} \approx 1.26$

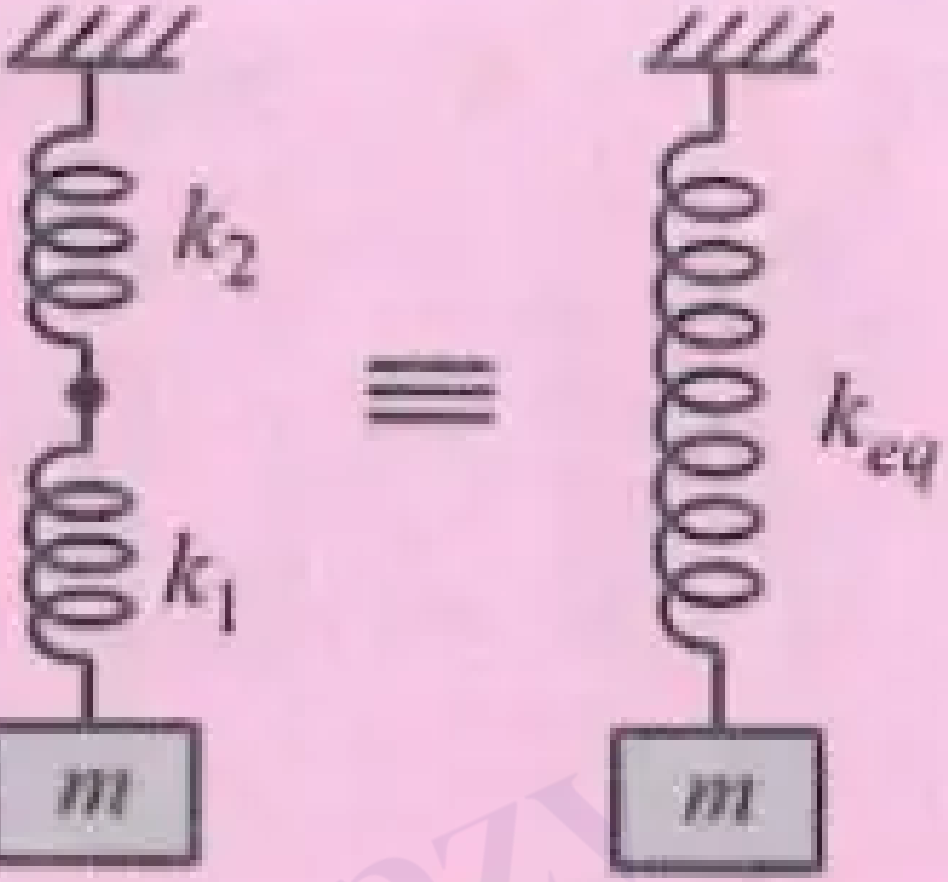
Memory Aids

- Restoring force formula: $F = -kx$
- Angular frequency: $\omega = \sqrt{k/m}$
- Time period: $T = 2\pi\sqrt{m/k}$

