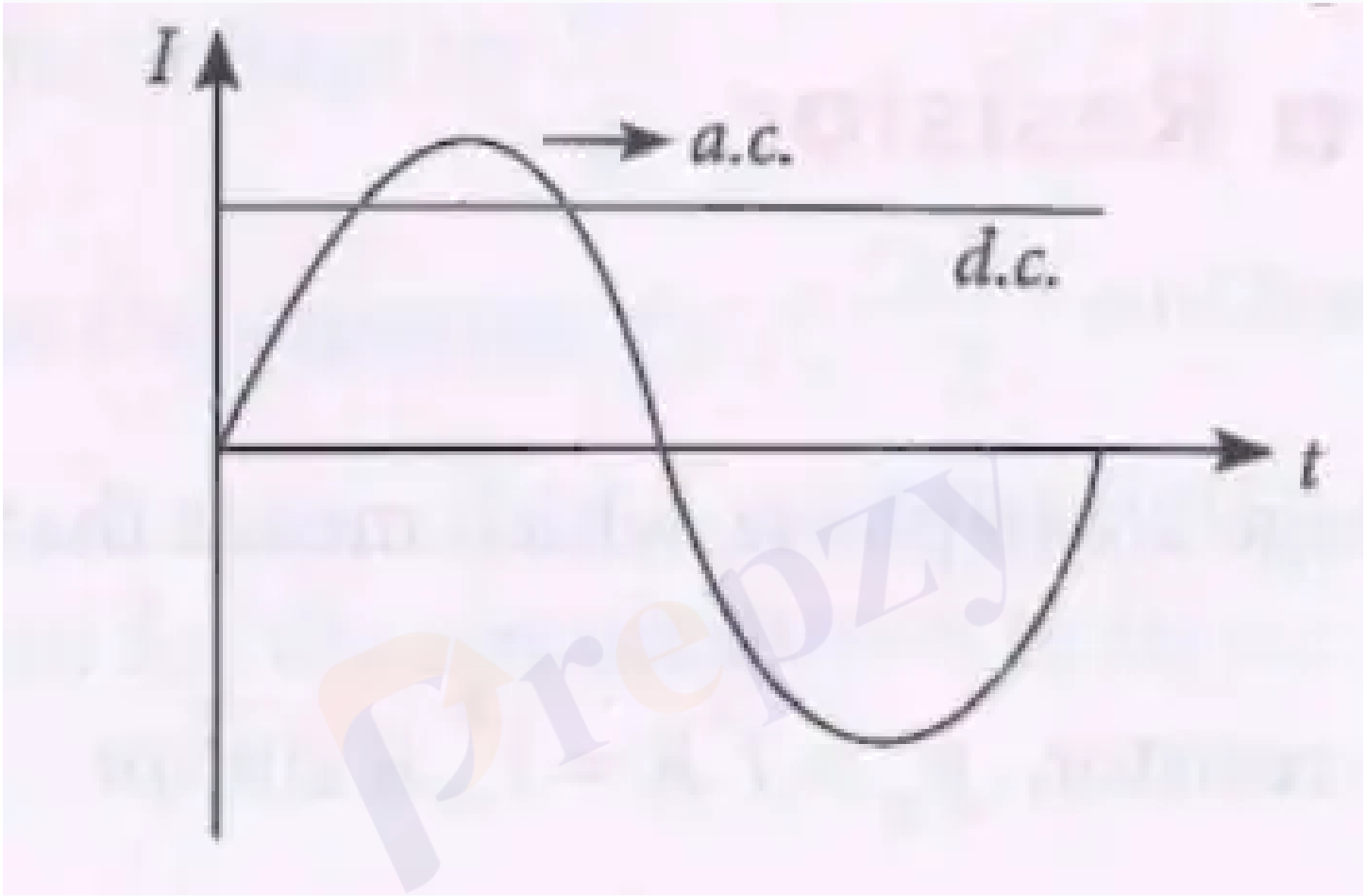
Table of Contents

1. Introduction
2. Fundamentals of Alternating Current
3. Average and RMS Values
4. Phase and Phase Difference
5. AC Voltage Applied to Resistor
6. Representation of AC Current and Voltage by Rotating Vectors–Phasors
7. AC Voltage Applied to an Inductor
8. Power in Inductive Load
9. Phasor and Waveform Representation
10. AC Voltage Applied to a Series LCR Circuit
11. Phasor and Phase Angle
12. Resonance
13. Quality Factor
14. Power in AC Circuit: The Power Factor
15. LC Oscillations
16. Transformers
17. Step-up Transformer
18. Step-down Transformer

Introduction

• Alternating current (AC) changes magnitude and reverses direction periodically.

• Direct current (DC) flows with constant magnitude and fixed direction.



• AC voltage drives AC current in circuits.

• AC voltage is preferred over DC because it can be easily transformed to different voltages using transformers.

Solved Examples

- Ex 1: Identify AC and DC from a graph.
  - AC is the wave crossing zero line periodically.
