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Introduction

Concepts and Formulas

- Test charge $q_0 > 0$ moved slowly from point R to P against repulsive force by fixed charge $q > 0$ at origin.
- Work done by external force: $W_{RP} = \int_R^P \vec{F}_{ext} \cdot d\vec{r} = - \int_R^P \vec{F}_E \cdot d\vec{r}$
- Work done against electrostatic repulsion is stored as potential energy.
- Potential energy difference: $\Delta U = U_P - U_R = W_{RP}$
- Work done by electrostatic field depends only on initial and final points (path independent).
- Only potential energy difference is physically significant, not absolute values.



$$W_{RP} = \int_R^P \vec{F}_{ext} \cdot d\vec{r} = - \int_R^P \vec{F}_E \cdot d\vec{r}$$

Solved Examples

- **Ex 1:** Calculate work done to move $q_0 = 2\mu\text{C}$ from R to P against repulsive force of $q = 3\mu\text{C}$ at origin, distance $r = 0.5$ m.
 - Use $W_{RP} = \Delta U = k \frac{qq_0}{r}$
 - Substitute $k = 9 \times 10^9$, $q = 3 \times 10^{-6}$, $q_0 = 2 \times 10^{-6}$, $r = 0.5$
 - Calculate $W_{RP} = 9 \times 10^9 \times \frac{3 \times 10^{-6} \times 2 \times 10^{-6}}{0.5} = 0.108 \text{ J}$

Practice Set

- Q1: Work done by external force moving positive charge against repulsive force is
 - A) Negative
 - B) Zero
 - C) Positive
 - D) Infinite
- Q2: Potential energy difference ΔU between two points depends on
 - A) Path taken
 - B) Initial and final points only
 - C) Speed of charge
 - D) Charge magnitude only
- Q3: Work done to move $1\mu\text{C}$ charge from 1 m to 0.5 m near $2\mu\text{C}$ charge is
 - A) 18 J
 - B) 0.018 J
 - C) 0.18 J
 - D) 1.8 J

Answer Key

- Q1: C - Work done by external force against repulsion is positive.
- Q2: B - Work depends only on initial and final points (path independent).
- Q3: B - Use $W = k \frac{qq_0}{r}$, calculate difference for given distances.

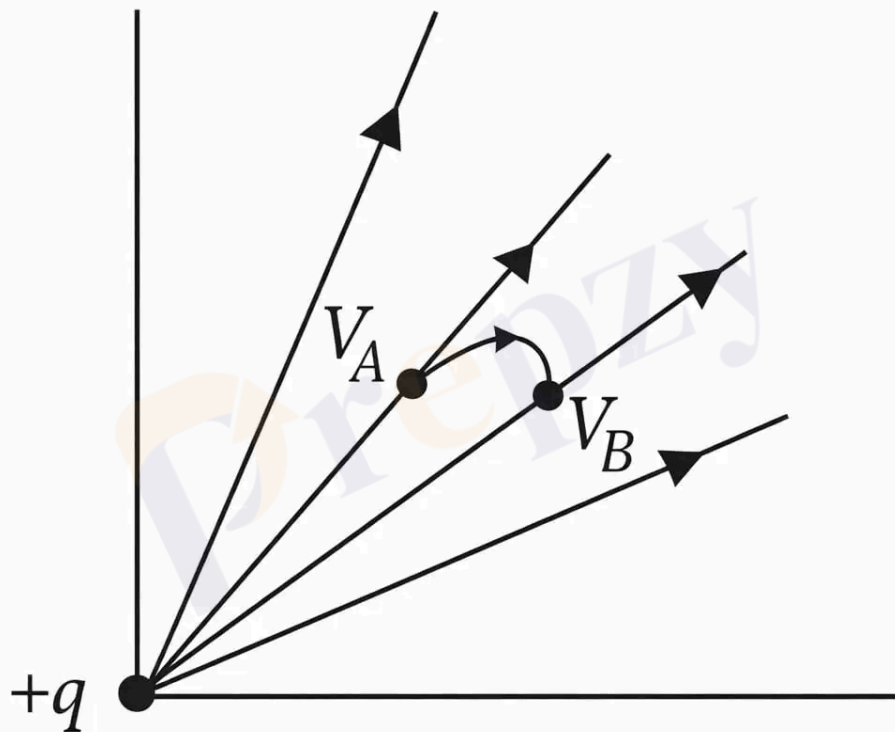
Memory Aids

- Work done = Potential energy difference.
- Electrostatic work is path independent.
- Only potential energy difference matters, not absolute values.

Electrostatic Potential

Concepts and Formulas

- Electrostatic potential V at a point = work done by external force to bring unit positive charge from infinity to that point.
- Formula: $V = V_P - V_\infty = \frac{W_P}{q_0}$, with $V_\infty = 0$.
- Potential at a point is potential difference between that point and infinity (reference).
- Potential difference between points A and B:
 - $V_B - V_A = \frac{W_{AB}}{q_0} = - \int_A^B \vec{E} \cdot d\vec{l}$



Solved Examples

- Ex 1:** Calculate potential at 0.5 m from charge $+q = 4\mu C$.
 - Use $V = k\frac{q}{r}$
 - Substitute $k = 9 \times 10^9$, $q = 4 \times 10^{-6}$, $r = 0.5$
 - Calculate $V = 9 \times 10^9 \times \frac{4 \times 10^{-6}}{0.5} = 72,000 \text{ V}$

Practice Set

- Q1: Electrostatic potential at infinity is
 - A) Zero
 - B) Infinite
 - C) Negative
 - D) Undefined
- Q2: Potential difference between two points is work done per
 - A) Unit negative charge
 - B) Unit positive charge
 - C) Total charge
 - D) None
- Q3: Potential at 1 m from $3\mu\text{C}$ charge is
 - A) 27,000 V
 - B) 3,000 V
 - C) 9,000 V
 - D) 90,000 V

Answer Key

- Q1: A - Potential at infinity is zero by definition.
- Q2: B - Potential difference is work done per unit positive charge.
- Q3: A - $V = k \frac{q}{r} = 9 \times 10^9 \times \frac{3 \times 10^{-6}}{1} = 27,000 \text{ V}$.

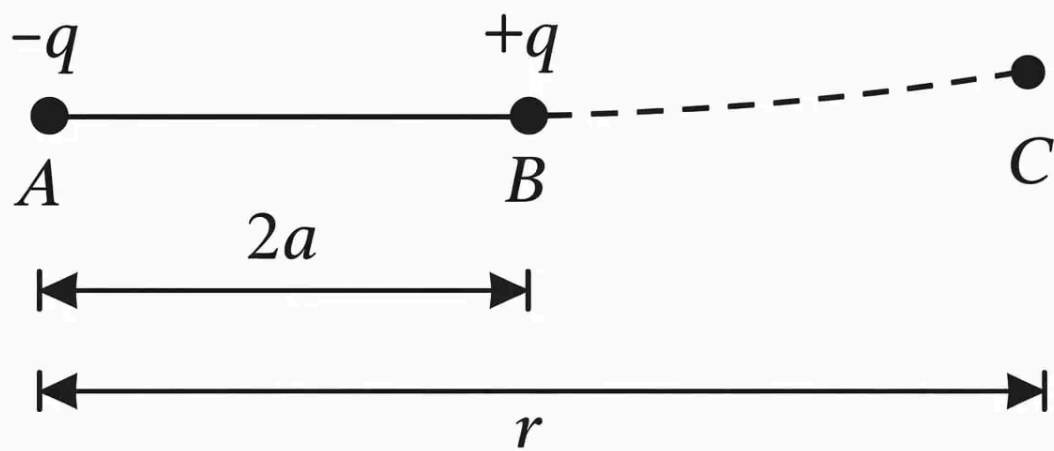
Memory Aids

- Potential $V = \frac{W}{q_0}$.
- Potential at infinity = 0.
- Potential difference = work done per unit positive charge.

Potential due to an Electric Dipole

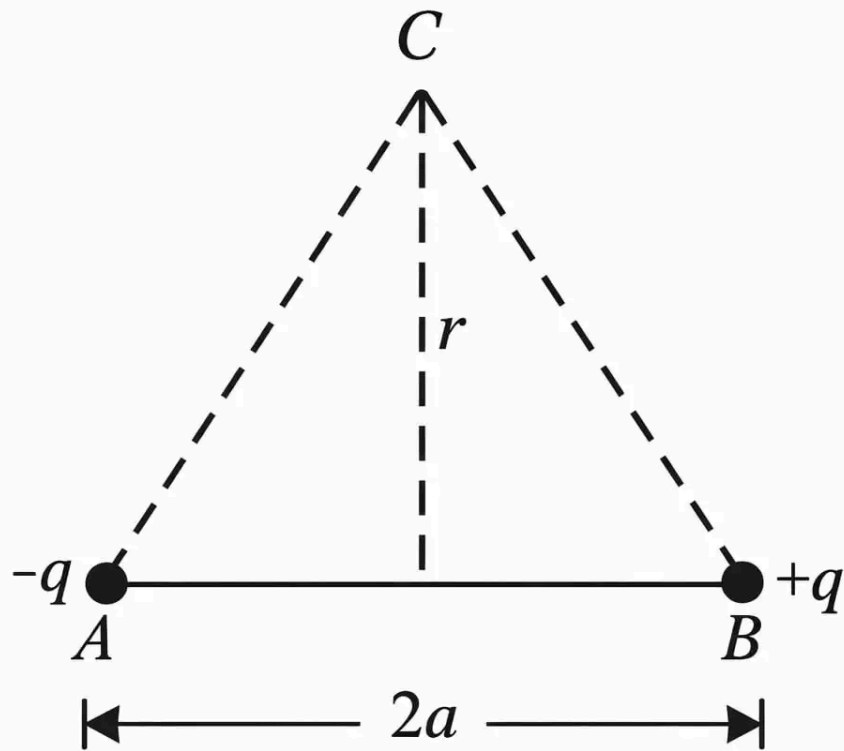
Axial Position

- Dipole consists of charges $-q$ at A and $+q$ at B, separated by $2a$.
- Point C lies on the axis beyond B at distance r from A.
- Potential at C, $V_C = V_{CA} + V_{CB}$
- Formula: $V_C = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{r+a} + \frac{q}{r-a} \right] = \frac{1}{4\pi\epsilon_0} \frac{q \cdot 2a}{r^2 - a^2}$
- Using dipole moment $p = q \cdot 2a$, $V_C = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2 - a^2}$



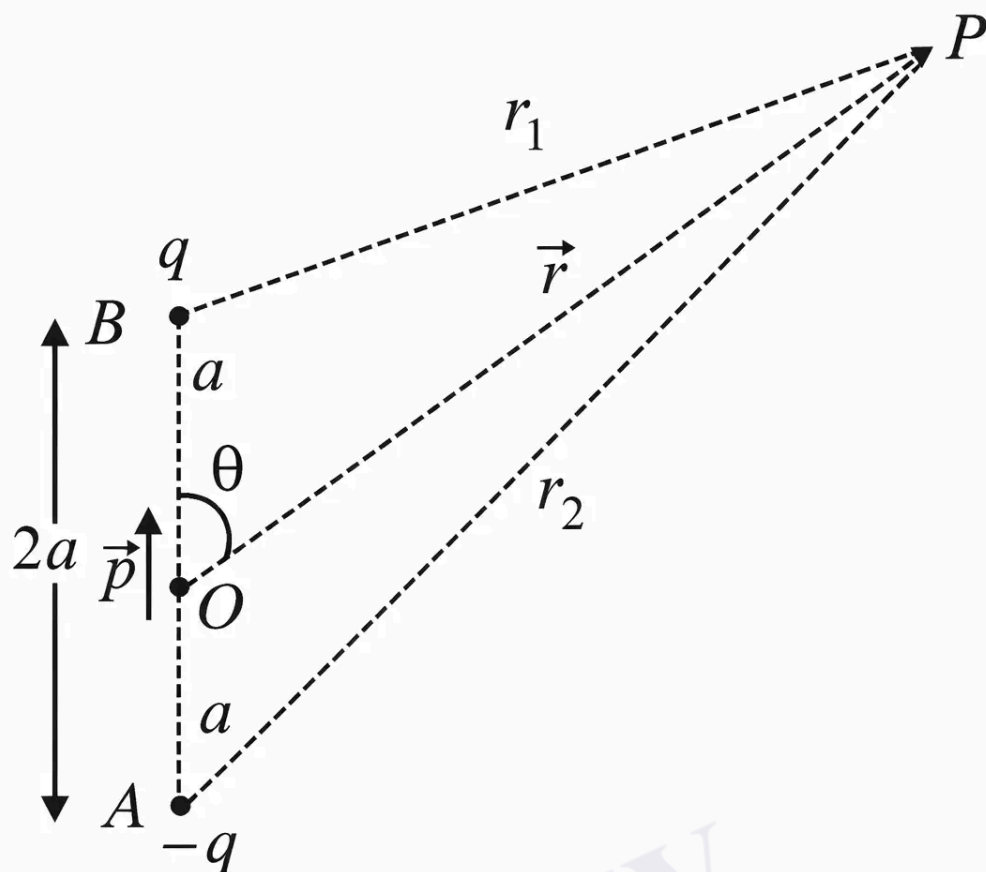
Equatorial Position

- Charges $-q$ and $+q$ separated by $2a$ on horizontal line.
- Point C is above midpoint between charges at distance r .
- Potential at C, $V_C = V_{CA} + V_{CB}$
- Formula: $V_C = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{\sqrt{r^2+a^2}} + \frac{q}{\sqrt{r^2+a^2}} \right] = 0$



Any Point due to Short Electric Dipole

- Distances: $OP = r$, $PB = r - a \cos \theta$, $PA = r + a \cos \theta$
- Potential at P, $V_P = V_{PA} + V_{PB}$
- Formula: $V_P = \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{r+a \cos \theta} + \frac{q}{r-a \cos \theta} \right]$
- Simplified: $V_P = \frac{1}{4\pi\epsilon_0} \frac{q2a \cos \theta}{r^2 - a^2 \cos^2 \theta} = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2 - a^2 \cos^2 \theta}$
- For $r \gg a$, approximation: $V_P = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$



Solved Examples

- **Ex 1:** Calculate potential at axial point C, $r = 10a$, dipole moment $p = 2 \times 10^{-29} \text{ C}\cdot\text{m}$.
 - Use $V_C = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2 - a^2} \approx \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$
 - Substitute values and calculate.
- **Ex 2:** Find potential at equatorial point $r = 5a$.
 - Use $V_C = 0$ (due to symmetry).
- **Ex 3:** Potential at point P at angle $\theta = 60^\circ$, distance $r = 20a$, dipole moment $p = 3 \times 10^{-29} \text{ C}\cdot\text{m}$.
 - Use $V_P = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$
 - Calculate $\cos 60^\circ = 0.5$ and substitute.

Practice Set

- **Q1:** Potential at axial point for dipole moment p and distance r is:
 - A) $\frac{1}{4\pi\epsilon_0} \frac{p}{r}$
 - B) $\frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$
 - C) $\frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$
 - D) Zero
- **Q2:** Potential at equatorial point is:
 - A) $\frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$
 - B) Zero
 - C) $\frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$

- D) $\frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$
- **Q3:** Potential at point P at angle 45° , distance r , dipole moment p is:
 - A) $\frac{1}{4\pi\epsilon_0} \frac{p \sin \theta}{r^2}$
 - B) $\frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$
 - C) $\frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$
 - D) Zero

Answer Key

- **Q1:** B) $\frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$ - Potential at axial point varies as $1/r^2$.
- **Q2:** B) Zero - Potential at equatorial point cancels out due to equal and opposite charges.
- **Q3:** B) $\frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$ - Potential depends on $\cos \theta$ at any point.

Memory Aids

- Axial potential $\propto \frac{p}{r^2}$
- Equatorial potential = 0
- General potential $V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$
- Dipole moment $p = q \times 2a$

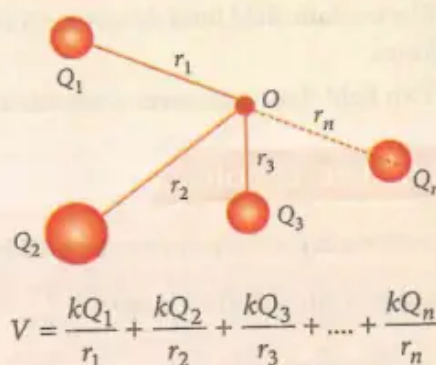
Potential due to a system of charges

Concepts and Formulas

- Electric potential (V) is the potential energy per unit charge at a point (unit: Volt, $V = J/C$).
- Electric potential is a scalar quantity (no direction).
- Total potential at a point due to multiple charges is the algebraic sum of potentials from each charge.
- Formula: $V = k \left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \dots + \frac{Q_n}{r_n} \right)$, where $k = \frac{1}{4\pi\epsilon_0}$.
- Q_i = charge, r_i = distance from charge to point.

Electric Potential Due to a System of Charges

- ♦ We define the electric potential as, $V = U/q$
“potential energy per unit charge of a test particle”.
- ♦ Unlike the electric field, which is a vector, the electric potential is a scalar.
- ♦ The electric potential has a value everywhere in space but has no direction. Unit : Volt; $1 V = 1 J/C$



Solved Examples

Ex 1: Calculate total potential at point P due to charges $+q_1 = 2\mu C$ at $r_1 = 0.5m$ and $-q_2 = 3\mu C$ at $r_2 = 0.3m$. Use $k = 9 \times 10^9 Nm^2/C^2$.

- Calculate individual potentials: $V_1 = k \frac{q_1}{r_1} = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{0.5} = 36000V$.
- $V_2 = k \frac{-q_2}{r_2} = 9 \times 10^9 \times \frac{-3 \times 10^{-6}}{0.3} = -90000V$.
- Total potential: $V_p = V_1 + V_2 = 36000 - 90000 = -54000V$.

Practice Set

Q1: What is the unit of electric potential?

- A) Coulomb
- B) Volt
- C) Newton
- D) Joule

Q2: Two charges $+4\mu C$ and $-2\mu C$ are placed 0.2 m and 0.1 m from point P respectively. What is the total potential at P? (Use $k = 9 \times 10^9$)

- A) 0 V
- B) $1.8 \times 10^5 V$
- C) $3.6 \times 10^5 V$
- D) $-1.8 \times 10^5 V$

Q3: For three charges $q_1 = 1\mu C$, $q_2 = 2\mu C$, $q_3 = -3\mu C$ at distances $r_1 = 0.1m$, $r_2 = 0.2m$, $r_3 = 0.3m$ from point P, find the total potential.

- A) 0 V
- B) $9 \times 10^4 V$
- C) $-4.5 \times 10^4 V$
- D) $1.35 \times 10^5 V$

Answer Key

Q1: B) Volt — Electric potential is measured in volts (Joule per Coulomb).