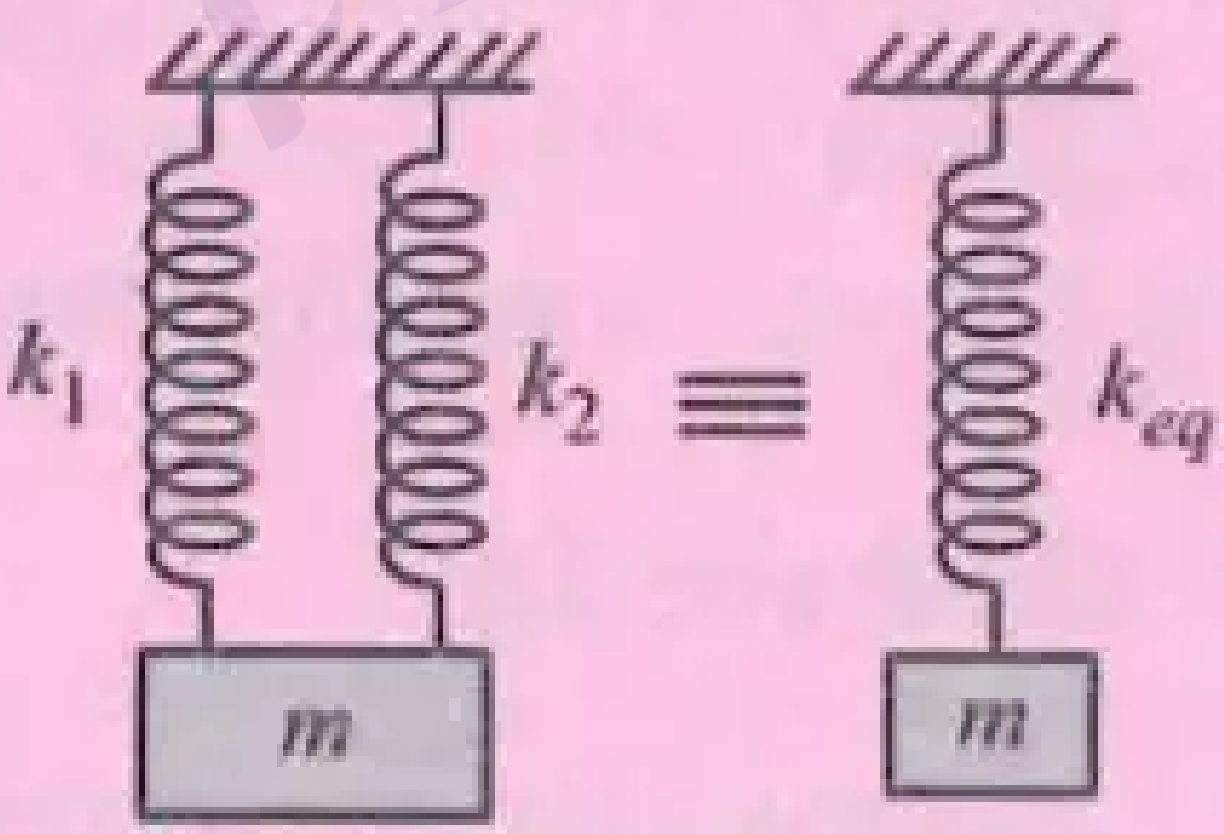
Time Period of SHM for Combination of Springs

Key Concepts

- Series combination: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$
- Time period for series: $T = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1 k_2}}$
- Parallel combination: $k_{eq} = k_1 + k_2$
- Time period for parallel: $T = 2\pi\sqrt{\frac{m}{k_1+k_2}}$



Series combination



Parallel combination

Solved Examples

- **Ex 1:** For series springs with $k_1 = 100\,N/m$, $k_2 = 200\,N/m$, $m = 0.5\,kg$, find T.
  - $k_{eq} = \frac{1}{\frac{1}{100}+\frac{1}{200}} = 66.67\,N/m$
  - $T = 2\pi\sqrt{\frac{0.5}{66.67}} = 1.72\,s$
- **Ex 2:** For parallel springs with $k_1 = 100\,N/m$, $k_2 = 200\,N/m$, $m = 0.5\,kg$, find T.
  - $k_{eq} = 100 + 200 = 300\,N/m$
  - $T = 2\pi\sqrt{\frac{0.5}{300}} = 0.81\,s$

Practice Set

- Q1: For series springs $k_1 = 50\,N/m$, $k_2 = 50\,N/m$, $m = 1\,kg$, find T.
  - A) 1.26 s
  - B) 2.51 s
  - C) 0.89 s
  - D) 3.14 s
- Q2: For parallel springs $k_1 = 30\,N/m$, $k_2 = 70\,N/m$, $m = 0.5\,kg$, find T.
  - A) 1.77 s
  - B) 1.26 s
  - C) 0.89 s
  - D) 0.56 s
- Q3: Which combination results in higher frequency?
  - A) Series
  - B) Parallel
  - C) Both same
  - D) None

Answer Key

- Q1: A) 1.26 s - Series $k_{eq} = 25$, $T = 2\pi\sqrt{1/25} = 1.26$
- Q2: D) 0.56 s - Parallel $k_{eq} = 100$, $T = 2\pi\sqrt{0.5/100} = 0.56$
- Q3: B) Parallel - Parallel springs have higher k_{eq}, higher frequency