  - DC is the constant line above zero.

Practice Set

- Q1: What is the main difference between AC and DC?
  - A. AC has constant magnitude, DC reverses direction.
  - B. AC reverses direction periodically, DC has constant direction.
  - C. Both have constant magnitude.
  - D. Both reverse direction periodically.
- Q2: Why is AC voltage preferred over DC voltage?
  - A. AC voltage is constant.
  - B. AC voltage can be easily transformed to different voltages.
  - C. DC voltage is safer.
  - D. DC voltage changes direction.
- Q3: Which of the following represents AC current?
  - A. Constant positive line.
  - B. Line crossing zero periodically.
  - C. Zero current.
  - D. Constant negative line.

Answer Key

- Q1: B - AC reverses direction periodically; DC does not.
- Q2: B - AC voltage can be transformed easily using transformers.
- Q3: B - AC current crosses zero line periodically.

Memory Aids

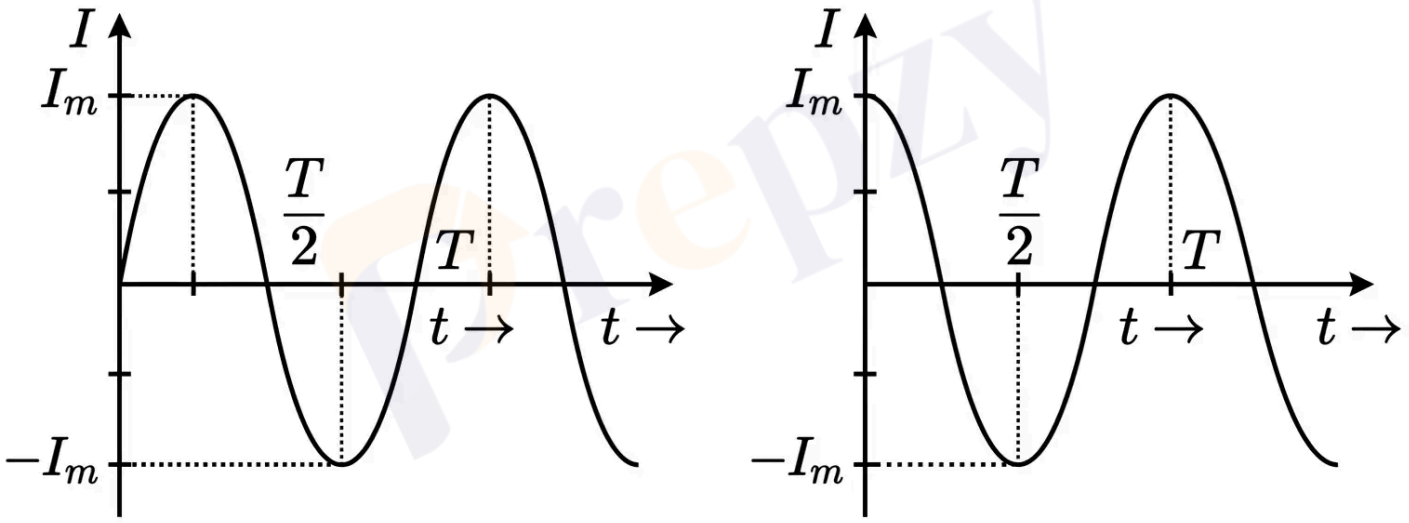
- AC = Alternates direction; DC = Direct constant flow.
- AC voltage preferred for easy voltage transformation.