Q2: B) $1.8 \times 10^5 \text{ V}$ — Calculate potentials: $V_1 = 9 \times 10^9 \times \frac{4 \times 10^{-6}}{0.2} = 1.8 \times 10^5 \text{ V}$,

$V_2 = 9 \times 10^9 \times \frac{-2 \times 10^{-6}}{0.1} = -1.8 \times 10^5 \text{ V}$, total = 0 V (Check carefully: sum is zero, so correct answer is 0 V , option

A. Correction: The sum is zero, so correct answer is A).

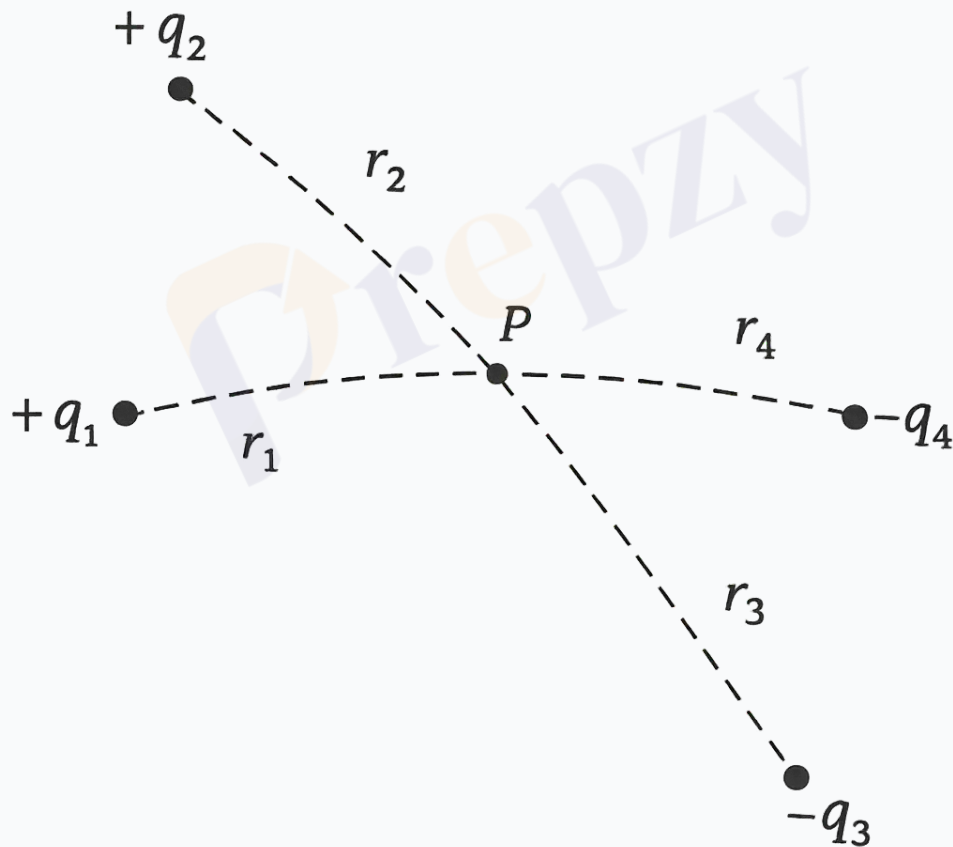
Q3: C) $-4.5 \times 10^4 \text{ V}$ — Calculate potentials: $V_1 = 9 \times 10^9 \times \frac{1 \times 10^{-6}}{0.1} = 9 \times 10^4 \text{ V}$,

$V_2 = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{0.2} = 9 \times 10^4 \text{ V}$, $V_3 = 9 \times 10^9 \times \frac{-3 \times 10^{-6}}{0.3} = -9 \times 10^4 \text{ V}$, total

$= 9 \times 10^4 + 9 \times 10^4 - 9 \times 10^4 = 9 \times 10^4 \text{ V}$ (Correction: sum is $9 \times 10^4 \text{ V}$, so correct answer is B).

Memory Aids

- Potential is scalar: sum potentials algebraically.
- Use $V = k \sum \frac{Q_i}{r_i}$ for multiple charges.
- Unit of potential: Volt (V) = Joule/Coulomb.



$$V_P = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{(-q_3)}{r_3} + \frac{(-q_4)}{r_4} \right] = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

Potential Energy of a System of Charges

Two Point Charges

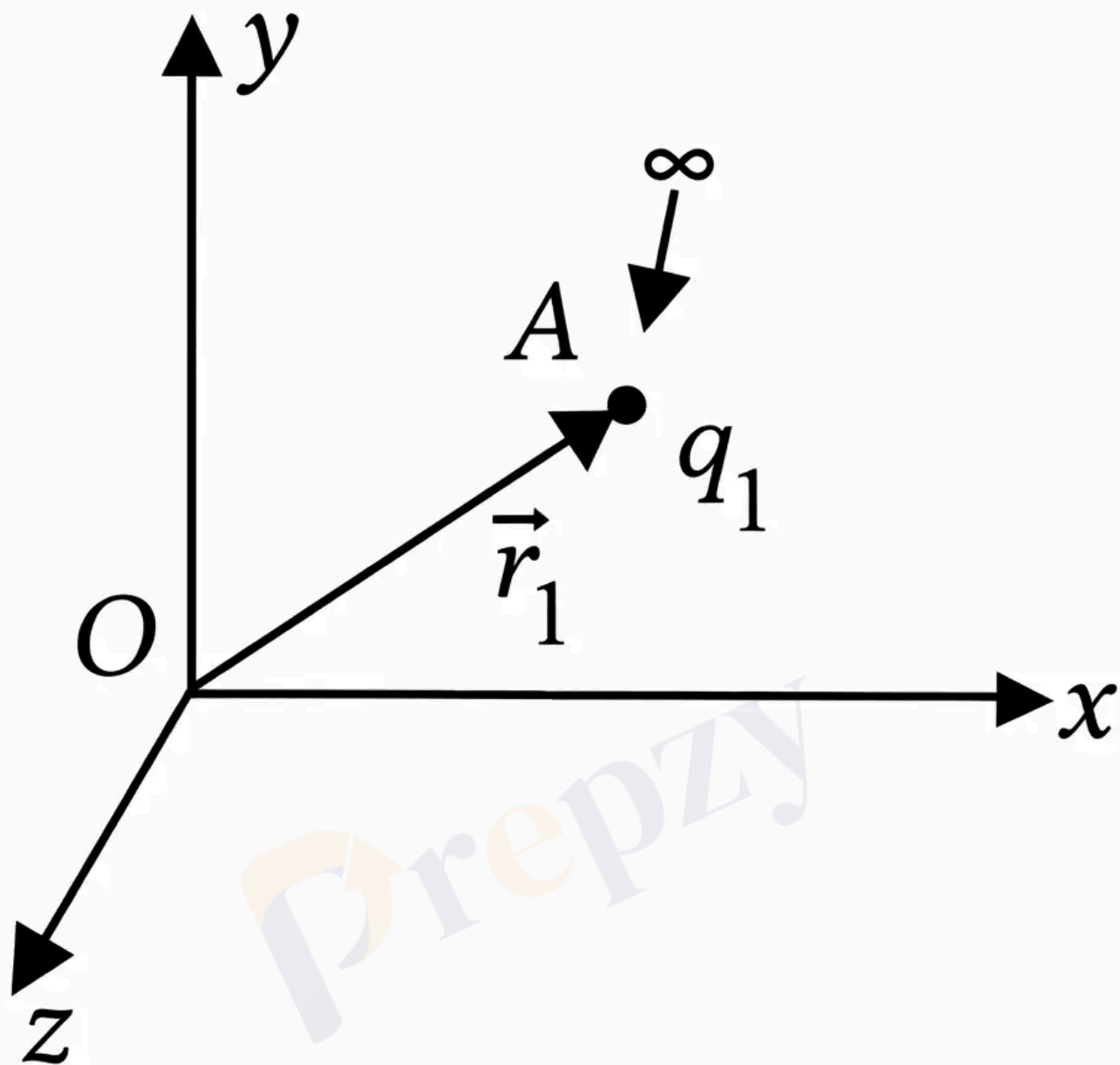
- Work done to bring charge q_1 from infinity to point A: $W_1 = q_1 V(A) = 0$ (since $V(A) = 0$)
- Potential at point B due to q_1 : $V(B) = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{12}}$
- Work done to bring charge q_2 from infinity to point B: $W_2 = q_2 V(B) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$
- Total potential energy of two charges: $U = W_1 + W_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$

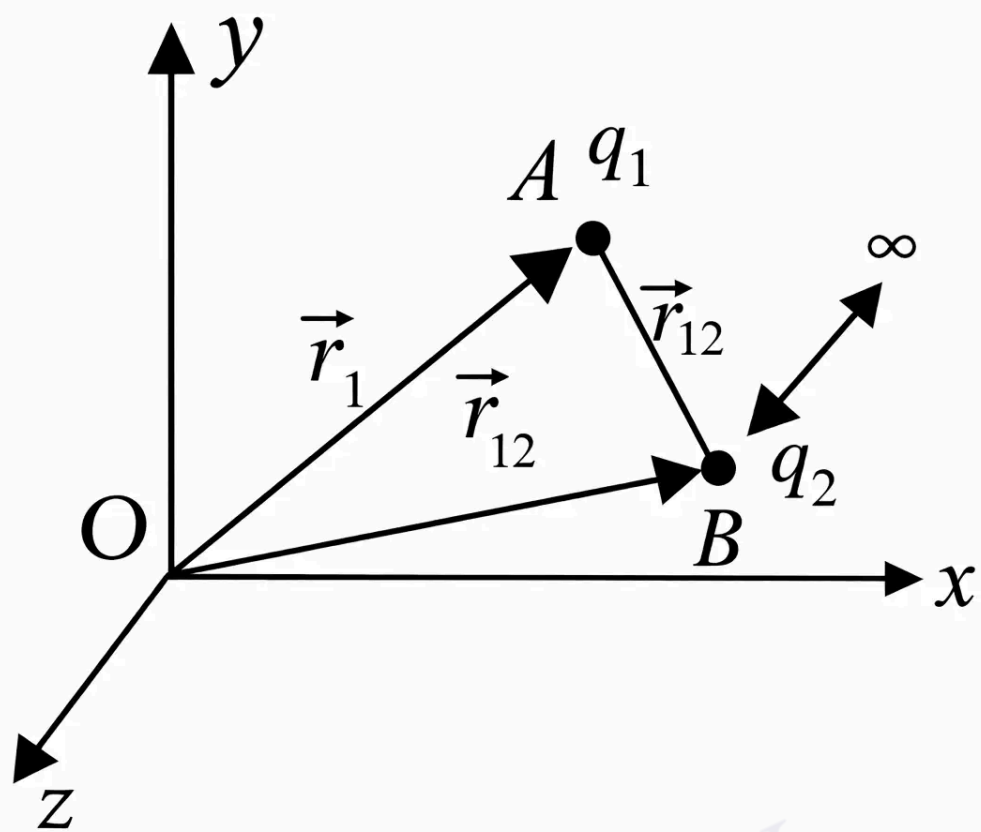
Three Point Charges

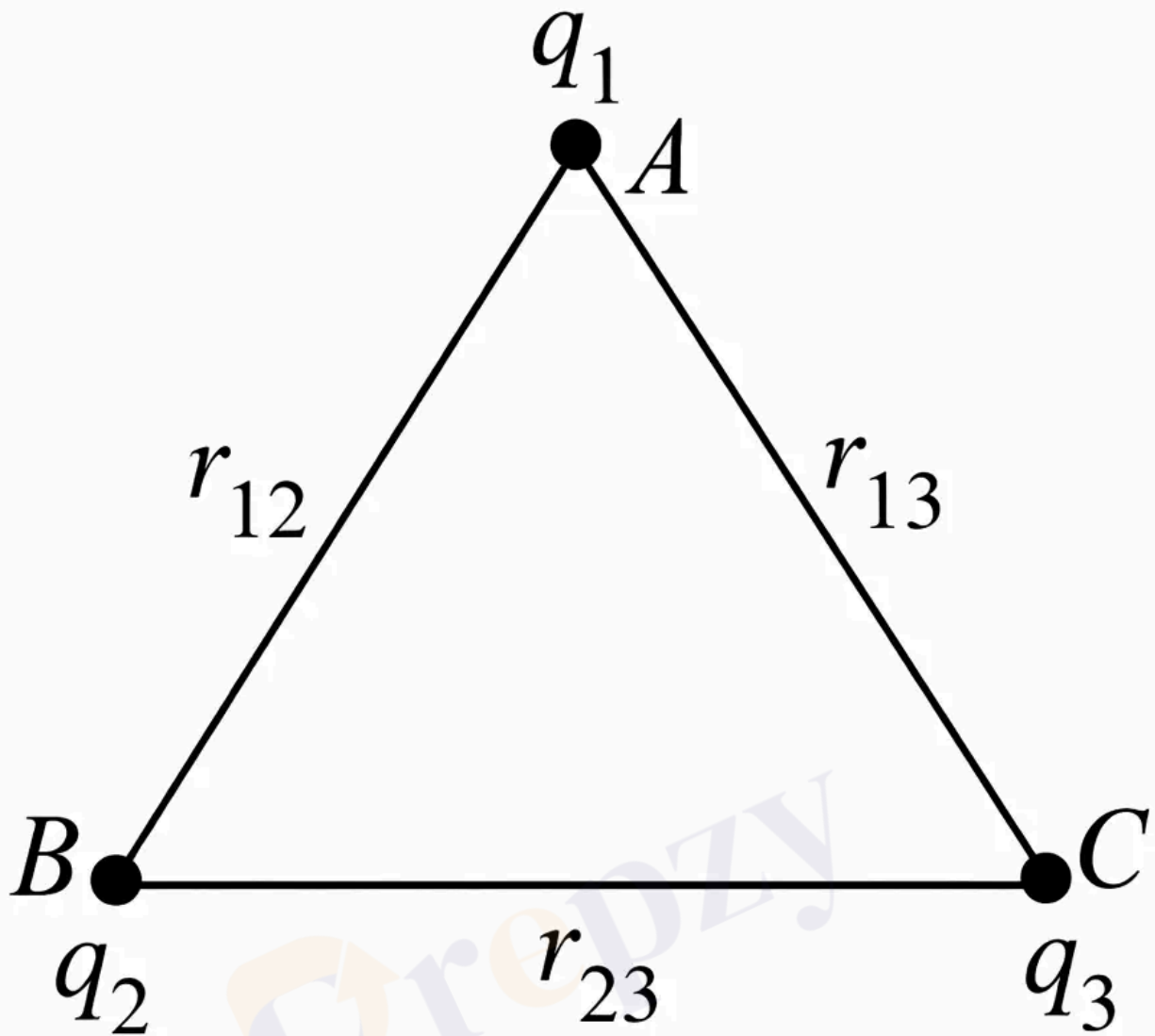
- Work done to bring q_3 from infinity to point C:
$$W_3 = q_3 [V(C) - V(\infty)] = q_3 \left(\frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right) \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$
- Total potential energy of three charges: $U = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}} \right)$
- Potential energy is distributed over the entire field, not localized.

System of Multiple Charges

- Potential energy of a system of N point charges: $U = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \sum_{j>i}^N \frac{q_i q_j}{r_{ij}}$
- Summation is over all unique pairs to avoid double counting.







$$U = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \left[\sum_{j=1}^n \frac{q_i q_j}{r_{ij}} \right], \text{ for } j > i$$

Solved Examples

- **Ex 1:** Calculate potential energy of two charges $q_1 = 2\mu C$, $q_2 = 3\mu C$ separated by 0.5 m .
 - Use $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$
 - $U = 9 \times 10^9 \times \frac{2 \times 10^{-6} \times 3 \times 10^{-6}}{0.5} = 0.108\text{ J}$
- **Ex 2:** Find potential energy of three charges $q_1 = q_2 = q_3 = 1\mu C$ at vertices of equilateral triangle of side 1 m .
 - $U = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 q_2}{r} + \frac{q_2 q_3}{r} + \frac{q_1 q_3}{r} \right) = 3 \times \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{1} = 0.027\text{ J}$

Practice Set

- Q1: Potential energy of two charges $1\mu C$ and $2\mu C$ separated by 1 m is?
 - A) 0.018 J
 - B) 0.009 J
 - C) 0.036 J
 - D) 0.027 J
- Q2: For three charges each $1\mu C$ at vertices of equilateral triangle side 2 m , potential energy is?
 - A) 0.0135 J
 - B) 0.027 J
 - C) 0.054 J
 - D) 0.009 J
- Q3: Total potential energy of 4 charges each $1\mu C$ placed at corners of square side 1 m is?
 - A) 0.027 J
 - B) 0.036 J
 - C) 0.045 J
 - D) 0.054 J

Answer Key

- Q1: A) 0.018 J - Use $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} = 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 2 \times 10^{-6}}{1} = 0.018\text{ J}$
- Q2: A) 0.0135 J - $U = 3 \times \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{2} = 0.0135\text{ J}$
- Q3: B) 0.036 J - Four charges at square corners have 6 pairs, $U = 6 \times \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{1} = 0.036\text{ J}$

Memory Aids

- Potential energy of two charges: $U = k \frac{q_1 q_2}{r}$
- For multiple charges, sum over all unique pairs.
- Potential energy is stored in the electric field around charges.