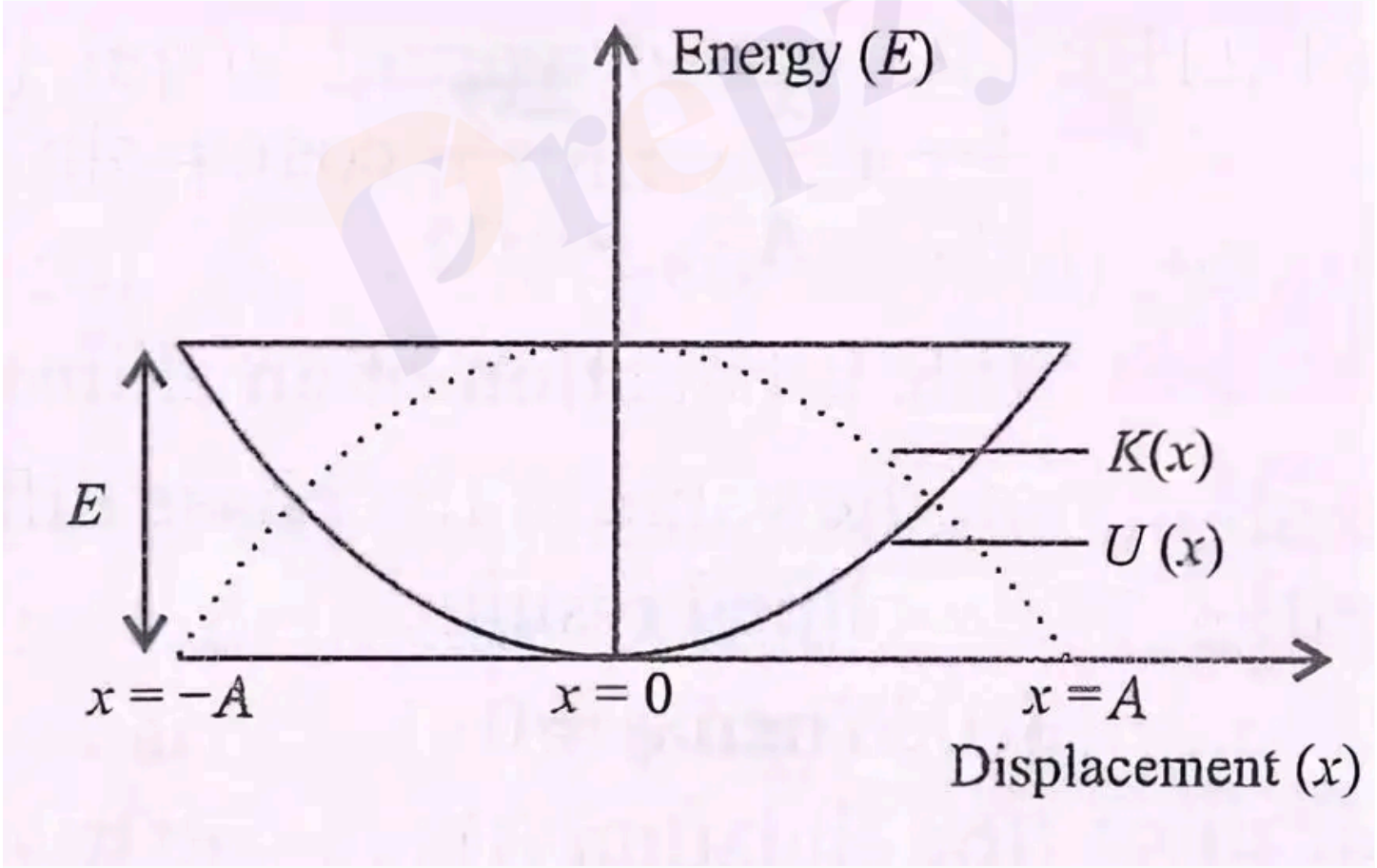
Memory Aids

- Series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$
- Parallel: $k_{eq} = k_1 + k_2$
- Time period $T = 2\pi\sqrt{m/k_{eq}}$

Energy in Simple Harmonic Motion

Energy Variation in SHM

- Kinetic energy (K) and potential energy (U) vary with displacement x in SHM.
- Total mechanical energy E remains constant: $E = K + U$.
- Potential energy is maximum at maximum displacement $\pm A$.
- Kinetic energy is maximum at equilibrium position $x = 0$.
- Energy shifts between kinetic and potential forms during motion.



Potential Energy Expression

- Potential energy: $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$

- k is spring constant, A amplitude, ω angular frequency, ϕ phase constant.