Fundamentals of Alternating Current

- **AC voltage or current can be represented as sine or cosine functions:**

$$I = I_m \sin \omega t \text{ or } I = I_m \cos \omega t$$

- I: **Instantaneous current at time** t
- I_m: **Peak (maximum) current**
- ω: **Angular frequency,** $\omega = \frac{2\pi}{T} = 2\pi\nu$
- T: **Time period,** ν: **Frequency**



I as a sine function of t I as a cosine function of t

Solved Examples

- Ex 1: Find angular frequency if $T = 0.02$ s.
  - $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.02} = 314.16$ rad/s

Practice Set

- Q1: What is the peak value in $I = I_m \sin \omega t$?
  - A. Instantaneous current

    - B. Average current
    - C. Maximum current I_m
    - D. Zero current
- Q2: Angular frequency ω is given by?
    - A. $2\pi T$
    - B. $\frac{2\pi}{T}$
    - C. $\frac{T}{2\pi}$
    - D. πT
- Q3: If frequency $\nu = 50$ Hz, find ω.
    - A. 100 rad/s
    - B. 314 rad/s
    - C. 50 rad/s
    - D. 25 rad/s

Answer Key

- Q1: C - I_m is the peak current.
- Q2: B - $\omega = \frac{2\pi}{T}$
- Q3: B - $\omega = 2\pi \times 50 = 314$ rad/s

Memory Aids

- $\omega = 2\pi\nu$, remember 2π radians = 1 cycle.
- Peak current = I_m, instantaneous = varies with time.

Average and RMS Values

• Average (mean) value over half cycle:

$$\langle I \rangle = \frac{2I_m}{\pi}$$

• Root mean square (rms) value:

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

• RMS value produces same heat in resistor as DC of same value.

• RMS value is also called virtual or apparent value.

Solved Examples

- Ex 1: Calculate rms current if $I_m = 10$ A.
  - $I_{rms} = \frac{10}{\sqrt{2}} = 7.07$ A

Practice Set

- Q1: Average value of AC current over half cycle is?
  - A. I_m
  - B. $\frac{2I_m}{\pi}$
  - C. $\frac{I_m}{\sqrt{2}}$
  - D. Zero
- Q2: RMS value of current is?
  - A. I_m
  - B. $\frac{2I_m}{\pi}$
  - C. $\frac{I_m}{\sqrt{2}}$
  - D. Zero
- Q3: If $I_m = 14$ A, find I_{rms}.
  - A. 9.9 A
  - B. 7 A
  - C. 14 A

- D. 20 A

Answer Key

- Q1: B - Average over half cycle is $\frac{2I_m}{\pi}$.
- Q2: C - RMS value is $\frac{I_m}{\sqrt{2}}$.
- Q3: A - $I_{rms} = \frac{14}{\sqrt{2}} = 9.9$ A.

Memory Aids

- Average value = $\frac{2}{\pi}$ times peak.
- RMS value = peak divided by $\sqrt{2}$.