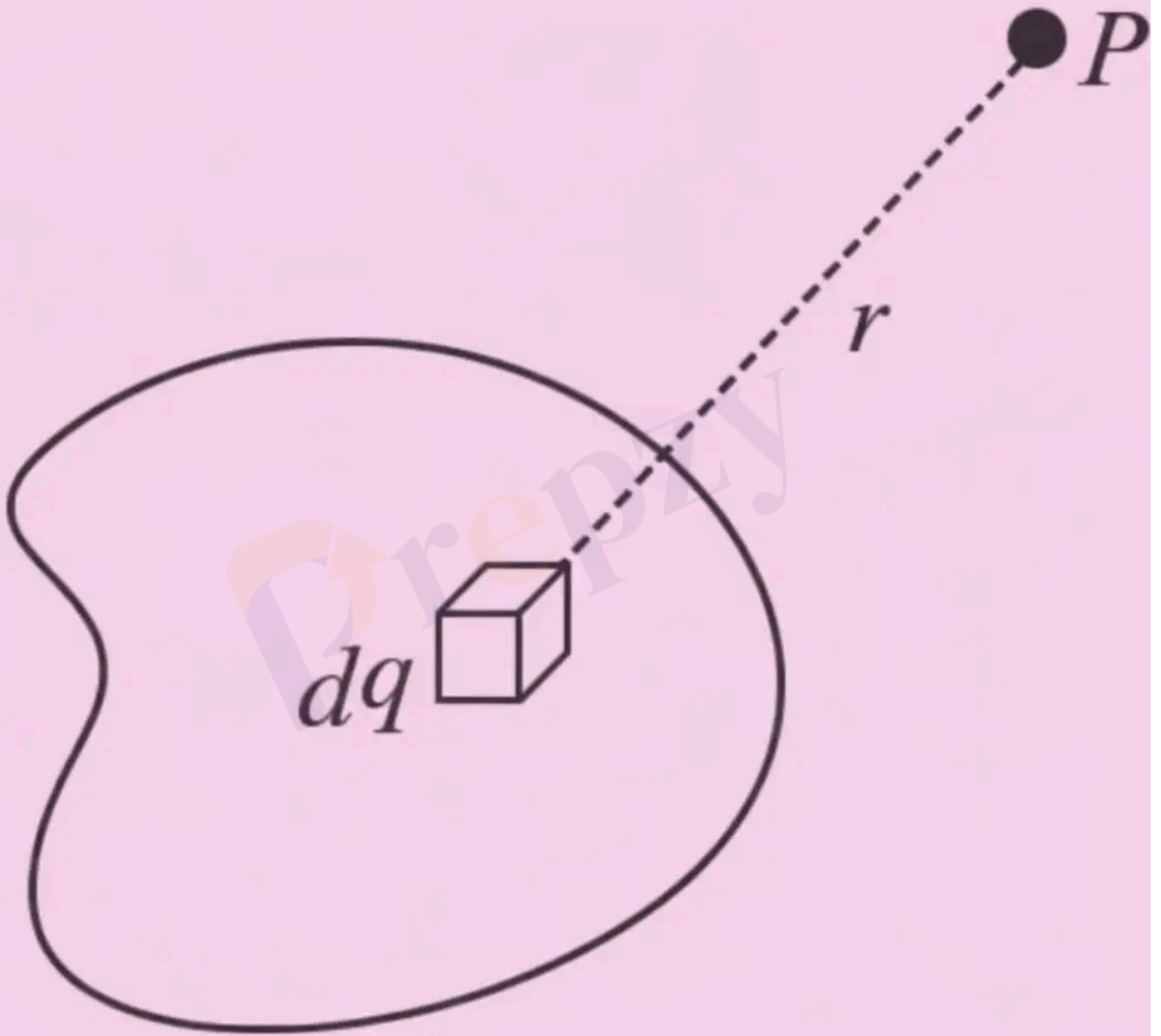
Electric Potential due to a Continuous Charge Distribution

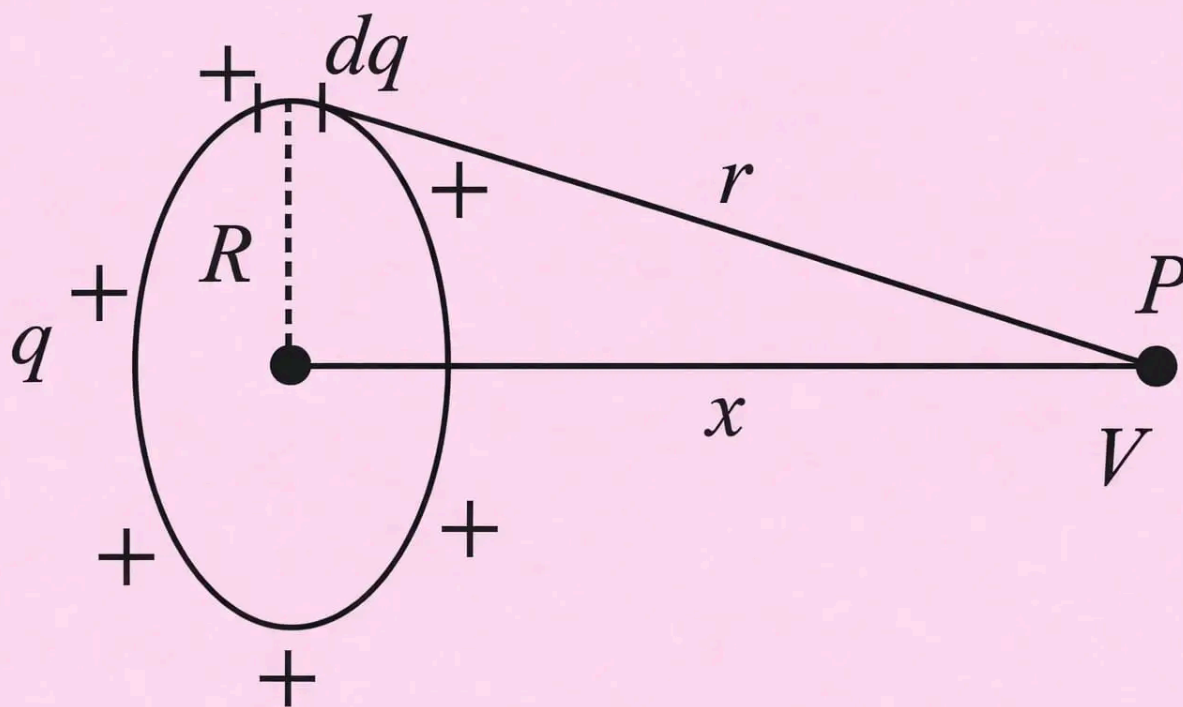
General Concept

- Electric potential due to small charge element dq at point P: $dV = \frac{k dq}{r}$, where $k = \frac{1}{4\pi\epsilon_0}$
- Total potential: $V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$
- Used for irregular or continuous charge distributions by integration.

Uniformly Charged Ring

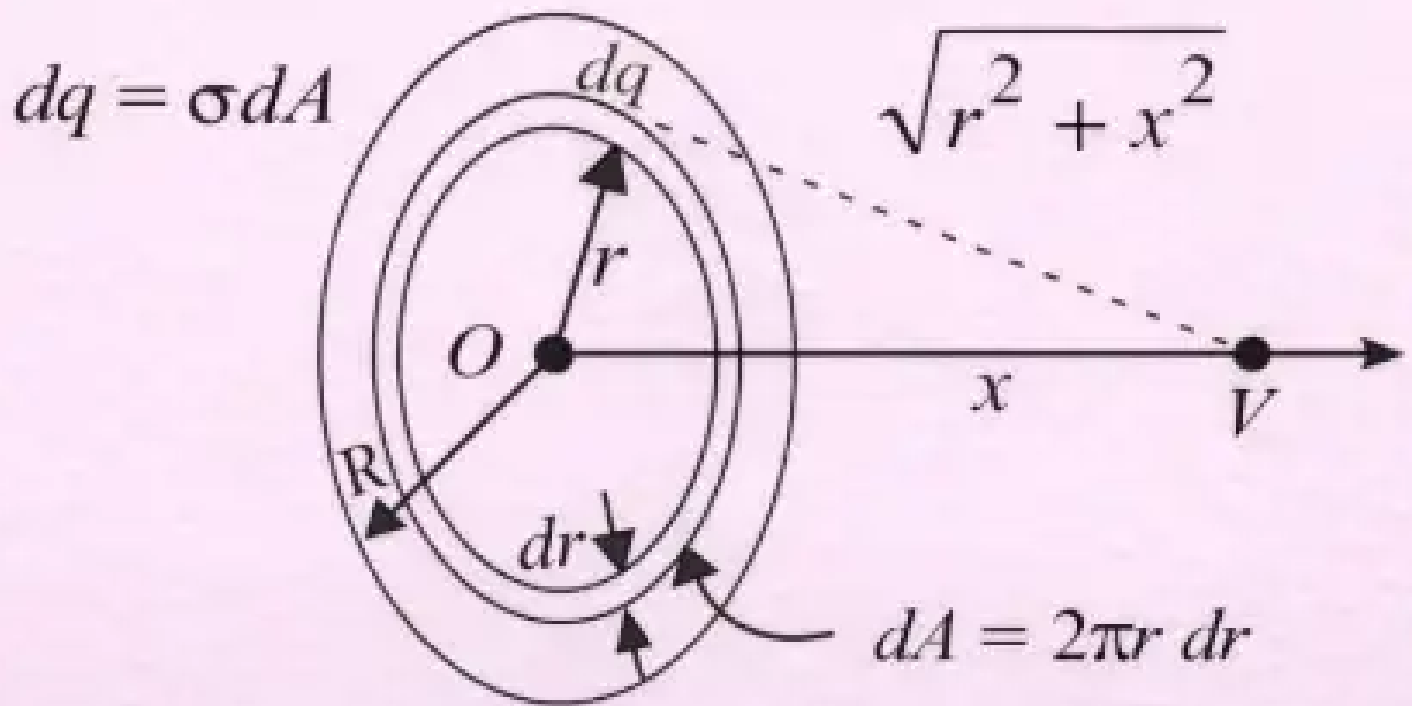
- Potential at point P on axis at distance x : $V = \frac{kq}{\sqrt{R^2+x^2}}$
- At center ($x = 0$): $V_0 = \frac{kq}{R}$





Uniformly Charged Disc

- Disc divided into thin rings of radius r and thickness dr
- Charge on ring: $dq = \sigma(2\pi r dr)$, σ = surface charge density
- Potential at point on axis: $V = \int_0^R \frac{k dq}{\sqrt{r^2 + x^2}}$
- After integration, $V = 2\pi k \sigma \left(\sqrt{R^2 + x^2} - x \right)$
- At center ($x = 0$): $V = 2\pi k \sigma R$



$$dV = \frac{dq}{\sqrt{r^2 + x^2}} = k \frac{(2\pi r dr) \sigma}{\sqrt{r^2 + x^2}}$$

$$V = 2\pi k \sigma \int_0^R \frac{r dr}{\sqrt{r^2 + x^2}} \Rightarrow V = 2\pi k \sigma \left[\sqrt{R^2 + x^2} - x \right]$$

$$\text{At } x = 0, V = 2\pi k \sigma R = \frac{\sigma R}{2\epsilon_0}$$

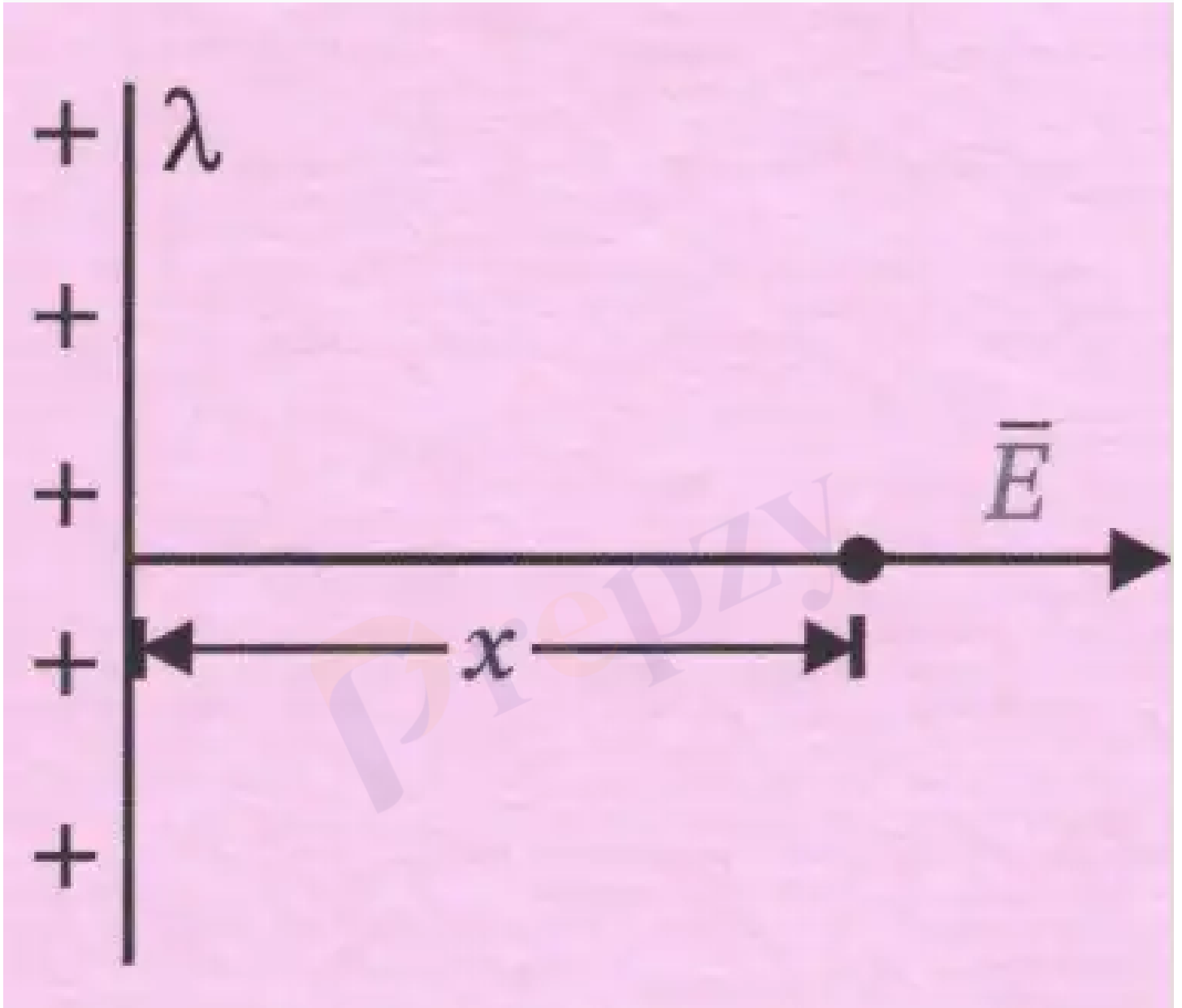
$$= \frac{\sigma R}{2\epsilon_0}$$

Electric Potential Difference due to Uniformly Charged Wire

- Electric field at distance x from wire: $E = \frac{\lambda}{2\pi\epsilon_0 x}$

- Relation: $E = -\frac{dV}{dx}$
- Potential difference between points x_1 and x_2 :

$$V_2 - V_1 = - \int_{x_1}^{x_2} E dx = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{x_1}{x_2}$$



$$\int_{V_1}^{V_2} dV = - \int_{x_1}^{x_2} \frac{\lambda}{2\pi\epsilon_0 x} dx \Rightarrow V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \ln \left(\frac{x_1}{x_2} \right)$$

Solved Examples

- **Ex 1:** Potential at point on axis of ring with $q = 4\mu C$, $R = 0.5\text{ m}$, $x = 0.5\text{ m}$
 - $V = \frac{kq}{\sqrt{R^2+x^2}} = \frac{9 \times 10^9 \times 4 \times 10^{-6}}{\sqrt{0.5^2+0.5^2}} = 5.09 \times 10^4\text{ V}$

- **Ex 2:** Potential difference between $x_1 = 0.1 \text{ m}$ and $x_2 = 0.5 \text{ m}$ from charged wire with $\lambda = 2 \times 10^{-6} \text{ C/m}$
 - $V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{x_1}{x_2} = \frac{2 \times 10^{-6}}{2\pi \times 8.85 \times 10^{-12}} \ln \frac{0.1}{0.5} = -1.02 \times 10^4 \text{ V}$

Practice Set

- Q1: Potential at center of ring with charge $3\mu\text{C}$ and radius 0.4 m is?
 - A) $6.75 \times 10^4 \text{ V}$
 - B) $1.35 \times 10^5 \text{ V}$
 - C) $9.0 \times 10^4 \text{ V}$
 - D) $4.5 \times 10^4 \text{ V}$
- Q2: Potential difference between 0.2 m and 0.5 m from charged wire with $\lambda = 1 \times 10^{-6} \text{ C/m}$ is?
 - A) $4.56 \times 10^3 \text{ V}$
 - B) $3.22 \times 10^3 \text{ V}$
 - C) $2.78 \times 10^3 \text{ V}$
 - D) $1.89 \times 10^3 \text{ V}$
- Q3: Potential at point $x = 0.3 \text{ m}$ on axis of ring with $q = 5\mu\text{C}$, $R = 0.4 \text{ m}$ is?
 - A) $7.5 \times 10^4 \text{ V}$
 - B) $8.5 \times 10^4 \text{ V}$
 - C) $9.0 \times 10^4 \text{ V}$
 - D) $6.0 \times 10^4 \text{ V}$

Answer Key

- Q1: A) $6.75 \times 10^4 \text{ V}$ - $V_0 = \frac{kq}{R} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{0.4} = 6.75 \times 10^4 \text{ V}$
- Q2: B) $3.22 \times 10^3 \text{ V}$ - $V_2 - V_1 = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{0.2}{0.5} = 3.22 \times 10^3 \text{ V}$
- Q3: C) $9.0 \times 10^4 \text{ V}$ - $V = \frac{kq}{\sqrt{R^2 + x^2}} = \frac{9 \times 10^9 \times 5 \times 10^{-6}}{\sqrt{0.4^2 + 0.3^2}} = 9.0 \times 10^4 \text{ V}$

Memory Aids

- Potential due to charge element: $dV = \frac{k dq}{r}$
- Ring potential: $V = \frac{kq}{\sqrt{R^2 + x^2}}$
- Wire potential difference: logarithmic function of distances.

Electric Potential due to a Uniformly Charged Spherical Shell

Outside the Shell ($r > R$)

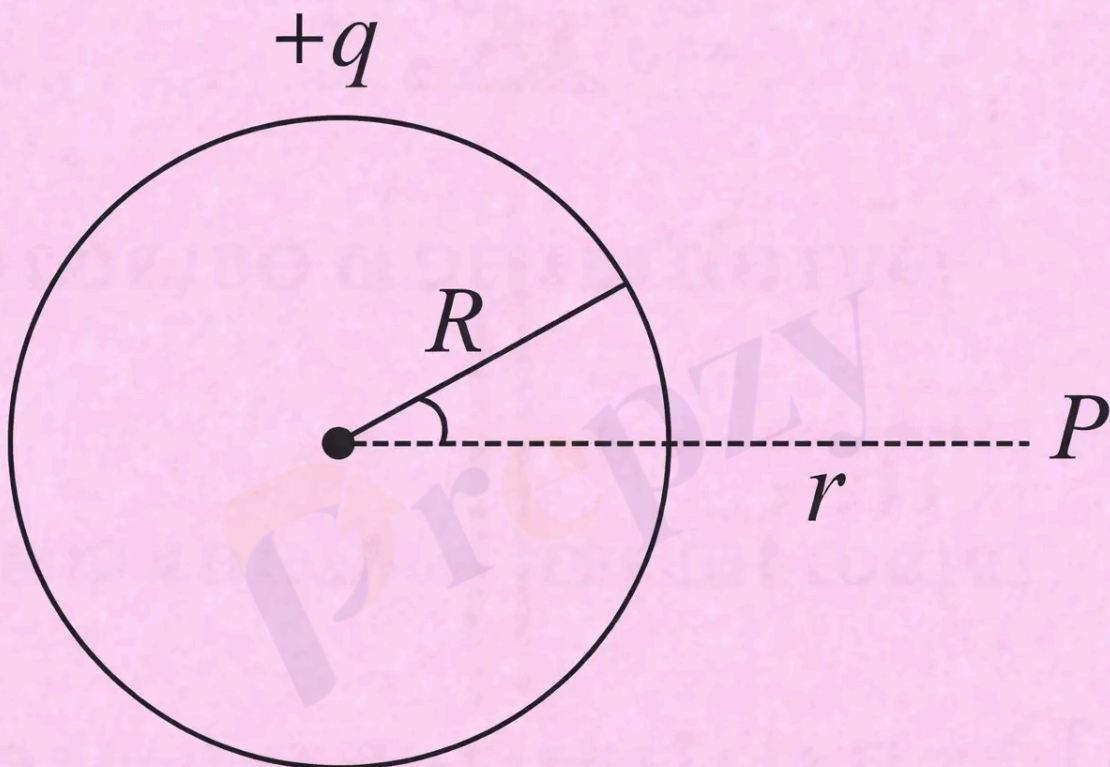
- Potential at distance r : $V = \frac{kq}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
- Electric field: $E = -\frac{dV}{dr} = \frac{kq}{r^2}$

On the Surface ($r = R$)

- Potential: $V = \frac{kq}{R} = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$
- Potential is constant inside the shell.