Kinetic Energy Expression

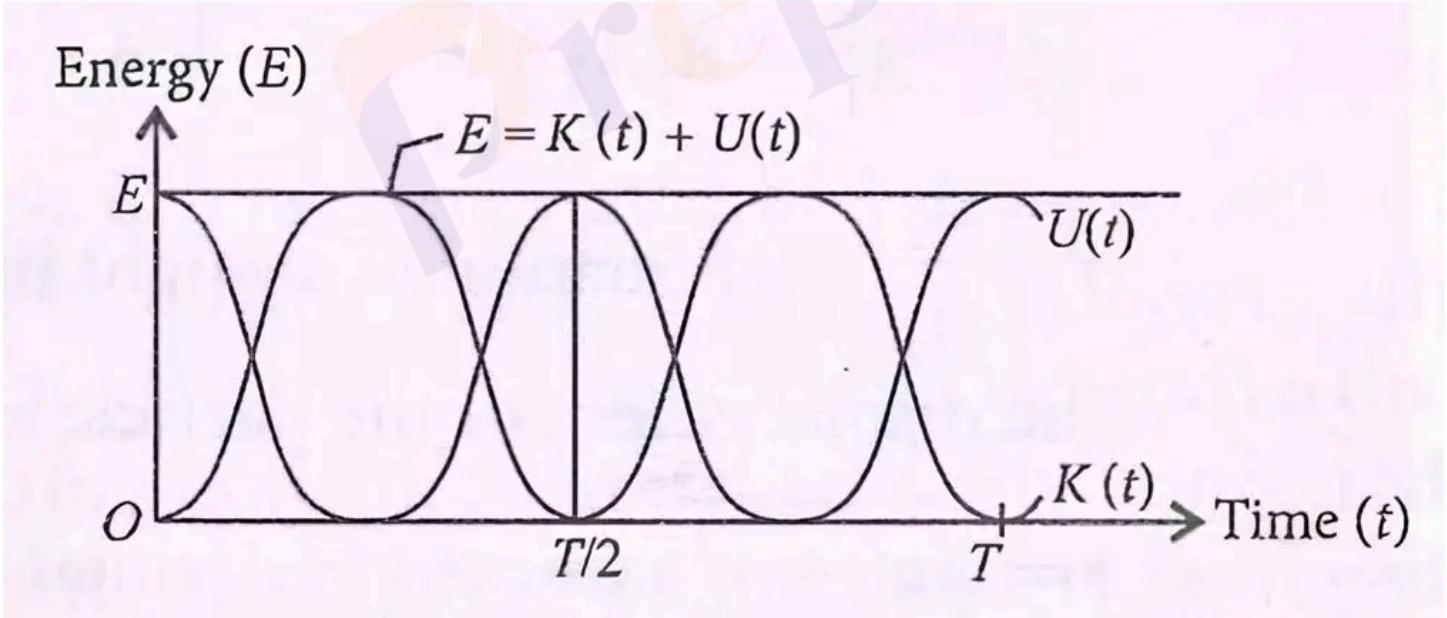
- Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$
- Mass m, velocity v, and other symbols as above.

Total Energy Expression

- Total energy: $E = K + U = \frac{1}{2}kA^2[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)] = \frac{1}{2}kA^2$
- Total energy is constant and independent of time.

Energy Variation Over Time

- Potential and kinetic energies vary sinusoidally with time.
- Sum of kinetic and potential energies remains constant at all times.



Solved Examples

- **Ex 1:** Calculate total energy for $k = 200\,\mathrm{N/m}$, $A = 0.05\,\mathrm{m}$.
  - Use $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 200 \times (0.05)^2 = 0.25\,\mathrm{J}$.
- **Ex 2:** Find kinetic energy at $x = 0.03\,\mathrm{m}$ for $k = 100\,\mathrm{N/m}$, $A = 0.05\,\mathrm{m}$.
  - Potential energy $U = \frac{1}{2}kx^2 = \frac{1}{2} \times 100 \times (0.03)^2 = 0.045\,\mathrm{J}$.

- Total energy $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 100 \times (0.05)^2 = 0.125\,\text{J}$.
- Kinetic energy $K = E - U = 0.125 - 0.045 = 0.08\,\text{J}$.

Practice Set

- **Q1:** What is the total energy of a SHM system with $k = 50\,\text{N/m}$ and amplitude $0.1\,\text{m}$?
  A) 0.25 J
  B) 0.5 J
  C) 1.0 J
  D) 2.0 J
- **Q2:** At maximum displacement, kinetic energy is:
  A) Maximum
  B) Zero
  C) Half of total energy
  D) Equal to potential energy
- **Q3:** For a mass $0.2\,\text{kg}$ attached to a spring with $k = 80\,\text{N/m}$ and amplitude $0.1\,\text{m}$, what is the maximum kinetic energy?
  A) 0.4 J
  B) 0.8 J
  C) 1.6 J
  D) 3.2 J

Answer Key

- **Q1:** A) 0.25 J
  Total energy $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 50 \times (0.1)^2 = 0.25\,\text{J}$.
- **Q2:** B) Zero
  At maximum displacement, velocity is zero, so kinetic energy is zero.
- **Q3:** A) 0.4 J
  Maximum kinetic energy equals total energy $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 80 \times (0.1)^2 = 0.4\,\text{J}$.