Phase and Phase Difference

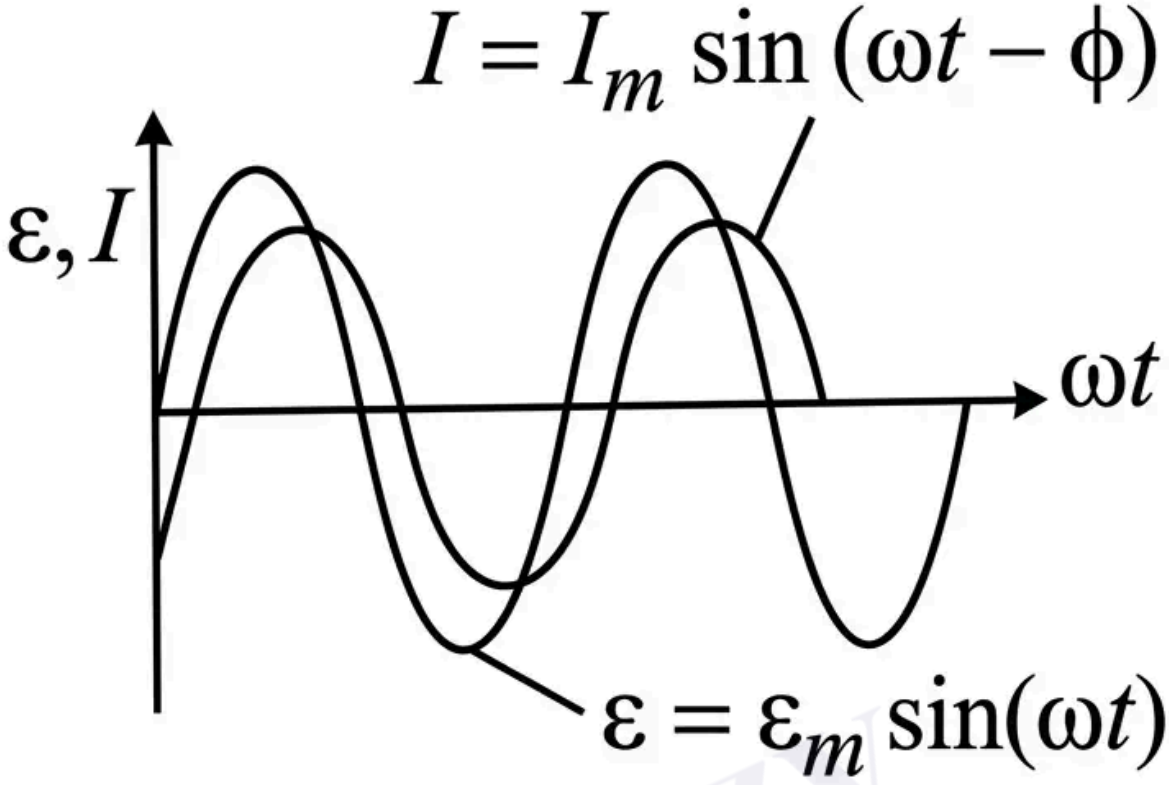
- **AC current with phase:** $I = I_m \sin(\omega t \pm \phi)$
- **Initial phase ϕ is constant; instantaneous phase ωt changes with time.**
- **Phase difference between voltage $\varepsilon = \varepsilon_0 \sin(\omega t + \phi_1)$ and current**