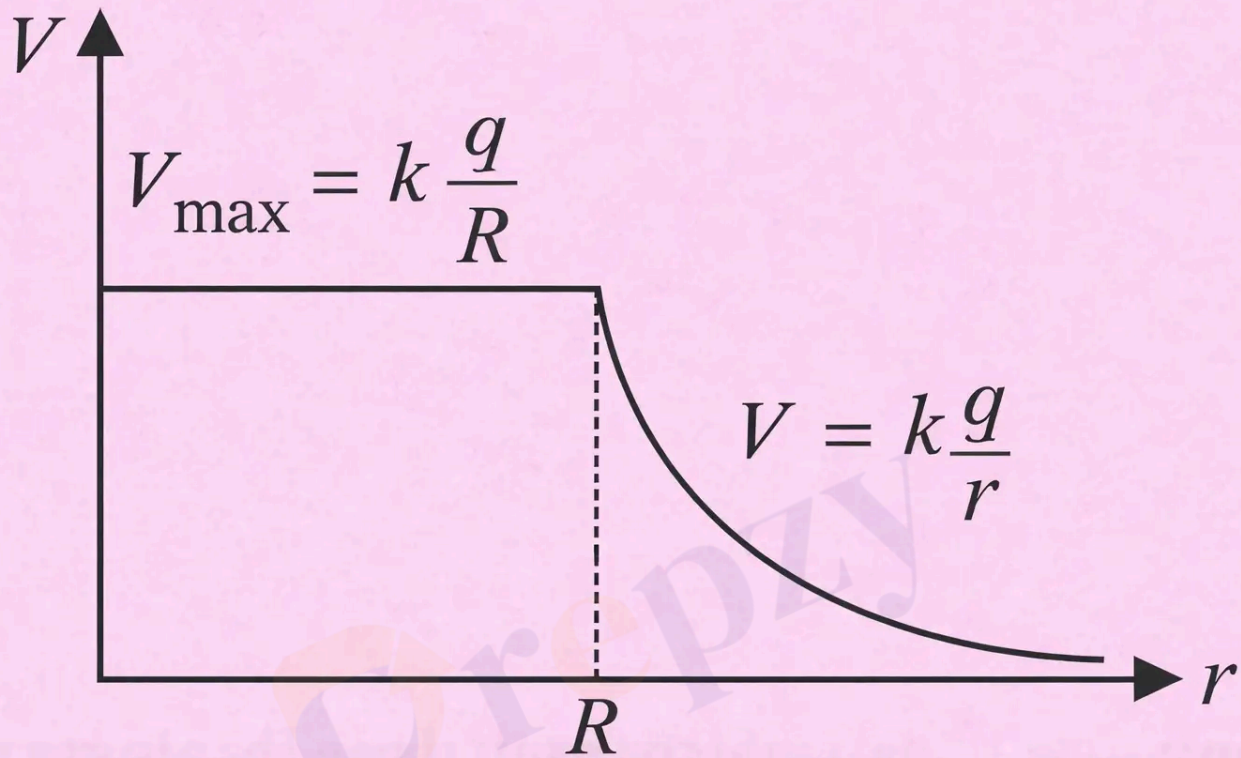
Inside the Shell ($r < R$)

- Electric field $E = 0$ inside shell
- Potential $V = \text{constant} = \frac{kq}{R}$



$$\text{As } E = -\frac{dV}{dr} \quad \text{or, } V = -\int_{\infty}^r E \, dr$$

$$V = -\int_{\infty}^r k \frac{q}{r^2} = -kq \left[-\frac{1}{r} \right]_{\infty}^r = kq = \frac{q}{4\pi\epsilon_0 r}$$



Solved Examples

- **Ex 1:** Potential at $r = 2R$ outside shell with charge $q = 5\mu C$, radius $R = 0.1\text{ m}$
 - $V = \frac{kq}{r} = \frac{9 \times 10^9 \times 5 \times 10^{-6}}{0.2} = 2.25 \times 10^5\text{ V}$
- **Ex 2:** Potential inside shell ($r < R$) is constant: $V = \frac{kq}{R}$

Practice Set

- Q1: Potential at surface of shell with $q = 3\mu C$, $R = 0.2\text{ m}$ is?
 - A) $1.35 \times 10^5\text{ V}$
 - B) $6.75 \times 10^4\text{ V}$
 - C) $9.0 \times 10^4\text{ V}$
 - D) $4.5 \times 10^4\text{ V}$

- Q2: Electric field inside shell is?
 - A) Zero
 - B) Constant non-zero
 - C) Varies with r
 - D) Infinite
- Q3: Potential at $r = 3R$ outside shell with charge $q = 2\mu C$, $R = 0.1\text{ m}$ is?
 - A) $6 \times 10^4\text{ V}$
 - B) $3 \times 10^4\text{ V}$
 - C) $9 \times 10^4\text{ V}$
 - D) $1.5 \times 10^5\text{ V}$

Answer Key

- Q1: A) $1.35 \times 10^5\text{ V}$ - $V = \frac{kq}{R} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{0.2} = 1.35 \times 10^5\text{ V}$
- Q2: A) Zero - Electric field inside a charged shell is zero.
- Q3: B) $3 \times 10^4\text{ V}$ - $V = \frac{kq}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 3 \times 10^4\text{ V}$

Memory Aids

- Outside shell: $V \propto \frac{1}{r}$
- Inside shell: $V = \text{constant}$, $E = 0$
- Potential continuous at surface $r = R$

Electric Potential due to a Uniformly Charged Dielectric Solid Sphere

Charge Distribution and Density

- Total charge: q , radius: R
- Charge density: $\rho = \frac{q}{\frac{4}{3}\pi R^3}$

Potential Outside Sphere ($r > R$)

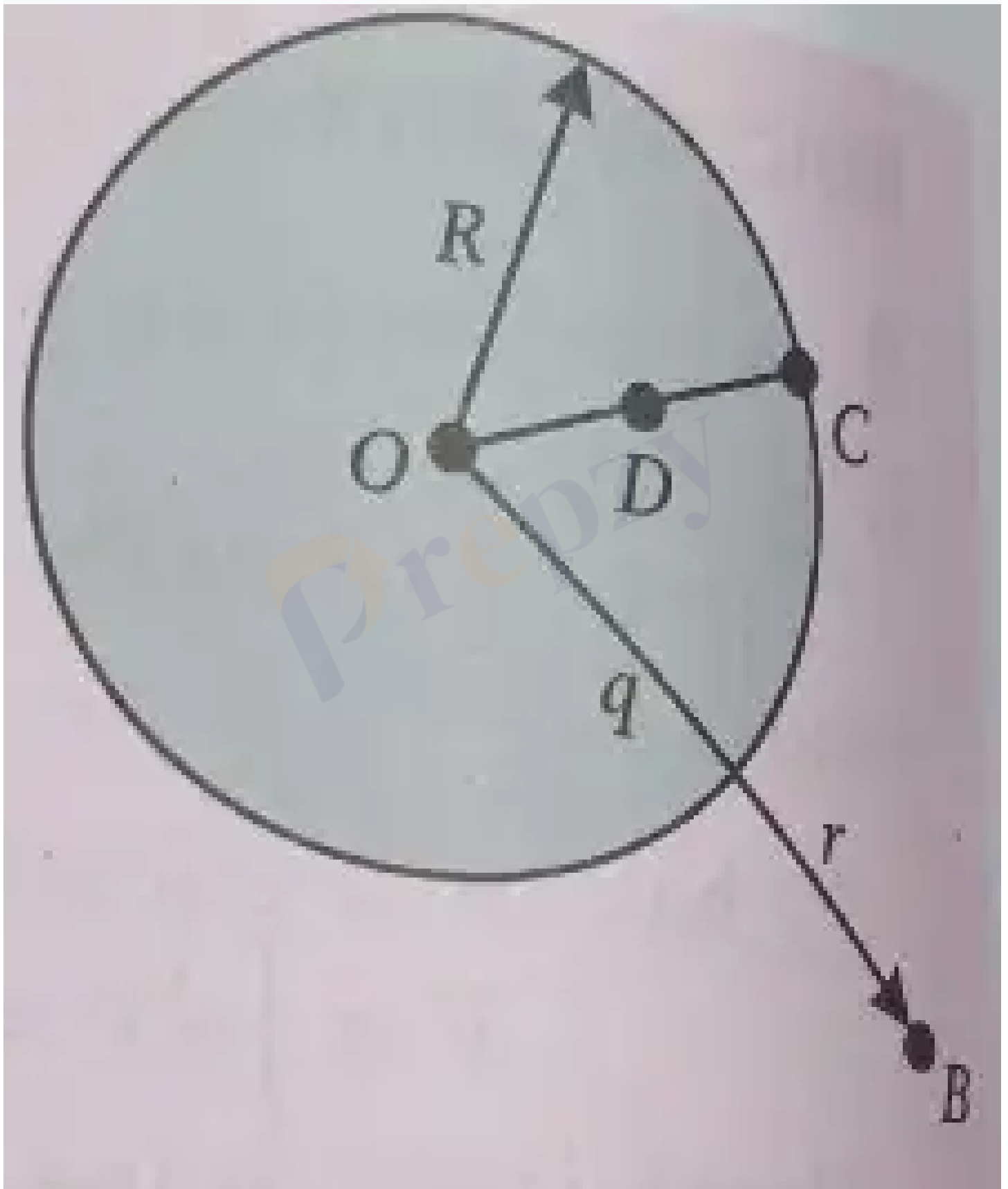
- $V_B = \frac{kq}{r}$

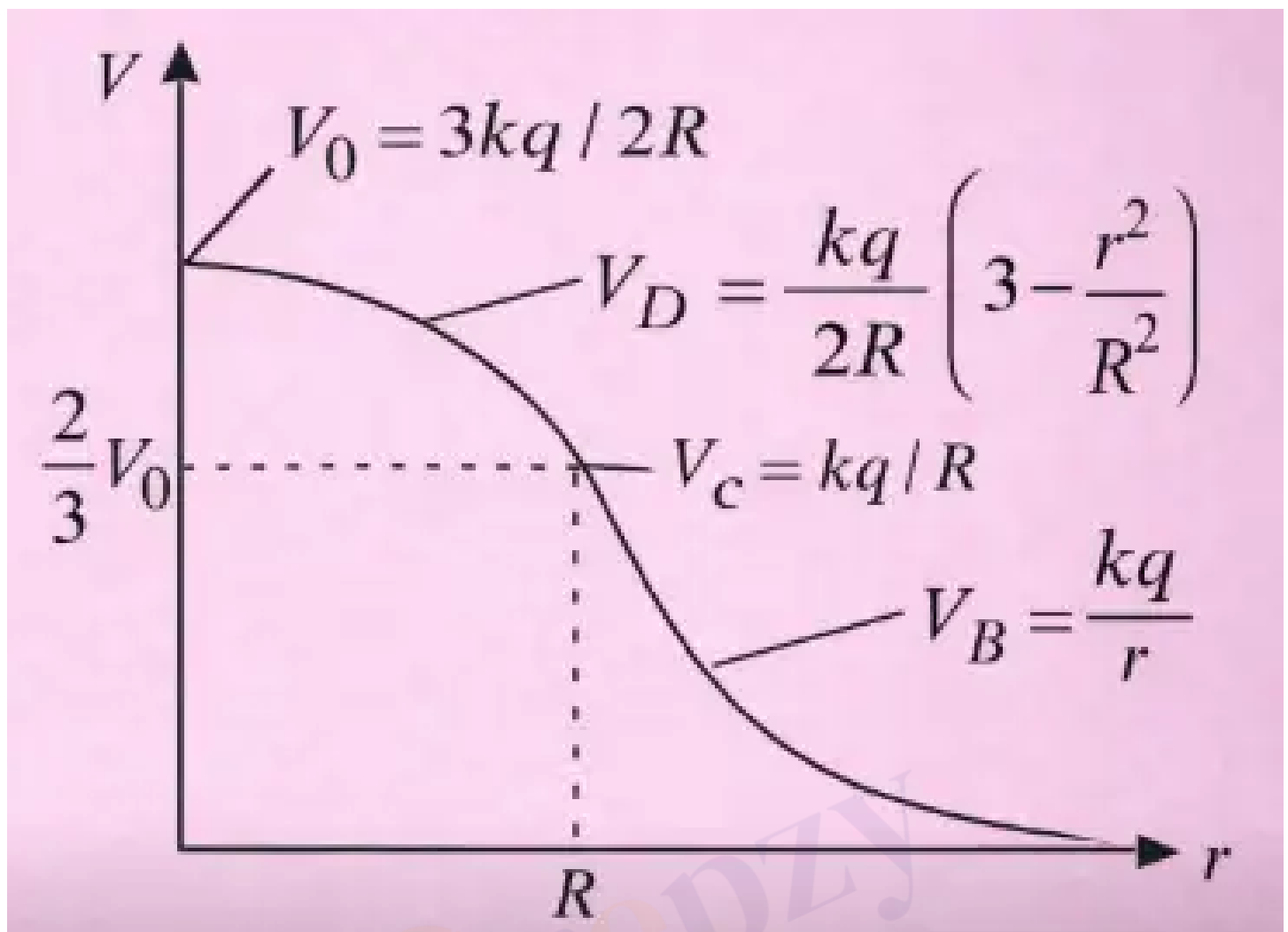
Potential On Surface ($r = R$)

- $V_C = \frac{kq}{R}$

Potential Inside Sphere ($r < R$)

- $V_D = \frac{kq}{2R} \left(3 - \frac{r^2}{R^2} \right)$
- Potential at center $V_0 = \frac{3kq}{2R}$





$$\therefore V_D - V_C = - \int_R^r E dr = - \int_R^r \frac{\rho}{3\epsilon_0} r dr = \frac{\rho}{3\epsilon_0} \left(\frac{R^2 - r^2}{2} \right)$$

$$V_D = \frac{q}{\left(\frac{4\pi}{3} R^3 \right) (3\epsilon_0)} \left(\frac{R^2 - r^2}{2} \right) + \frac{kq}{R} = \left(kq \left(\frac{r^2 - R^2}{R^2} \right) \right)$$

When $r=0$, then potential at centre, $V_0 = \frac{3kq}{2R} V_C$

Solved Examples

- **Ex 1:** Calculate potential at center of sphere with $q = 6\mu C$, $R = 0.1 \text{ m}$
 - $V_0 = \frac{3kq}{2R} = \frac{3 \times 9 \times 10^9 \times 6 \times 10^{-6}}{2 \times 0.1} = 8.1 \times 10^5 \text{ V}$
- **Ex 2:** Potential at $r = 0.05 \text{ m}$ inside sphere

$$\circ V_D = \frac{kq}{2R} \left(3 - \frac{r^2}{R^2} \right) = \frac{9 \times 10^9 \times 6 \times 10^{-6}}{2 \times 0.1} \left(3 - \frac{0.05^2}{0.1^2} \right) = 7.29 \times 10^5 \text{ V}$$

Practice Set

- Q1: Charge density of sphere with $q = 4\mu\text{C}$, $R = 0.2 \text{ m}$ is?
 - A) $1.43 \times 10^{-4} \text{ C/m}^3$
 - B) $2.39 \times 10^{-4} \text{ C/m}^3$
 - C) $3.82 \times 10^{-4} \text{ C/m}^3$
 - D) $4.77 \times 10^{-4} \text{ C/m}^3$
- Q2: Potential at center of sphere with $q = 3\mu\text{C}$, $R = 0.1 \text{ m}$ is?
 - A) $4.05 \times 10^5 \text{ V}$
 - B) $6.75 \times 10^5 \text{ V}$
 - C) $9.0 \times 10^5 \text{ V}$
 - D) $1.35 \times 10^5 \text{ V}$
- Q3: Potential at $r = 0.05 \text{ m}$ inside sphere with $q = 5\mu\text{C}$, $R = 0.1 \text{ m}$ is?
 - A) $3.75 \times 10^5 \text{ V}$
 - B) $4.5 \times 10^5 \text{ V}$
 - C) $5.06 \times 10^5 \text{ V}$
 - D) $6.0 \times 10^5 \text{ V}$

Answer Key

- Q1: A) $1.43 \times 10^{-4} \text{ C/m}^3$ - $\rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{4 \times 10^{-6}}{\frac{4}{3}\pi (0.2)^3} = 1.43 \times 10^{-4} \text{ C/m}^3$
- Q2: B) $6.75 \times 10^5 \text{ V}$ - $V_0 = \frac{3kq}{2R} = \frac{3 \times 9 \times 10^9 \times 3 \times 10^{-6}}{2 \times 0.1} = 6.75 \times 10^5 \text{ V}$
- Q3: C) $5.06 \times 10^5 \text{ V}$ - $V_D = \frac{kq}{2R} \left(3 - \frac{r^2}{R^2} \right) = \frac{9 \times 10^9 \times 5 \times 10^{-6}}{2 \times 0.1} \left(3 - \frac{0.05^2}{0.1^2} \right) = 5.06 \times 10^5 \text{ V}$

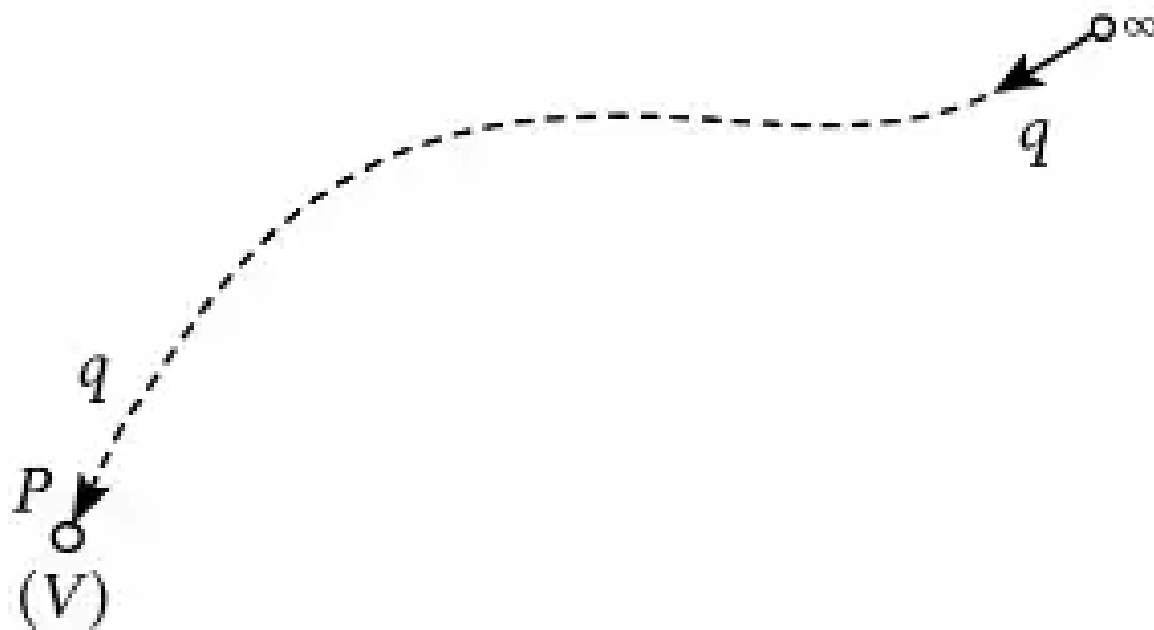
Memory Aids

- Charge density: $\rho = \frac{q}{\frac{4}{3}\pi R^3}$
- Potential inside sphere varies as $V_D = \frac{kq}{2R} \left(3 - \frac{r^2}{R^2} \right)$
- Potential at center is maximum.