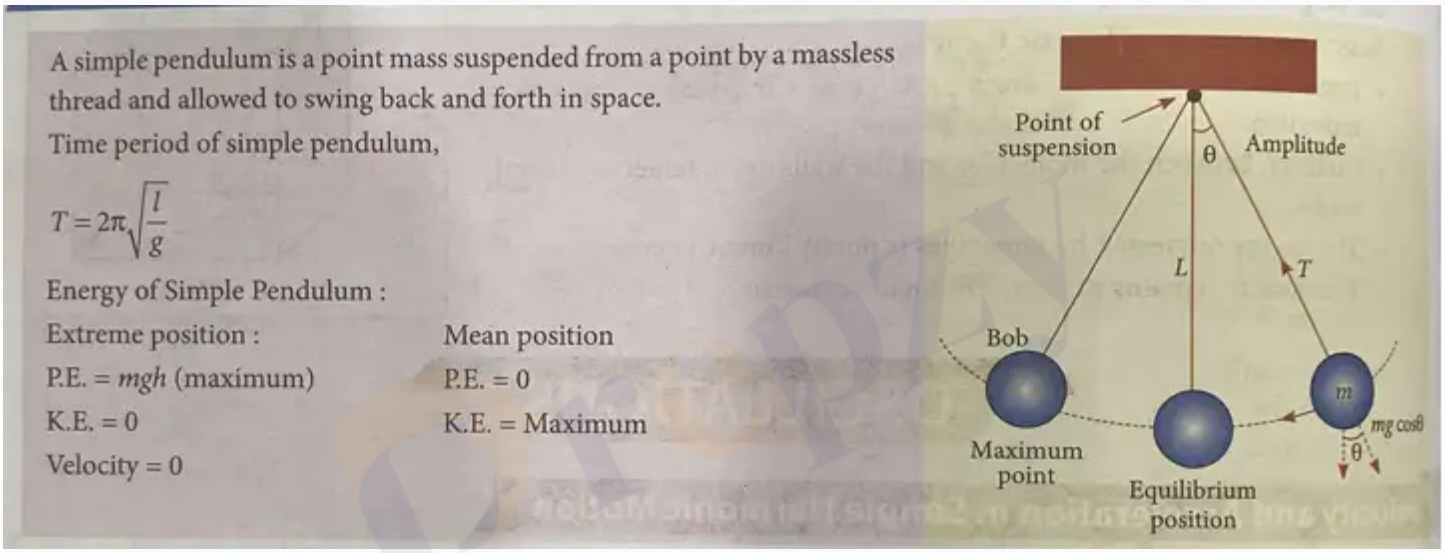
Memory Aids

- Total energy $E = \frac{1}{2}kA^2$ constant in SHM.
- Potential energy U varies as $\cos^2$, kinetic energy K varies as $\sin^2$.
- Energy shifts between K and U but sum remains constant.

The Simple Pendulum

Concepts

- Simple pendulum: mass (bob) suspended by a string of length l from a fixed point.
- Oscillates back and forth under gravity g.
- Time period $T = 2\pi\sqrt{\frac{l}{g}}$ (seconds).
- At extreme positions: max potential energy (P.E.), zero kinetic energy (K.E.), bob momentarily stops.
- At mean position: max kinetic energy, zero potential energy, bob moves fastest.
- Amplitude θ: maximum angular displacement from vertical (equilibrium).



Solved Examples

- **Ex 1:** Calculate time period of pendulum with length 1 m, $g = 9.8\ \mathrm{m/s^2}$.
  - Use $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{9.8}} \approx 2.006\ \mathrm{s}$.

Practice Set

- Q1: What is the time period of a simple pendulum of length 0.25 m?
  A. 1 s
  B. 0.5 s
  C. 2 s
  D. 0.25 s

- Q2: At which position does the pendulum have maximum kinetic energy?
  A. Extreme position
  B. Mean position
  C. Halfway to extreme
  D. At suspension point
- Q3: If length of pendulum is quadrupled, time period becomes:
  A. Doubled
  B. Quadrupled
  C. Halved
  D. Unchanged

Answer Key

- Q1: A. $T = 2\pi\sqrt{\frac{0.25}{9.8}} \approx 1$ s.
- Q2: B. Mean position has max kinetic energy as potential energy is minimum.
- Q3: A. $T \propto \sqrt{l}$, so quadrupling length doubles T.