Potential Energy of a Single Charge

Concepts

- Potential energy (U) of charge q at point P in external field with potential V : $U = qV$
- Work done by external agent to bring charge from infinity (where potential = 0) to P : $W = q(V - 0) = qV$
- Potential at infinity is zero by definition



Solved Examples

- **Ex 1:** Calculate potential energy of charge $q = 2\mu\text{C}$ at point with potential $V = 5\text{ V}$.
 - Use $U = qV = 2 \times 10^{-6} \times 5 = 10 \times 10^{-6}\text{ J} = 10\mu\text{J}$

Practice Set

- Q1: Potential energy of a charge $3\mu\text{C}$ at a point with potential 10 V is?
 - A) $30\mu\text{J}$
 - B) 3 J
 - C) 0
 - D) $300\mu\text{J}$
- Q2: Work done to bring 1 C charge from infinity to a point at 12 V is?
 - A) 12 J
 - B) 1 J
 - C) 0
 - D) 24 J
- Q3: If potential at point P is zero, potential energy of any charge at P is?
 - A) Zero
 - B) Infinite
 - C) Depends on charge
 - D) Negative

Answer Key

- Q1: A) $30\mu\text{J}$ - $U = qV = 3 \times 10^{-6} \times 10 = 30 \times 10^{-6} \text{ J}$
- Q2: A) 12 J - Work done $W = qV = 1 \times 12 = 12 \text{ J}$
- Q3: A) Zero - Potential energy $U = qV = q \times 0 = 0$

Memory Aids

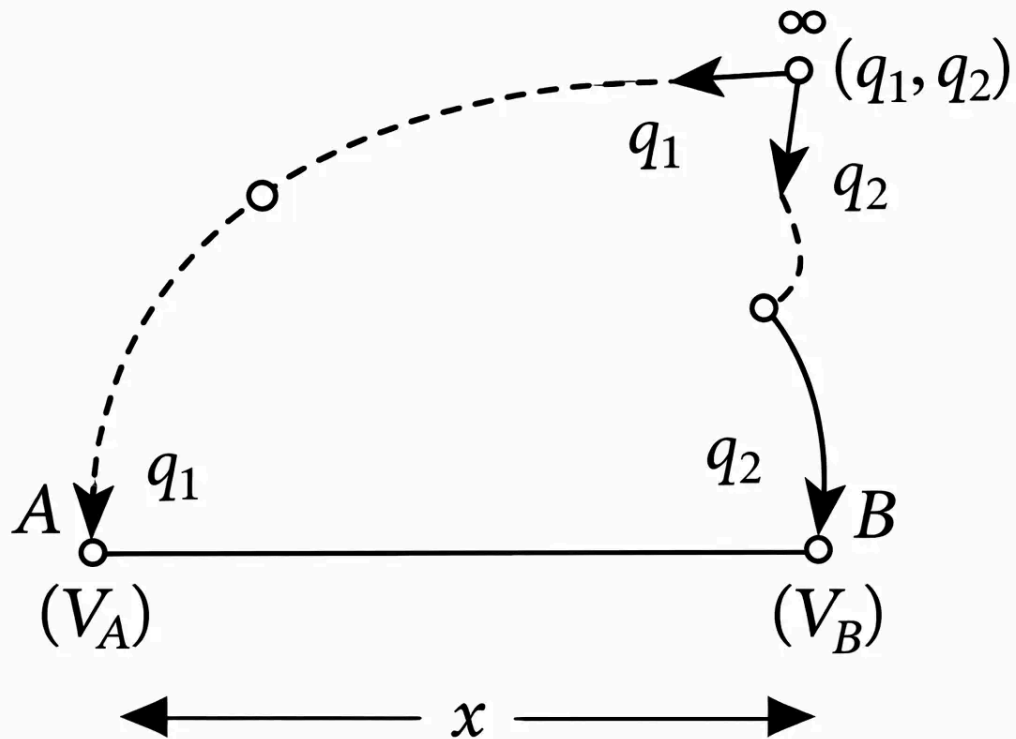
- Potential energy $U = qV$
- Potential at infinity = 0
- Work done = potential energy

Potential Energy of Two Charges

Concepts

- Work done to bring q_1 from infinity to point A with potential V_A : $W_1 = q_1 V_A$
- Potential at B due to q_1 at A: $V_B = \frac{1}{4\pi\epsilon_0} \frac{q_1}{x}$, where $x = AB$
- Work done to bring q_2 from infinity to B: $W_2 = q_2 \left(V_B + \frac{1}{4\pi\epsilon_0} \frac{q_1}{x} \right) = q_2 V_B + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{x}$
- Total work done (potential energy) in assembling two charges in external field:

$$W = W_1 + W_2 = q_1 V_A + q_2 V_B + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{x}$$



Solved Examples

- **Ex 1:** Calculate total potential energy for $q_1 = 2\mu\text{C}$, $q_2 = 3\mu\text{C}$, $V_A = 5\text{ V}$, $V_B = 4\text{ V}$, and $x = 0.1\text{ m}$.
 - Use $W = q_1 V_A + q_2 V_B + \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{x}$
 - Calculate $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$
 - Calculate interaction term: $9 \times 10^9 \times \frac{2 \times 10^{-6} \times 3 \times 10^{-6}}{0.1} = 0.54\text{ J}$
 - Total $W = 2 \times 10^{-6} \times 5 + 3 \times 10^{-6} \times 4 + 0.54 = 0.00001 + 0.000012 + 0.54 \approx 0.54\text{ J}$

Practice Set

- Q1: Work done to bring $q_1 = 1\mu\text{C}$ to point A with $V_A = 3\text{ V}$ is?
 - A) $3\mu\text{J}$
 - B) 3 J
 - C) 0
 - D) $30\mu\text{J}$
- Q2: Potential at B due to $q_1 = 2\mu\text{C}$ at distance $x = 0.2\text{ m}$ is?
 - A) $4.5 \times 10^4\text{ V}$
 - B) $9 \times 10^4\text{ V}$
 - C) $1.8 \times 10^5\text{ V}$
 - D) $9 \times 10^3\text{ V}$
- Q3: Total work done to assemble $q_1 = 1\mu\text{C}$, $q_2 = 1\mu\text{C}$, $V_A = 2\text{ V}$, $V_B = 3\text{ V}$, $x = 0.1\text{ m}$ is?
 - A) 0.09 J

- B) 0.1 J
- C) 0.12 J
- D) 0.15 J

Answer Key

- Q1: A) $3\mu\text{J}$ - $W_1 = q_1 V_A = 1 \times 10^{-6} \times 3 = 3 \times 10^{-6} \text{ J}$
- Q2: D) $9 \times 10^3 \text{ V}$ - $V_B = \frac{1}{4\pi\epsilon_0} \frac{q_1}{x} = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{0.2} = 9 \times 10^3 \text{ V}$
- Q3: C) 0.12 J - Calculate total work using formula, interaction term dominates

Memory Aids

- Total work = sum of individual works + interaction energy
- Potential due to point charge: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
- Use $9 \times 10^9 \text{ Nm}^2/\text{C}^2$ for $\frac{1}{4\pi\epsilon_0}$

Potential Energy of a Dipole

Concepts

- Potential energy of dipole in external field: $U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos \theta$
- θ is angle between dipole moment $\mathbf{p}$ and external electric field $\mathbf{E}$
- Work done to rotate dipole from θ_1 to θ_2 : $W_{\theta_1 \rightarrow \theta_2} = U_{\theta_2} - U_{\theta_1} = pE(\cos \theta_1 - \cos \theta_2)$
- Stable equilibrium at $\theta = 0^\circ$ with minimum potential energy $U = -pE$
- Unstable equilibrium at $\theta = 180^\circ$ with maximum potential energy $U = pE$

Solved Examples

- **Ex 1:** Calculate potential energy of dipole with $p = 2 \times 10^{-29} \text{ C m}$, $E = 10^5 \text{ V/m}$, at $\theta = 60^\circ$.
 - Use $U = -pE \cos \theta = -2 \times 10^{-29} \times 10^5 \times 0.5 = -1 \times 10^{-24} \text{ J}$

Practice Set

- Q1: Potential energy of dipole with $p = 1 \times 10^{-29} \text{ C m}$, $E = 2 \times 10^5 \text{ V/m}$, at $\theta = 0^\circ$ is?
 - A) $-2 \times 10^{-24} \text{ J}$
 - B) $2 \times 10^{-24} \text{ J}$
 - C) 0
 - D) $-1 \times 10^{-24} \text{ J}$
- Q2: Work done to rotate dipole from 0° to 180° with $p = 3 \times 10^{-29} \text{ C m}$, $E = 10^5 \text{ V/m}$ is?
 - A) $6 \times 10^{-24} \text{ J}$

- B) 0
- C) $-6 \times 10^{-24} \text{ J}$
- D) $3 \times 10^{-24} \text{ J}$
- Q3: At which angle is dipole in stable equilibrium?
 - A) 0°
 - B) 90°
 - C) 180°
 - D) 45°

Answer Key

- Q1: A) $-2 \times 10^{-24} \text{ J}$ - $U = -pE \cos 0^\circ = -pE$
- Q2: A) $6 \times 10^{-24} \text{ J}$ - Work done $= pE(\cos 0^\circ - \cos 180^\circ) = pE(1 - (-1)) = 2pE$
- Q3: A) 0° - Minimum potential energy, stable equilibrium

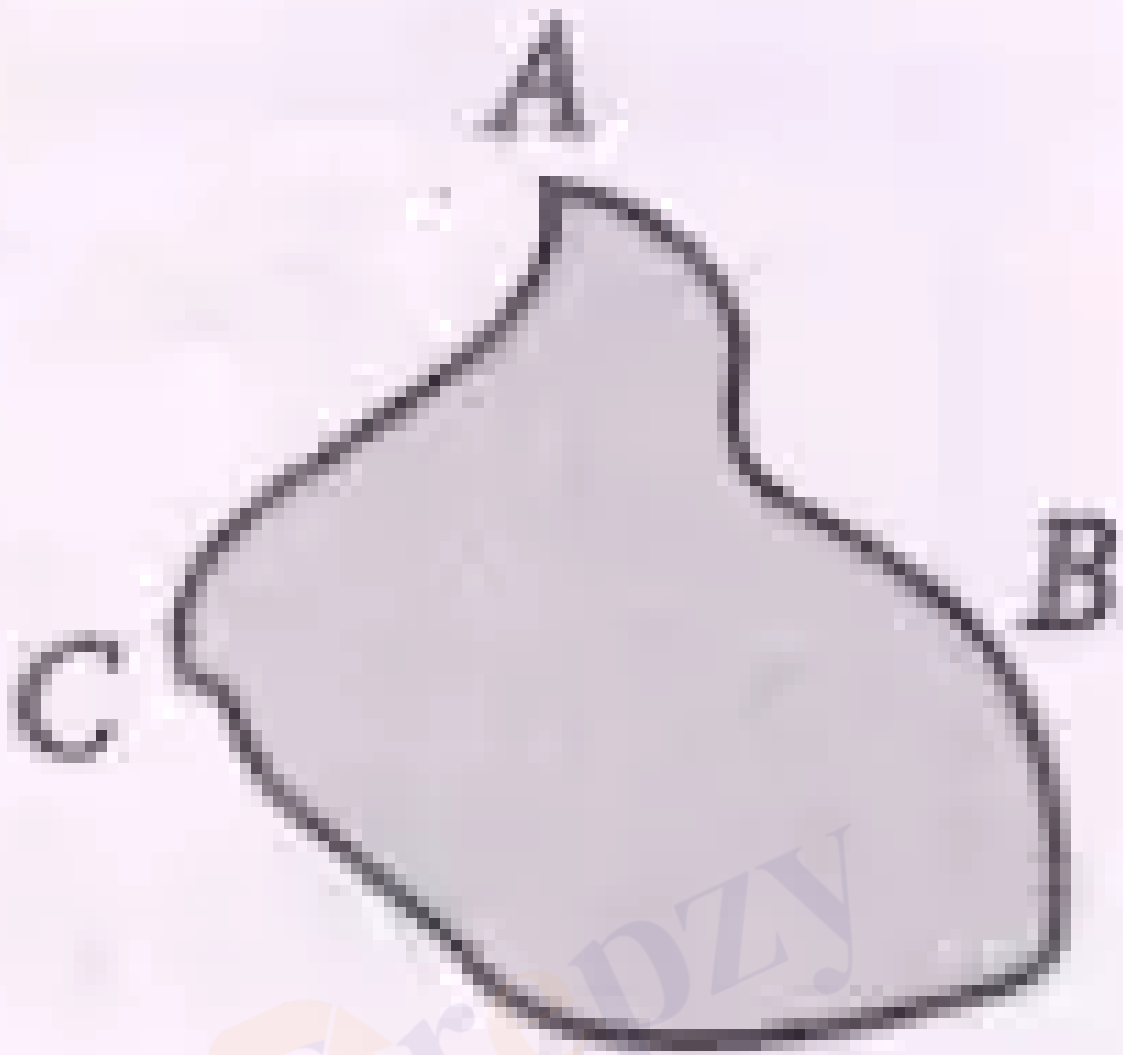
Memory Aids

- Dipole energy: $U = -pE \cos \theta$
- Stable at 0° , unstable at 180°
- Work done = change in potential energy

Electrostatics of Conductors

Properties of Conductors in Electrostatics

- Conductors have many free electrons that move freely inside.
- In electrostatics, conductors are equipotential surfaces (constant potential everywhere on the surface).
- Charge resides only on the outer surface of a conductor.
- Potential inside a charged conducting sphere is constant.
- If a cavity inside a charged conductor has no charge, all charge stays on the outer surface.
- Electric field is always perpendicular to the conductor's surface.
- Electric field near the surface: $\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$, where σ = surface charge density, $\hat{n}$ = outward normal unit vector.



Solved Examples

Ex 1: Calculate electric field intensity near a conductor surface with surface charge density $\sigma = 5 \times 10^{-6} \text{ C/m}^2$.
Given $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.

- Use formula: $E = \frac{\sigma}{\epsilon_0}$
- $E = \frac{5 \times 10^{-6}}{8.85 \times 10^{-12}} = 5.65 \times 10^5 \text{ N/C}$

Practice Set

- Q1: What is true about charge distribution on a charged conductor?
 - A. Charge is inside the conductor
 - B. Charge is uniformly distributed inside
 - C. Charge resides only on the outer surface
 - D. Charge is zero on the surface
- Q2: Electric field near a charged conductor surface depends on?

- A. Volume charge density
- B. Surface charge density
- C. Potential inside conductor
- D. Electric potential energy
- Q3: If surface charge density $\sigma = 1 \times 10^{-6} \text{ C/m}^2$, what is electric field near surface? ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$)
 - A. $1.13 \times 10^5 \text{ N/C}$
 - B. $8.85 \times 10^5 \text{ N/C}$
 - C. $1.0 \times 10^6 \text{ N/C}$
 - D. $1.13 \times 10^6 \text{ N/C}$

Answer Key

- Q1: C - Charge resides only on the outer surface of a conductor in electrostatics.
- Q2: B - Electric field near surface depends on surface charge density σ .
- Q3: A - $E = \frac{\sigma}{\epsilon_0} = \frac{1 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.13 \times 10^5 \text{ N/C}$.

Memory Aids

- Conductors = Equipotential surfaces.
- Charge only on outer surface.
- Electric field $E = \sigma/\epsilon_0$ perpendicular outward.
- Potential inside conductor = constant.

Electrostatic Shielding

Concept and Application

- Electrostatic shielding protects a region from external electric fields.
- A cavity inside a conductor with no internal charge is field-free (electric field = 0).
- Field inside cavity is always zero regardless of external charges.
- Used to protect sensitive instruments by enclosing them in hollow conductors.

Solved Examples

Ex 1: Why is the electric field zero inside a hollow conductor cavity?

- Because charges reside on outer surface, no field penetrates inside.
- Electric field inside cavity = 0 due to electrostatic shielding.

Practice Set

- Q1: What happens to electric field inside a hollow conductor cavity?
 - A. It is maximum
 - B. It is zero
 - C. It depends on external charges
 - D. It varies with time
- Q2: Electrostatic shielding is used to protect?
 - A. Magnetic fields
 - B. Sensitive electronic instruments
 - C. Mechanical parts
 - D. Heat
- Q3: If a cavity inside a conductor has no charge, where does the charge reside?
 - A. Inside cavity
 - B. On inner surface of cavity
 - C. On outer surface of conductor
 - D. Uniformly inside conductor

Answer Key

- Q1: B - Electric field inside hollow cavity is zero due to shielding.
- Q2: B - Shielding protects sensitive electronic instruments from electric fields.
- Q3: C - Charge resides only on outer surface if cavity has no charge.

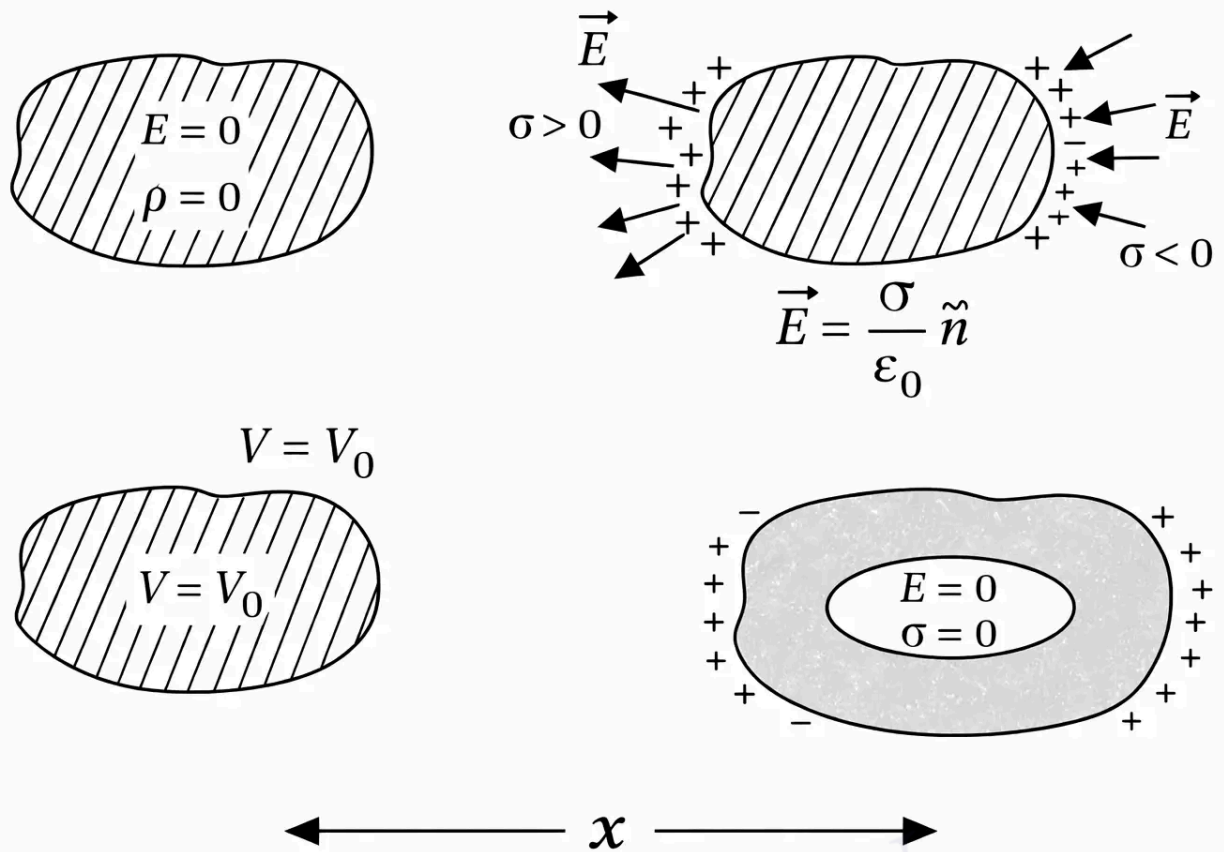
Memory Aids

- Shielding = zero field inside cavity.
- Charge stays outside cavity.
- Protect instruments by hollow conductor enclosure.

Summary of Electrostatic Properties

Key Points

- Inside neutral conductor: Electric field $E = 0$, charge density $\rho = 0$.
- On charged surface: Electric field $E = \frac{\sigma}{\epsilon_0} \hat{n}$, points outward for positive charge.
- Inside charged conductor: Potential $V = V_0$ constant.
- Inside hollow conductor: Electric field and surface charge density are zero; charges on outer surface.



Solved Examples

Ex 1: State electric field inside a neutral conductor.

- Electric field inside neutral conductor is zero.

Practice Set

- Q1: Electric field inside a neutral conductor is?
 - A. Zero
 - B. Maximum
 - C. Depends on charge
 - D. Infinite
- Q2: Potential inside a charged conductor is?
 - A. Zero
 - B. Constant
 - C. Varies with position
 - D. Negative
- Q3: Where do charges reside in a hollow conductor?
 - A. Inside cavity
 - B. On inner surface
 - C. On outer surface

- D. Uniformly inside

Answer Key

- Q1: A - Electric field inside neutral conductor is zero.
- Q2: B - Potential inside charged conductor is constant.
- Q3: C - Charges reside on outer surface of hollow conductor.

Memory Aids

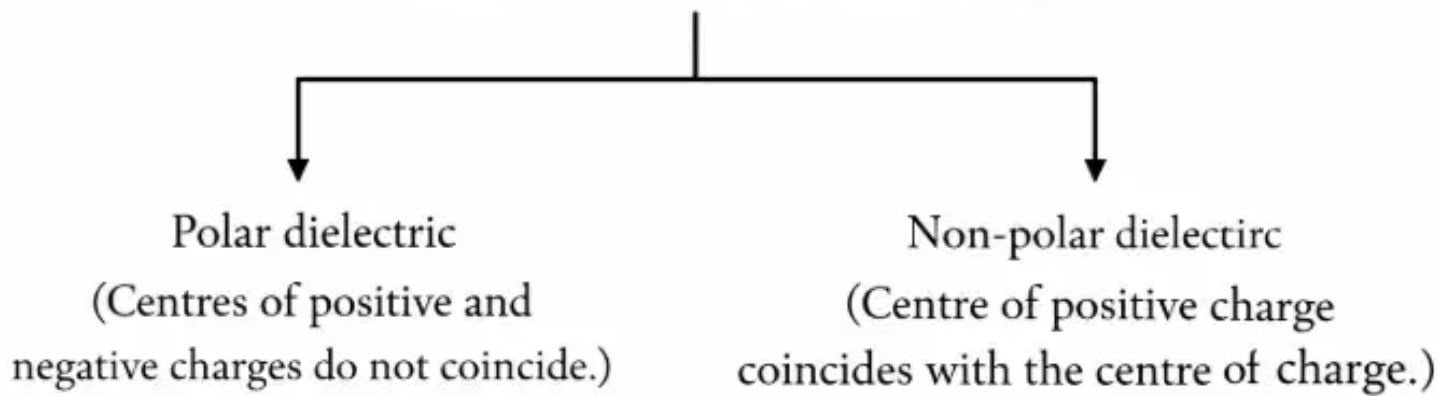
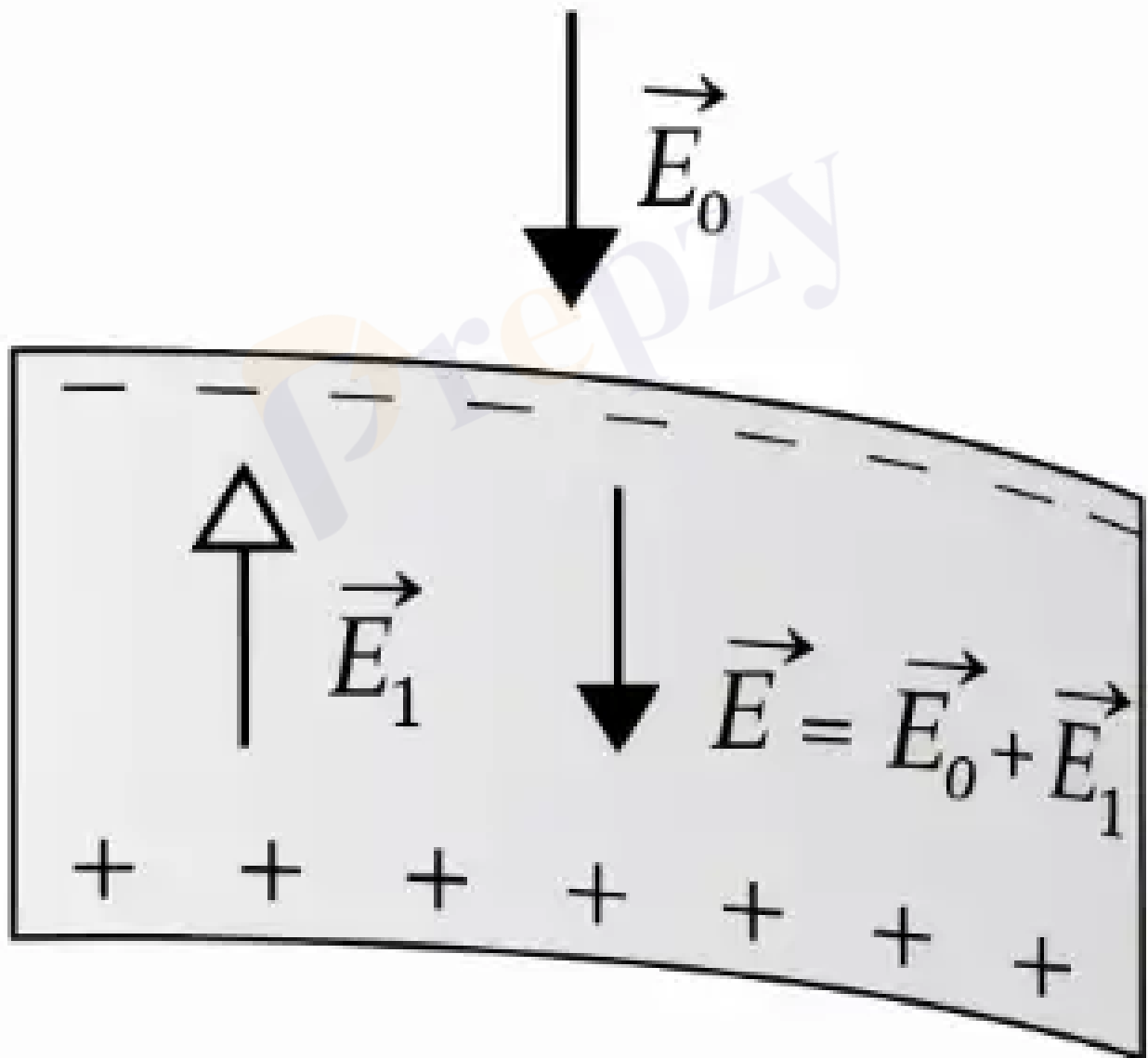
- Neutral conductor: $E = 0, \rho = 0$.
- Charged surface: $E = \frac{\sigma}{\epsilon_0} \hat{n}$.
- Inside conductor: Potential constant.
- Hollow conductor: Charges outside, field zero inside.

Dielectrics and Polarisation

Types of Dielectrics

- Dielectrics are insulating materials that support electric fields but do not conduct electricity.
- Polar dielectrics: positive and negative charges are separated; centers do not coincide.
- Non-polar dielectrics: positive and negative charges overlap; centers coincide.
- Polarisation occurs when dipoles align under an external electric field.
- Polarisation field $\vec{E}_i$ is induced inside the dielectric opposing the external field $\vec{E}_0$.
- Net electric field inside dielectric: $\vec{E} = \vec{E}_0 + \vec{E}_i$.
- Polarisation vector $\vec{P}$ = dipole moment per unit volume.
- For linear isotropic dielectrics: $\vec{P} = \chi_e \vec{E}_i$, where χ_e is electrical susceptibility.

Dielectric (Insulating materials)



Solved Examples

- **Ex 1:** Calculate induced electric field E_i if $E_0 = 1000 \text{ V/m}$ and relative permittivity $\epsilon_r = 5$.
 - Use $E_i = \left(1 - \frac{1}{\epsilon_r}\right) E_0 = \left(1 - \frac{1}{5}\right) \times 1000 = 800 \text{ V/m}$.
- **Ex 2:** Find induced charge q_i if free charge $q = 10 \mu\text{C}$ and $\epsilon_r = 4$.
 - Use $q_i = q \left(1 - \frac{1}{\epsilon_r}\right) = 10 \times \left(1 - \frac{1}{4}\right) = 7.5 \mu\text{C}$.

Practice Set

- Q1: What is the polarisation vector $\vec{P}$ in a dielectric?
 - A. Electric field inside dielectric
 - B. Dipole moment per unit volume
 - C. Induced charge density
 - D. External electric field
- Q2: If $\epsilon_r = 3$ and $E_0 = 600 \text{ V/m}$, what is E_i ?
 - A. 200 V/m
 - B. 400 V/m
 - C. 600 V/m
 - D. 800 V/m
- Q3: For a dielectric with $\chi_e = 2$ and $E_i = 500 \text{ V/m}$, find P .
 - A. 1000 C/m³
 - B. 250 C/m³
 - C. 500 C/m³
 - D. 2000 C/m³

Answer Key

- Q1: B - Polarisation $\vec{P}$ is the dipole moment per unit volume.
- Q2: B - $E_i = \left(1 - 1/3\right) \times 600 = 400 \text{ V/m}$.
- Q3: A - $P = \chi_e E_i = 2 \times 500 = 1000 \text{ C/m}^3$.

Memory Aids

- Polarisation $\vec{P}$ = dipole moment density.
- Induced field $E_i = \left(1 - 1/\epsilon_r\right) E_0$.
- Net field inside dielectric is reduced by E_i .

Induced Electric Charge and Net Field

Key Concepts

- Net electric field inside dielectric: $E = E_0 - E_i$.
- Relation: $E_0/\epsilon_r = E_0 - E_i$.
- Induced field: $E_i = \left(1 - \frac{1}{\epsilon_r}\right)E_0$.
- Induced surface charge density: $\sigma_i = \left(1 - \frac{1}{\epsilon_r}\right)\sigma$.
- Induced charge: $q_i = q\left(1 - \frac{1}{\epsilon_r}\right)$.

Solved Examples

- **Ex 1:** Calculate induced charge q_i if $q = 20 \mu C$ and $\epsilon_r = 5$.
 - $q_i = 20 \times (1 - 1/5) = 16 \mu C$.
- **Ex 2:** Find net field E if $E_0 = 1000 \text{ V/m}$ and $\epsilon_r = 4$.
 - $E = E_0 - E_i = E_0 - (1 - 1/4)E_0 = E_0/4 = 250 \text{ V/m}$.

Practice Set

- Q1: If $\epsilon_r = 2$ and $q = 10 \mu C$, find induced charge q_i .
 - A. $5 \mu C$
 - B. $10 \mu C$
 - C. $15 \mu C$
 - D. $20 \mu C$
- Q2: Net electric field E inside dielectric is related to E_0 by?
 - A. $E = E_0 + E_i$
 - B. $E = E_0 - E_i$
 - C. $E = E_i - E_0$
 - D. $E = E_0/\epsilon_r$
- Q3: For $\epsilon_r = 3$ and $E_0 = 900 \text{ V/m}$, find E_i .
 - A. 300 V/m
 - B. 600 V/m
 - C. 900 V/m
 - D. 1200 V/m

Answer Key

- Q1: A - $q_i = q(1 - 1/2) = 5 \mu C$.
- Q2: B - Net field $E = E_0 - E_i$.
- Q3: B - $E_i = (1 - 1/3) \times 900 = 600 \text{ V/m}$.

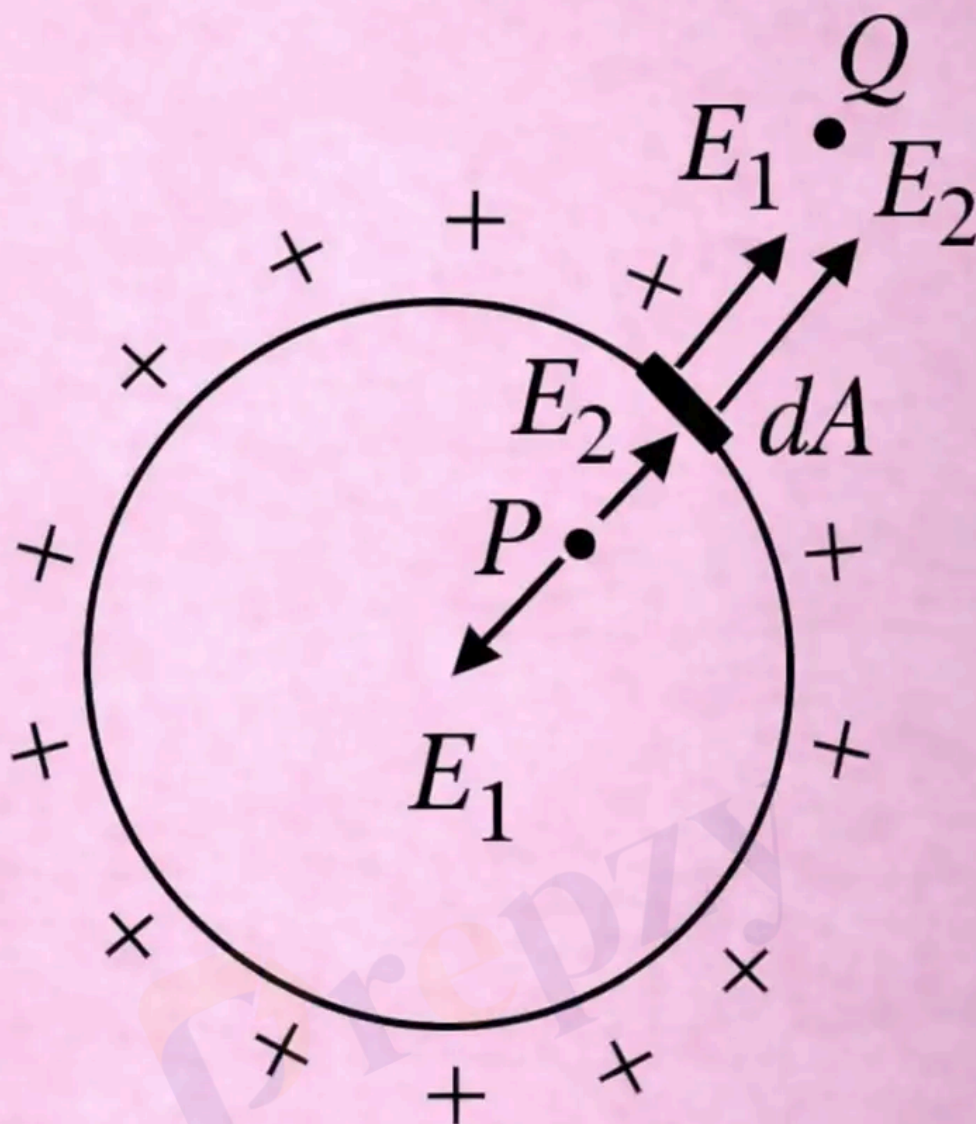
Memory Aids

- Induced charge $q_i = q(1 - 1/\epsilon_r)$.
- Net field reduced by induced field inside dielectric.
- Use $E = E_0 - E_i$ for net field.

Electrostatic Pressure on Charged Conductor

Key Points

- Force on surface element dA : $dF = (\sigma dA)E_2 = \frac{\sigma^2 dA}{2\epsilon_0}$.
- Electrostatic pressure $P = \frac{dF}{dA} = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2$ using $E = \frac{\sigma}{\epsilon_0}$.
- In presence of dielectric, pressure $P = \frac{1}{2}\epsilon_0 K E^2$, where K is dielectric constant.



Solved Examples

- **Ex 1:** Calculate electrostatic pressure if $\sigma = 2 \times 10^{-6} \text{ C/m}^2$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.
 - $P = \frac{\sigma^2}{2\epsilon_0} = \frac{(2 \times 10^{-6})^2}{2 \times 8.85 \times 10^{-12}} = 226 \text{ N/m}^2$.
- **Ex 2:** Find pressure in dielectric with $K = 4$, $E = 10^4 \text{ V/m}$, and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.
 - $P = \frac{1}{2} \epsilon_0 K E^2 = 0.5 \times 8.85 \times 10^{-12} \times 4 \times (10^4)^2 = 1.77 \text{ N/m}^2$.

Practice Set

- Q1: Electrostatic pressure on charged conductor surface is given by?
 - A. $\frac{\sigma}{\epsilon_0}$

- B. $\frac{\sigma^2}{2\epsilon_0}$
- C. $\frac{\sigma^2}{\epsilon_0}$
- D. $\frac{\sigma}{2\epsilon_0}$
- Q2: If $E = 5 \times 10^3 \text{ V/m}$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$, find pressure.
 - A. 1.1 N/m^2
 - B. 0.5 N/m^2
 - C. 0.25 N/m^2
 - D. 2.2 N/m^2
- Q3: In dielectric with dielectric constant $K = 3$, pressure is?
 - A. $\frac{1}{2}\epsilon_0 E^2$
 - B. $\epsilon_0 E^2$
 - C. $\frac{3}{2}\epsilon_0 E^2$
 - D. $\frac{1}{6}\epsilon_0 E^2$

Answer Key

- Q1: B - Pressure = $\frac{\sigma^2}{2\epsilon_0}$ from surface charge and field.
- Q2: A - $P = \frac{1}{2}\epsilon_0 E^2 = 0.5 \times 8.85 \times 10^{-12} \times (5 \times 10^3)^2 = 1.1 \text{ N/m}^2$.
- Q3: C - Pressure in dielectric = $\frac{1}{2}\epsilon_0 K E^2 = \frac{3}{2}\epsilon_0 E^2$.