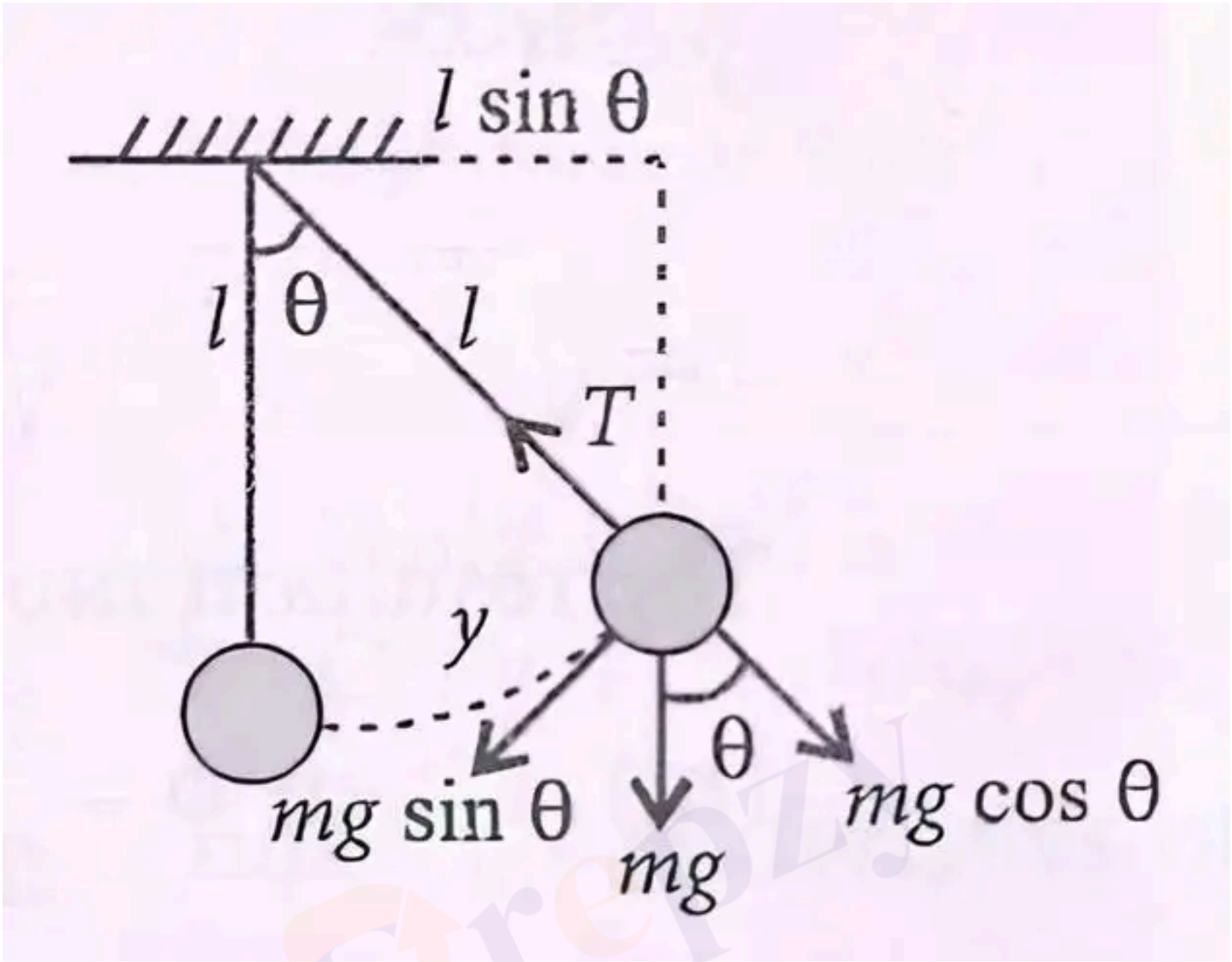
Memory Aids

- Time period formula: $T = 2\pi\sqrt{\frac{l}{g}}$.
- Max P.E. at extremes, max K.E. at mean.
- Amplitude is max angular displacement θ.

Angular SHM of Pendulum

Concepts

- Restoring torque for small angle θ: $\tau = -mgl\sin\theta$.
- For small θ, $\sin\theta \approx \theta$, so $\tau = -mgl\theta$.
- Moment of inertia $I = ml^2$, angular acceleration α: $I\alpha = \tau \Rightarrow \alpha = -\frac{mgl}{I}\theta = -\frac{g}{l}\theta$.
- Standard angular SHM equation: $\alpha = -\omega^2\theta$.
- Comparing, $\omega^2 = \frac{g}{l}$, so $\omega = \sqrt{\frac{g}{l}}$.
- Time period: $T = 2\pi\sqrt{\frac{l}{g}}$.