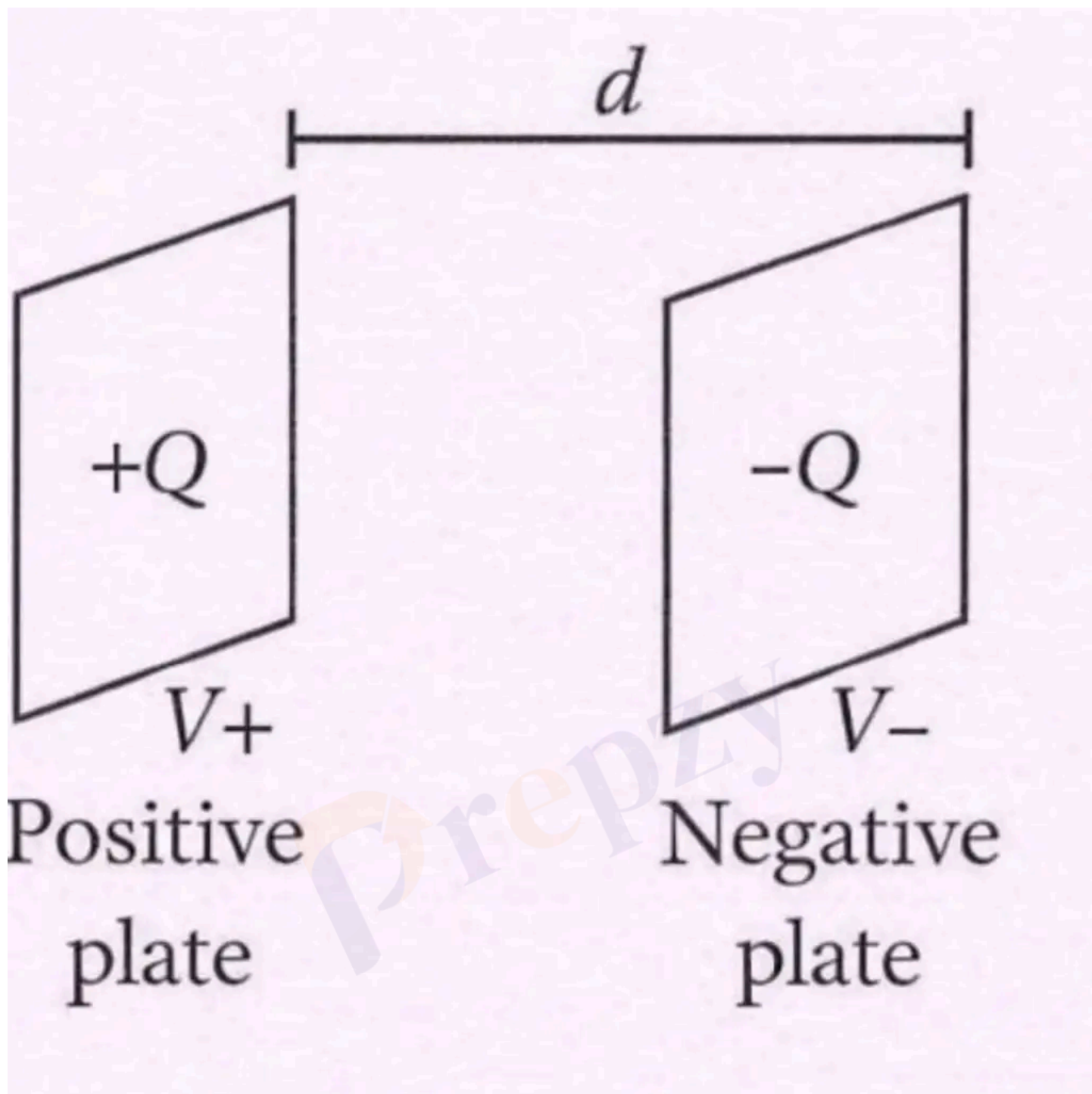
Memory Aids

- Pressure = $\frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2$.
- Dielectric increases pressure by factor K .
- Use $E = \frac{\sigma}{\epsilon_0}$ to relate field and charge density.

The Parallel Plate Capacitor

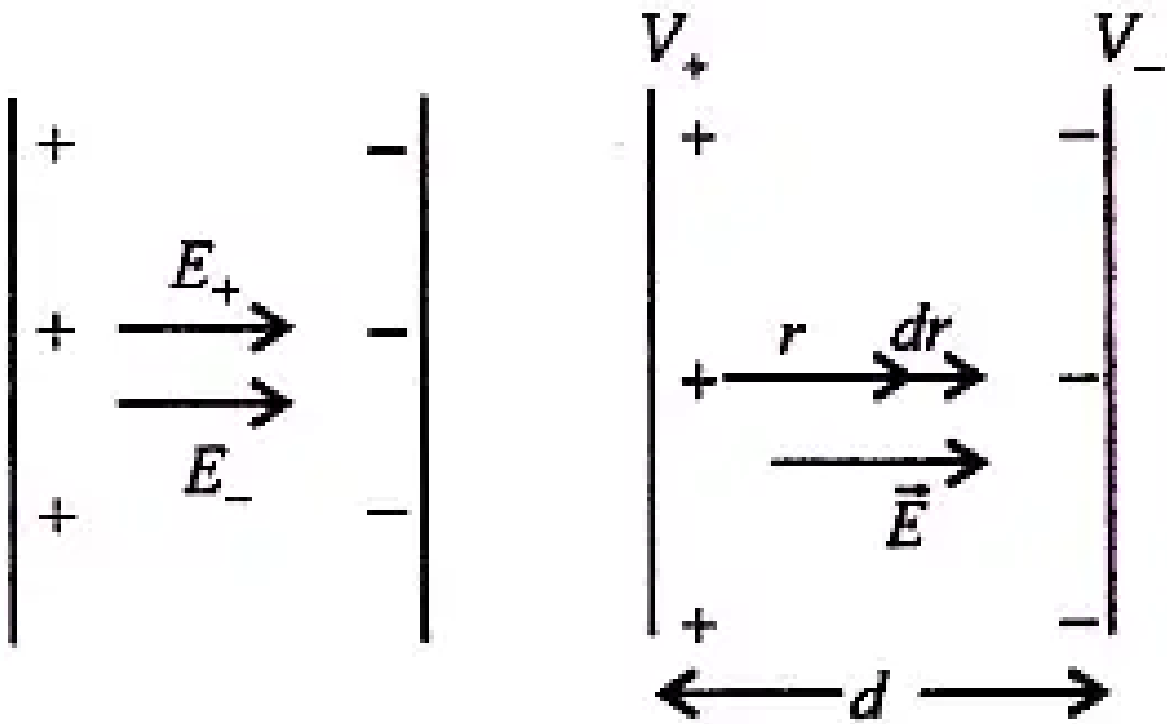
Basic Concept

- A capacitor consists of two conductors (plates) placed close but not touching.
- Plates are parallel with area A and separated by distance d .
- One plate carries positive charge $+Q$, the other negative charge $-Q$.
- Voltage difference between plates is $V = V_+ - V_-$.
- Capacitance C relates charge and voltage: $Q = CV$.
- Electric field E is uniform between plates if plate size $\gg d$, ignoring fringing.



Electric Field and Potential Difference

- Electric field between plates: $E = \frac{Q}{A\epsilon_0}$, where ϵ_0 is permittivity of free space.
- Potential difference: $V = Ed = \frac{Qd}{A\epsilon_0}$.
- Capacitance formula: $C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$.
- Fringing effect causes field lines to spread near edges but is usually neglected.



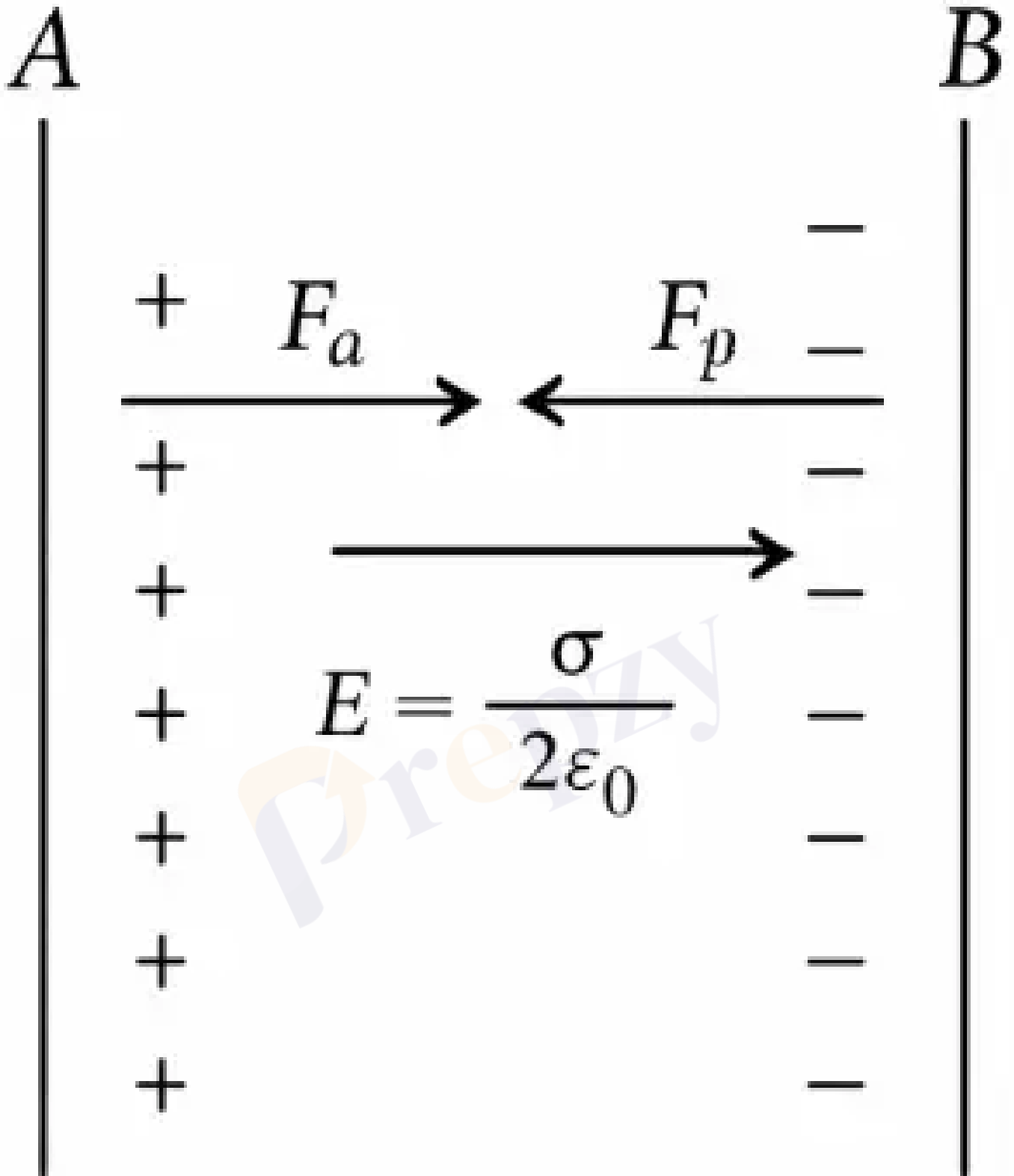
$$dV = -\vec{E} \cdot d\vec{r} = -E dr \cos 0^\circ = -E dr$$

$$\int_{V_+}^{V_-} dV = -E \int_0^d dr; V_- - V_+ = -E d$$

$$\therefore V = E d, V = \frac{Qd}{\epsilon_0 A} \text{ and capacitance, } C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

Force Between Plates

- Plates exert attractive force due to opposite charges.
- Electric field from one plate acts on charge of the other.
- Electric field due to one plate: $E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0}$, where σ is charge density.
- Force on a plate: $F = QE = \frac{Q^2}{2A\epsilon_0}$.



Solved Examples

- **Ex 1:** Calculate capacitance of plates with area 0.01 m^2 , separation 0.001 m , $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.
 - Use $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.01}{0.001} = 8.85 \times 10^{-11} \text{ F}$.
- **Ex 2:** Find force between plates if $Q = 1 \mu\text{C}$, $A = 0.01 \text{ m}^2$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.
 - Calculate $F = \frac{Q^2}{2A\epsilon_0} = \frac{(1 \times 10^{-6})^2}{2 \times 0.01 \times 8.85 \times 10^{-12}} = 5.65 \text{ N}$.

Practice Set

- **Q1:** What is the capacitance of a parallel plate capacitor with plate area $2 \times 10^{-4} \text{ m}^2$ and plate separation 0.5 mm ?
 - A) $3.54 \times 10^{-12} \text{ F}$
 - B) $7.08 \times 10^{-12} \text{ F}$
 - C) $1.77 \times 10^{-11} \text{ F}$
 - D) $8.85 \times 10^{-12} \text{ F}$
- **Q2:** If charge on the positive plate is $2 \mu\text{C}$ and plate area is 0.02 m^2 , what is the electric field between plates?
 - A) $1.13 \times 10^5 \text{ V/m}$
 - B) $5.65 \times 10^4 \text{ V/m}$
 - C) $2.26 \times 10^5 \text{ V/m}$
 - D) $4.52 \times 10^4 \text{ V/m}$
- **Q3:** Force between plates with charge $3 \mu\text{C}$, area 0.01 m^2 , and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ is?
 - A) 5.1 N
 - B) 15.3 N
 - C) 25.5 N
 - D) 45.9 N

Answer Key

- **Q1:** B. $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-4}}{0.5 \times 10^{-3}} = 7.08 \times 10^{-12} \text{ F}.$
- **Q2:** A. $E = \frac{Q}{A\epsilon_0} = \frac{2 \times 10^{-6}}{0.02 \times 8.85 \times 10^{-12}} = 1.13 \times 10^5 \text{ V/m}.$
- **Q3:** B. $F = \frac{Q^2}{2A\epsilon_0} = \frac{(3 \times 10^{-6})^2}{2 \times 0.01 \times 8.85 \times 10^{-12}} = 15.3 \text{ N}.$

Memory Aids

- Capacitance C increases with plate area A and decreases with distance d .
- Electric field $E = \frac{Q}{A\epsilon_0}$ between plates.
- Force between plates $F = \frac{Q^2}{2A\epsilon_0}$ is attractive.
- Ignore fringing effect for quick calculations.

Effect of Dielectric on Capacitance

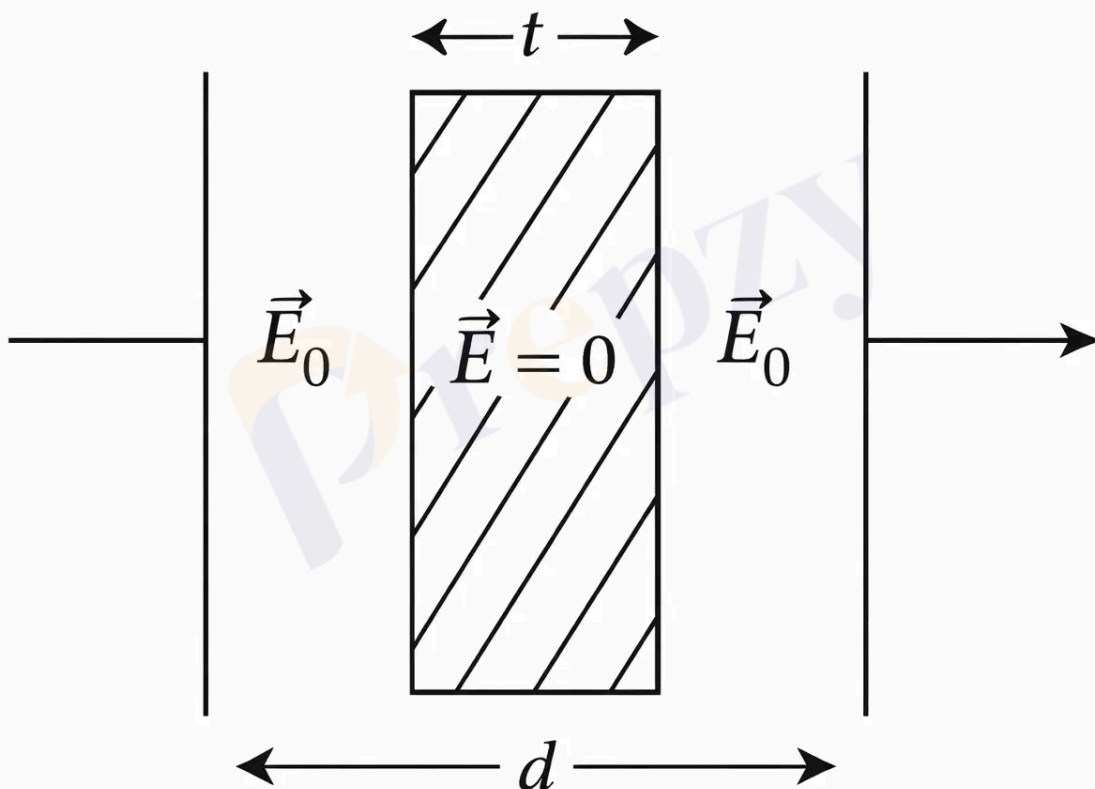
Basic Concepts

- Vacuum between plates: Electric field $E_0 = \frac{\sigma}{\epsilon_0}$, potential difference $V_0 = E_0 d$
- Capacitance without dielectric: $C_0 = \frac{Q}{V_0} = \frac{\epsilon_0 A}{d}$
- Dielectric inserted fully between plates polarizes, creating surface charges σ_P and $-\sigma_P$
- Electric field inside dielectric: $E = \frac{\sigma - \sigma_P}{\epsilon_0}$
- Potential difference with dielectric: $V = Ed = \frac{\sigma - \sigma_P}{\epsilon_0} d$

- For linear dielectrics, σ_P proportional to σ , so $\sigma - \sigma_P = \frac{\sigma}{K}$, where $K > 1$
- Potential difference: $V = \frac{\sigma d}{\epsilon_0 K} = \frac{Qd}{A\epsilon_0 K}$
- Capacitance with dielectric: $C = \frac{Q}{V} = \frac{\epsilon_0 K A}{d}$
- Permittivity of medium: $\epsilon = \epsilon_0 K$
- Dielectric constant: $K = \frac{\epsilon}{\epsilon_0} = \frac{C}{C_0}$

Effect of Conducting Slab

- Conducting slab thickness $t < d$ inserted between plates
- Electric field inside conductor is zero ($E = 0$)
- Potential difference reduces: $V = E_0(d - t) = \frac{Q}{A\epsilon_0}(d - t)$
- Capacitance: $C = \frac{\epsilon_0 A}{d - t} = \frac{C_0}{1 - \frac{t}{d}}$



Capacitance of Partially Filled Capacitor

- Dielectric thickness f partially fills the gap
- Capacitance: $C = \frac{K\epsilon_0 A}{K(d - t) + t}$

Solved Examples

- **Ex 1:** Calculate capacitance with dielectric $K = 5$, plate area $A = 0.01 \text{ m}^2$, plate separation $d = 1 \text{ mm}$, and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$.
 - Use $C = \frac{\epsilon_0 K A}{d}$
 - $C = \frac{8.85 \times 10^{-12} \times 5 \times 0.01}{1 \times 10^{-3}} = 4.425 \times 10^{-10} \text{ F}$
- **Ex 2:** Conducting slab thickness $t = 0.2 \text{ mm}$ inserted in capacitor with $d = 1 \text{ mm}$, $A = 0.01 \text{ m}^2$, find new capacitance.
 - $C = \frac{\epsilon_0 A}{d-t} = \frac{8.85 \times 10^{-12} \times 0.01}{1 \times 10^{-3} - 0.2 \times 10^{-3}} = 1.106 \times 10^{-10} \text{ F}$

Practice Set

- **Q1:** What is the dielectric constant K of vacuum?
 - A. 0
 - B. 1
 - C. ϵ_0
 - D. Infinity
- **Q2:** Capacitance of a capacitor with dielectric $K = 4$, plate area 2 m^2 , plate separation 0.5 mm , and $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ is?
 - A. $1.42 \times 10^{-7} \text{ F}$
 - B. $7.08 \times 10^{-9} \text{ F}$
 - C. $7.08 \times 10^{-7} \text{ F}$
 - D. $1.42 \times 10^{-8} \text{ F}$
- **Q3:** A conducting slab of thickness 0.1 mm is inserted in a capacitor with plate separation 1 mm and capacitance without slab 10 pF . New capacitance is?
 - A. 11.1 pF
 - B. 9.0 pF
 - C. 10.1 pF
 - D. 8.9 pF

Answer Key

- **Q1:** B. 1 — Vacuum has dielectric constant 1 by definition.
- **Q2:** C. $7.08 \times 10^{-7} \text{ F}$ — Use $C = \frac{\epsilon_0 K A}{d}$.
- **Q3:** A. 11.1 pF — New capacitance $C = \frac{C_0}{1 - \frac{t}{d}} = \frac{10}{1 - 0.1} = 11.1 \text{ pF}$.

Memory Aids

- Dielectric constant $K = \frac{C}{C_0}$
- Capacitance with dielectric $C = \epsilon_0 K \frac{A}{d}$
- Conducting slab reduces effective distance: $d - t$

Combination of Capacitors

Capacitors in Series

In this arrangement of capacitors, charge has no alternative path(s) to flow.

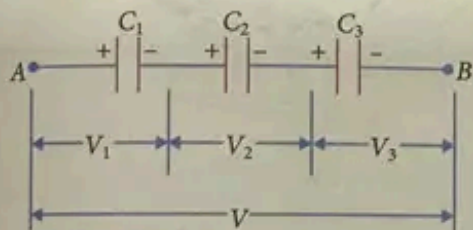
- The charge on each capacitor is equal

$$\text{i.e., } Q = C_1 V_1 = C_2 V_2 = C_3 V_3$$

- The total potential difference across AB is shared by the capacitors in the inverse ratio of their respective capacitance $V = V_1 + V_2 + V_3$

If C_s is the net capacitance of the series combination, then

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Capacitors in Parallel

In such an arrangement of capacitors, charge has an alternative path(s) to flow.

- The potential difference across each capacitor is same and equal to the total potential difference applied.

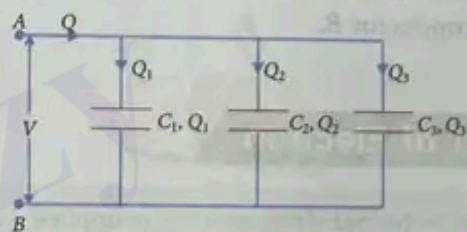
$$\text{i.e., } V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$$

- The total charge Q is shared by the capacitor in the direct ratio of their respective capacitances.

$$Q = Q_1 + Q_2 + Q_3$$

If C_p is the net capacitance for the parallel combination of capacitors, then :

$$C_p V = C_1 V + C_2 V + C_3 V \Rightarrow C_p = C_1 + C_2 + C_3$$



Capacitors in Series

- Connected end-to-end; charge path is single.
- Charge Q on each capacitor is the same.
- Total voltage V is sum of voltages across capacitors: $V = V_1 + V_2 + V_3 + \dots$
- Equivalent capacitance C_s given by:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

- Voltage divides inversely with capacitance: smaller capacitance gets higher voltage.
- Equivalent capacitance is always less than the smallest individual capacitance.
- For n identical capacitors each of capacitance C , $C_s = \frac{C}{n}$.

Capacitors in Parallel

- Connected side-by-side; multiple charge paths.
- Voltage V across each capacitor is the same.
- Total charge Q is sum of individual charges: $Q = Q_1 + Q_2 + Q_3 + \dots$
- Equivalent capacitance C_p is sum of capacitances:

$$C_p = C_1 + C_2 + C_3 + \dots$$

- Charge divides proportional to capacitance values.

Solved Examples

Ex 1

- Two capacitors $C_1 = 4\mu F$ and $C_2 = 6\mu F$ in series. Find C_s .
- Use formula: $\frac{1}{C_s} = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} \Rightarrow C_s = \frac{12}{5} = 2.4\mu F$.

Ex 2

- Two capacitors $C_1 = 3\mu F$ and $C_2 = 5\mu F$ in parallel. Find C_p .
- Use formula: $C_p = 3 + 5 = 8\mu F$.

Practice Set

Q1

- Two capacitors $2\mu F$ and $3\mu F$ are connected in series. What is the equivalent capacitance?
- A) $5\mu F$
- B) $1.2\mu F$
- C) $6\mu F$
- D) $0.6\mu F$

Q2

- Three capacitors $1\mu F$, $2\mu F$, and $3\mu F$ are connected in parallel. Find the total capacitance.
- A) $6\mu F$
- B) $0.6\mu F$
- C) $1.8\mu F$
- D) $3\mu F$

Q3

- Four identical capacitors each of $4\mu F$ are connected in series. What is the equivalent capacitance?
- A) $16\mu F$
- B) $1\mu F$
- C) $4\mu F$
- D) $8\mu F$

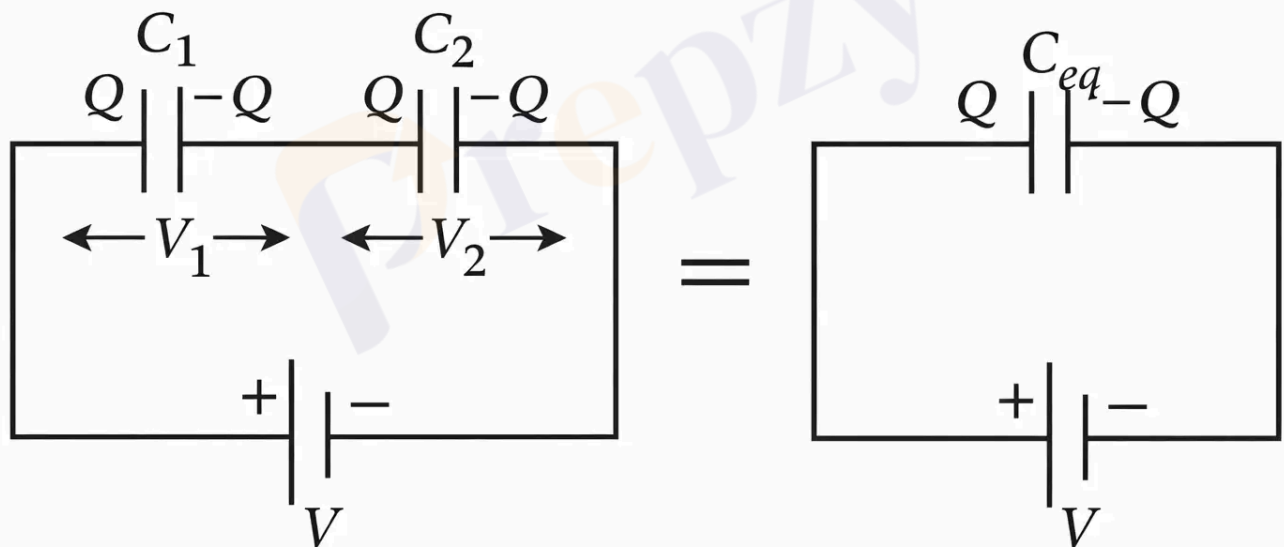
Answer Key

- Q1: B) $1.2 \mu F$ - Series formula $\frac{1}{C_s} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \Rightarrow C_s = 1.2 \mu F$.
- Q2: A) $6 \mu F$ - Parallel sum $C_p = 1 + 2 + 3 = 6 \mu F$.
- Q3: B) $1 \mu F$ - Series identical capacitors $C_s = \frac{C}{n} = \frac{4}{4} = 1 \mu F$.

Memory Aids

- Series: $\frac{1}{C_s} = \sum \frac{1}{C_i}$, less than smallest capacitor.
- Parallel: $C_p = \sum C_i$, sum of all capacitances.
- Charge same in series, voltage same in parallel.

Series Combination



Key Points

- Capacitors connected one after another.
- Charge Q on each capacitor is the same.
- Total voltage $V = V_1 + V_2 + \dots$
- Voltage across each capacitor: $V_i = \frac{Q}{C_i}$

- Equivalent capacitance for two capacitors:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

- General formula for n capacitors:

$$\frac{1}{C_{eq}} = \sum_{i=1}^n \frac{1}{C_i}$$

- For identical capacitors: $C_{eq} = \frac{C}{n}$
- Equivalent capacitance is always less than the smallest capacitor.

Solved Examples

Ex 1

- Two capacitors $5\mu F$ and $10\mu F$ in series. Find C_{eq} .
- $\frac{1}{C_{eq}} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10} \Rightarrow C_{eq} = \frac{10}{3} = 3.33\mu F$.

Practice Set

Q1

- Two capacitors $3\mu F$ and $6\mu F$ in series. Find C_{eq} .
- A) $9\mu F$
- B) $2\mu F$
- C) $1.5\mu F$
- D) $18\mu F$

Q2

- Three capacitors $2\mu F$, $4\mu F$, and $6\mu F$ in series. Find C_{eq} .
- A) $1\mu F$
- B) $12\mu F$
- C) $3\mu F$
- D) $0.5\mu F$

Q3

- Four identical capacitors each $8\mu F$ in series. Find C_{eq} .
- A) $32\mu F$
- B) $2\mu F$

- C) $8 \mu F$
- D) $4 \mu F$

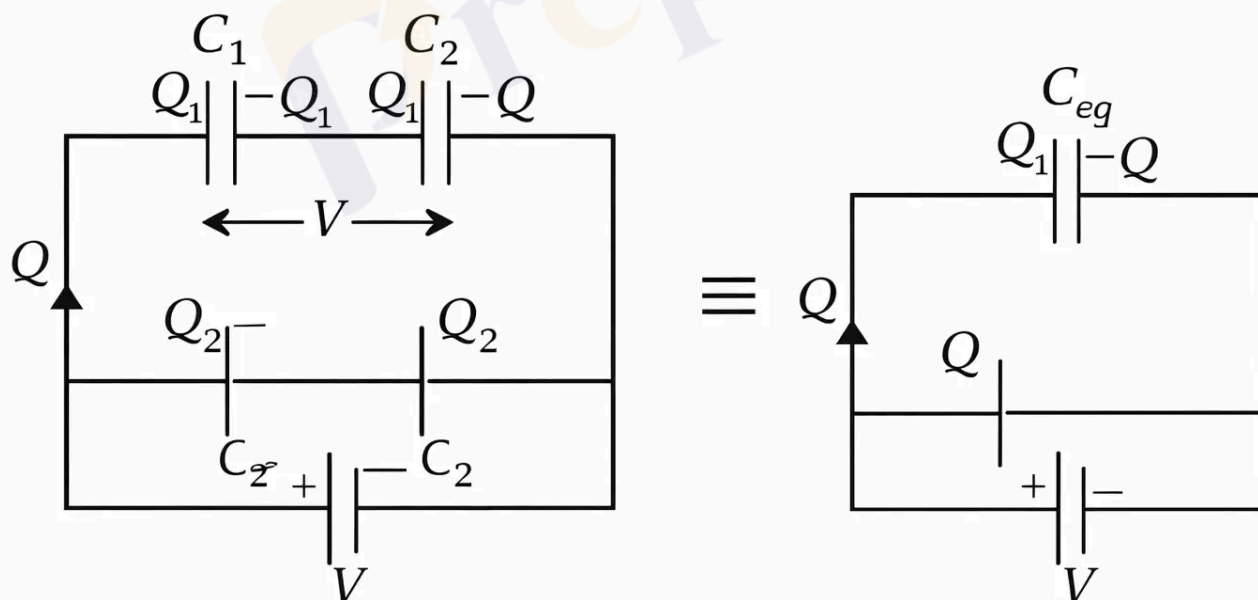
Answer Key

- Q1: B) $2 \mu F - \frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{1}{2} \Rightarrow C_{eq} = 2 \mu F$.
- Q2: A) $1 \mu F - \frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{6+3+2}{12} = \frac{11}{12} \Rightarrow C_{eq} \approx 1.09 \mu F$ (closest 1).
- Q3: B) $2 \mu F - C_{eq} = \frac{8}{4} = 2 \mu F$.

Memory Aids

- Series: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$, less than smallest capacitor.
- Charge same on all capacitors.
- Voltage divides inversely with capacitance.

Parallel Combination



Key Points

- Capacitors connected side-by-side.

- Voltage V across each capacitor is the same.
- Charge on each capacitor: $Q_i = C_i V$.
- Total charge $Q = \sum Q_i = V \sum C_i$.
- Equivalent capacitance:

$$C_{eq} = \sum C_i$$

- Charge divides according to capacitance values.

Solved Examples

Ex 1

- Two capacitors $2\mu F$ and $5\mu F$ in parallel. Find C_{eq} .
- $C_{eq} = 2 + 5 = 7\mu F$.

Practice Set

Q1

- Two capacitors $1\mu F$ and $4\mu F$ in parallel. Find C_{eq} .
- A) $5\mu F$
- B) $0.8\mu F$
- C) $3\mu F$
- D) $4\mu F$

Q2

- Three capacitors $3\mu F$, $6\mu F$, and $9\mu F$ in parallel. Find C_{eq} .
- A) $18\mu F$
- B) $6\mu F$
- C) $9\mu F$
- D) $3\mu F$

Q3

- Four capacitors $2\mu F$, $2\mu F$, $2\mu F$, and $2\mu F$ in parallel. Find C_{eq} .
- A) $8\mu F$
- B) $2\mu F$
- C) $4\mu F$
- D) $1\mu F$

Answer Key

- Q1: A) $5 \mu F$ - Sum of capacitances in parallel.
- Q2: A) $18 \mu F$ - $3 + 6 + 9 = 18 \mu F$.
- Q3: A) $8 \mu F$ - Sum of four identical capacitors.

Memory Aids

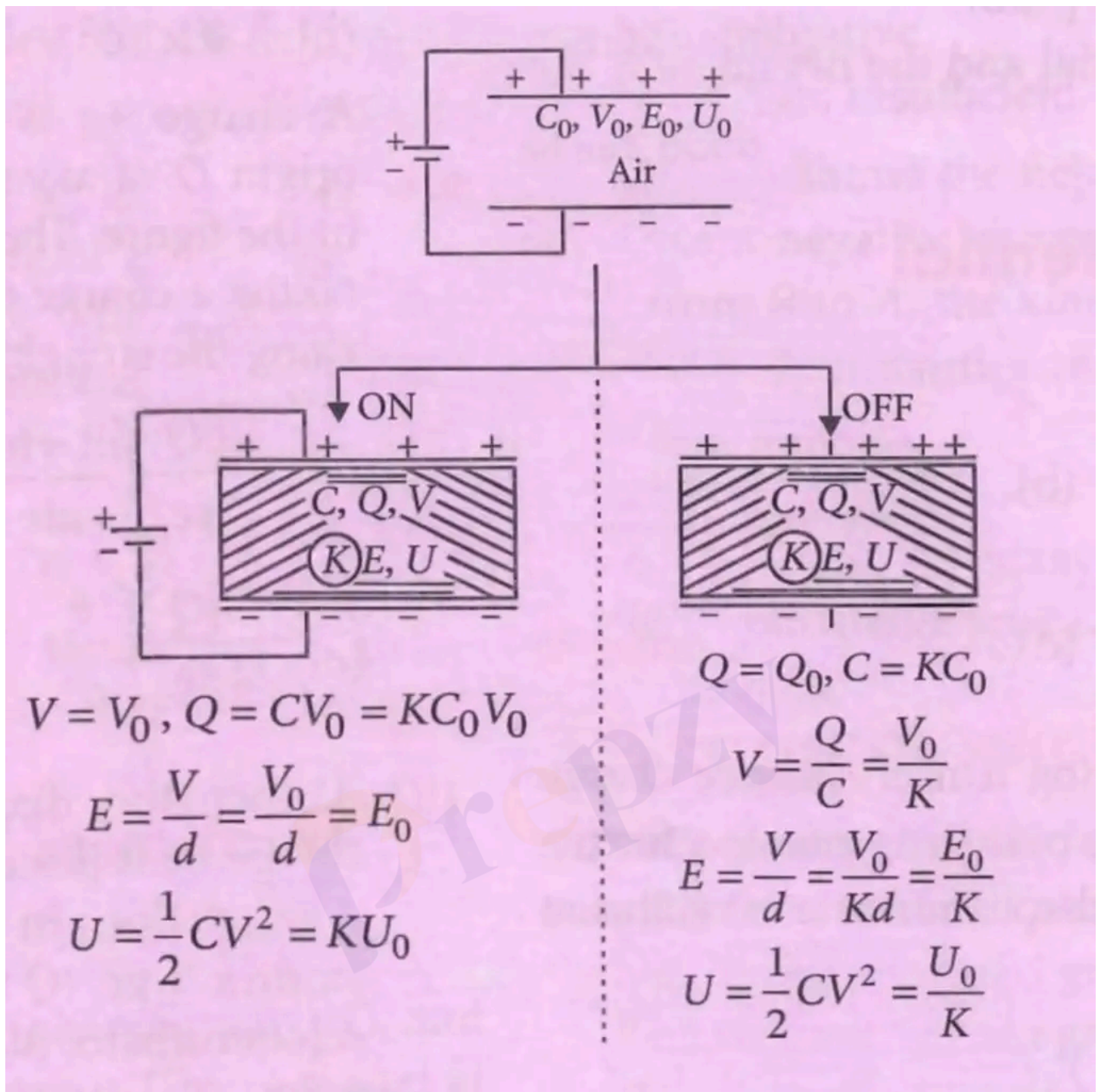
- Parallel: $C_{eq} = \sum C_i$, sum of capacitances.
- Voltage same across all capacitors.
- Charge divides by capacitance.

Energy Stored in a Capacitor

Concepts and Formulas

- If q is charge and V is potential difference at any instant, small work done to add charge dq is $dW = V \cdot dq = \frac{q}{C} dq$.
- Total work done to store maximum charge Q is $W = \int_0^Q \frac{q}{C} dq = \frac{1}{2} \frac{Q^2}{C}$.
- Energy stored in capacitor (electrostatic energy):
 $U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$.
- Energy density (energy per unit volume) in parallel plate capacitor:
 $u = \frac{1}{2} \epsilon_0 E^2$, where ϵ_0 is permittivity of free space and E is electric field.
- Effect of dielectric:
 - With dielectric connected to battery: Capacitance $C = KC_0$, voltage $V = V_0$, electric field $E = E_0$, energy stored $U = KU_0$ (increases).
 - With dielectric removed (isolated capacitor): Charge $Q = Q_0$ constant, voltage $V = \frac{V_0}{K}$, electric field $E = \frac{E_0}{K}$, energy stored $U = \frac{U_0}{K}$ (decreases).

$$W = \frac{1}{C} \int_0^Q q \, dq = \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q = \frac{1}{2} \cdot \frac{Q^2}{C}$$



Solved Examples

- Ex 1:** Calculate energy stored in a capacitor of capacitance $5 \mu F$ charged to 10 V.

Solution:

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 5 \times 10^{-6} \times 10^2 = 2.5 \times 10^{-4} \text{ J.}$$

- Ex 2:** Find energy density if electric field $E = 2 \times 10^6 \text{ V/m}$.

Solution:

$$u = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \times 8.85 \times 10^{-12} \times (2 \times 10^6)^2 = 1.77 \times 10^{-5} \text{ J/m}^3.$$

Practice Set

- Q1: Energy stored in a capacitor of $2 \mu F$ charged to 5 V is:
A) $25 \mu J$ B) $10 \mu J$ C) $5 \mu J$ D) $50 \mu J$
- Q2: Energy density in a capacitor depends on:
A) Capacitance only B) Electric field only C) Voltage only D) Charge only
- Q3: A capacitor with dielectric constant 4 is connected to battery. Energy stored becomes:
A) Same B) 4 times C) Half D) 2 times

Answer Key

- Q1: A) $25 \mu J$ - Using $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times 5^2 = 25 \times 10^{-6} \text{ J}$.
- Q2: B) Electric field only - Energy density formula $u = \frac{1}{2} \epsilon_0 E^2$ depends on electric field.
- Q3: B) 4 times - Energy stored increases by dielectric constant K when connected to battery.

Memory Aids

- Energy stored: $U = \frac{1}{2} CV^2$ or $\frac{1}{2} QV$.
- Energy density: $u = \frac{1}{2} \epsilon_0 E^2$.
- Dielectric effect: Energy U scales by dielectric constant K if battery connected.