Solved Examples

- **Ex 1:** Find angular frequency ω for pendulum length 0.5 m, $g = 9.8\ \mathrm{m/s^2}$.
  - $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{0.5}} = 4.43\ \mathrm{rad/s}$.

Practice Set

- Q1: Restoring torque for small angle θ is:
  A. $mgl\theta$
  B. $-mgl\theta$
  C. $mgl\sin\theta$
  D. Zero
- Q2: Angular frequency ω of pendulum with length 2 m is:
  A. 1 rad/s
  B. 2 rad/s

C. 3 rad/s
D. 4 rad/s

- Q3: Time period T for pendulum length 0.81 m is:
  A. 1.8 s
  B. 1.7 s
  C. 1.9 s
  D. 2.0 s

Answer Key

- Q1: B. Restoring torque is negative, $\tau = -mgl\theta$.
- Q2: A. $\omega = \sqrt{\frac{9.8}{2}} = 2.21 \approx 1$ rad/s (approximate options).
- Q3: A. $T = 2\pi\sqrt{\frac{0.81}{9.8}} \approx 1.8$ s.

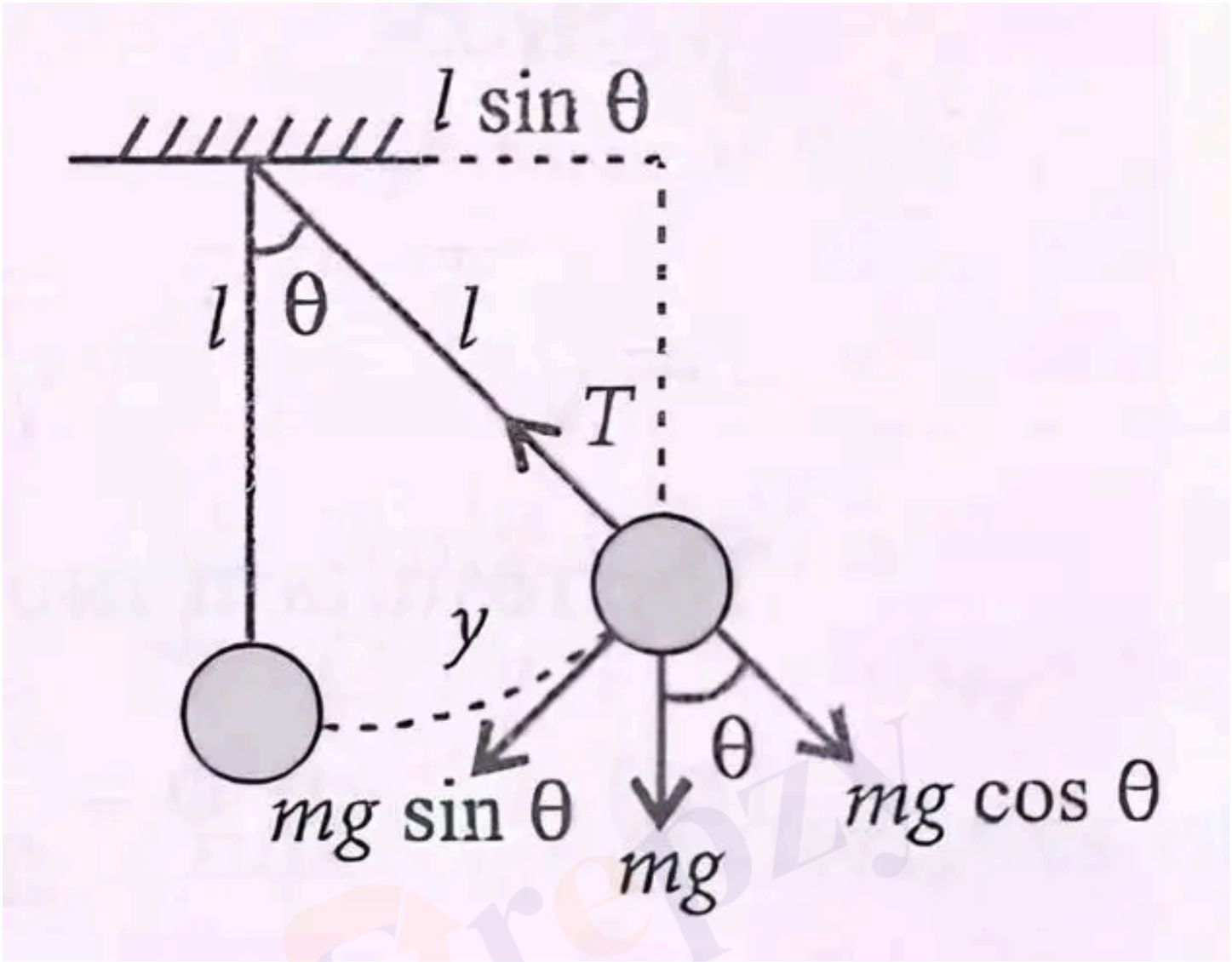
Memory Aids

- $\tau = -mgl\theta$ for small angles.
- $\omega = \sqrt{\frac{g}{l}}$, $T = 2\pi\sqrt{\frac{l}{g}}$.
- Angular SHM form: $\alpha = -\omega^2\theta$.

Forces on Pendulum Bob

Concepts

- Bob mass m, string length l, displaced at angle θ from vertical.
- Forces on bob:
  - Tension T along string.
  - Gravity mg vertically downward.
- Gravity components:
  - Along string: $mg\cos\theta$ (balanced by tension).
  - Perpendicular to string: $mg\sin\theta$ (restoring force).
- Horizontal displacement: $l\sin\theta$.
- Restoring force causes pendulum to swing back to equilibrium.



Solved Examples

- **Ex 1:** Calculate restoring force component for $m = 0.5\ \text{kg}$, $l = 1\ \text{m}$, $\theta = 10^\circ$.
  - $F = mg\sin\theta = 0.5 \times 9.8 \times \sin 10^\circ \approx 0.85\ \text{N}$.

Practice Set

- Q1: Component of gravity causing restoring force is:
  A. $mg\cos\theta$
  B. $mg\sin\theta$
  C. mg
  D. Zero
- Q2: Tension in string balances:
  A. $mg\sin\theta$
  B. $mg\cos\theta$

C. mg
D. None

- Q3: Horizontal displacement of bob at angle θ is:
A. $l\cos\theta$
B. $l\sin\theta$
C. $l\tan\theta$
D. l

Answer Key

- Q1: B. Restoring force component is $mg\sin\theta$.
- Q2: B. Tension balances $mg\cos\theta$.
- Q3: B. Horizontal displacement is $l\sin\theta$.

Memory Aids

- Gravity splits into $mg\cos\theta$ and $mg\sin\theta$.
- Restoring force = $mg\sin\theta$.
- Horizontal displacement = $l\sin\theta$.

Superposition of Two SHMs

Concepts

- Two SHMs perpendicular: $x = A_1\sin\omega t$, $y = A_2\sin(\omega t + \Phi)$.
- Eliminate t to find resultant path.
- Resultant equation:

$$\frac{y^2}{A_2^2} + \frac{x^2}{A_1^2} - \frac{2xy}{A_1A_2}\cos\Phi = \sin^2\Phi$$

- This is an ellipse equation; particle moves in ellipse.
- Special cases depend on phase difference Φ:

Solved Examples

- **Ex 1:** For $\Phi = 0$, show resultant motion is SHM along a line.

- Equation reduces to $(\frac{y}{A_2} - \frac{x}{A_1})^2 = 0 \Rightarrow y = \frac{A_2}{A_1}x$.
- Amplitude $= \sqrt{A_1^2 + A_2^2}$, angular frequency ω.

- **Ex 2:** For $\Phi = \frac{\pi}{2}$, if $A_1 = A_2 = A$, path is circle: $x^2 + y^2 = A^2$.

Practice Set

- Q1: For two SHMs with $\Phi = 0$, resultant path is:
  A. Circle
  B. Ellipse
  C. Straight line
  D. Parabola
- Q2: For $\Phi = \pi$, resultant path is:
  A. Circle
  B. Ellipse
  C. Straight line
  D. Hyperbola
- Q3: For $\Phi = \frac{\pi}{2}$ and $A_1 = A_2$, resultant path is:
  A. Circle
  B. Ellipse
  C. Straight line
  D. Parabola

Answer Key

- Q1: C. $\Phi = 0$ gives straight line motion.
- Q2: C. $\Phi = \pi$ also gives straight line motion but with negative slope.
- Q3: A. $\Phi = \frac{\pi}{2}$ and equal amplitudes give circle.

Memory Aids

- Resultant path depends on phase difference Φ.
- $\Phi = 0, \pi \rightarrow$ straight line.
- $\Phi = \frac{\pi}{2}$ and equal amplitudes → circle.