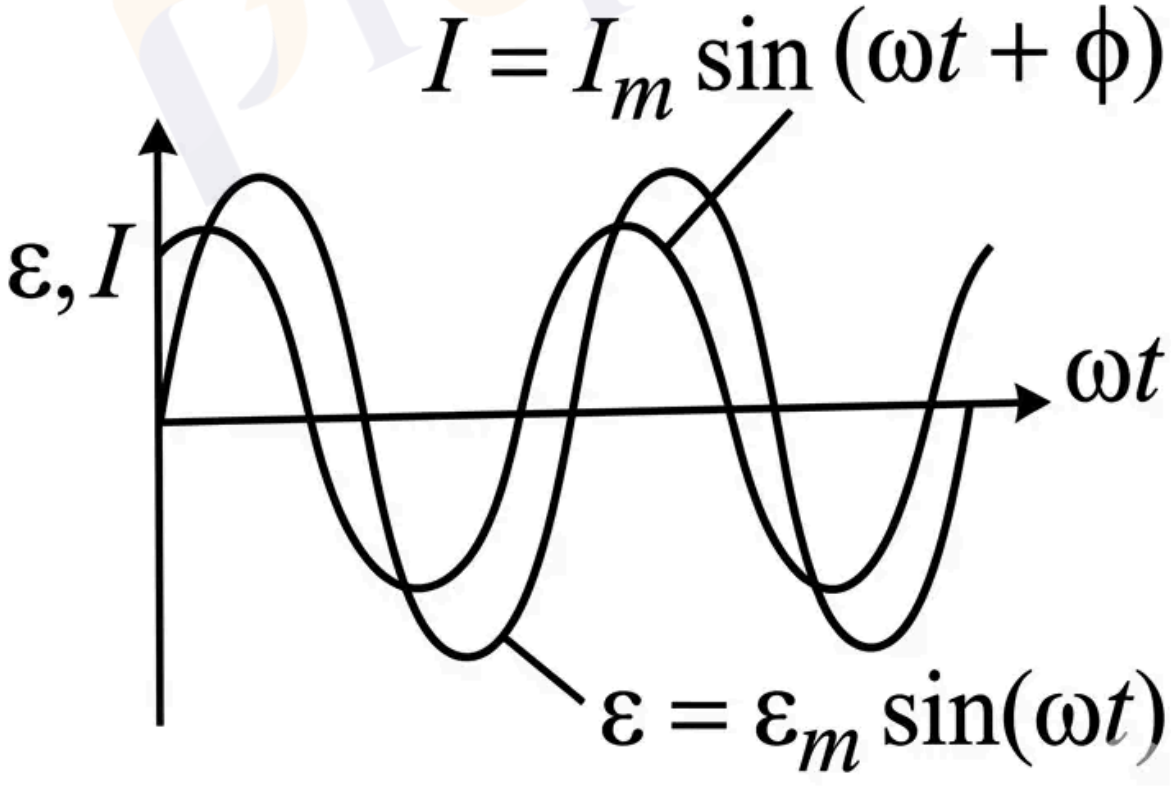
Prepzy

$I = I_m \sin(\omega t + \phi_2)$:

$\phi = \phi_2 - \phi_1$ **(current w.r.t voltage)**

$\phi = \phi_1 - \phi_2$ **(voltage w.r.t current)**


$I = I_m \sin(\omega t - \phi)$
ε, I
ωt
$\varepsilon = \varepsilon_m \sin(\omega t)$


$I = I_m \sin(\omega t + \phi)$
ε, I
ωt
$\varepsilon = \varepsilon_m \sin(\omega t)$

Solved Examples

- Ex 1: If current lags voltage by 30°, write current expression.
  - $I = I_m \sin(\omega t - 30^\circ)$

Practice Set

- Q1: If $\varepsilon = \varepsilon_m \sin \omega t$, current lags voltage by 45°, current is?
  - A. $I_m \sin(\omega t + 45^\circ)$
  - B. $I_m \sin(\omega t - 45^\circ)$
  - C. $I_m \sin \omega t$
  - D. Zero
- Q2: Phase difference ϕ is?
  - A. Difference of initial phases
  - B. Sum of initial phases
  - C. Product of initial phases
  - D. Zero
- Q3: If voltage leads current, what is the relation?
  - A. $\varepsilon = \varepsilon_m \sin(\omega t - \phi)$
  - B. $I = I_m \sin(\omega t - \phi)$
  - C. $I = I_m \sin(\omega t + \phi)$
  - D. $\varepsilon = \varepsilon_m \sin(\omega t + \phi)$

Answer Key

- Q1: B - Current lags voltage by subtracting phase.
- Q2: A - Phase difference is difference of initial phases.
- Q3: A - Voltage leads current means voltage phase is ahead.

Memory Aids

- Lead means reaches max earlier; lag means reaches max later.
- Phase difference = difference of initial phases.

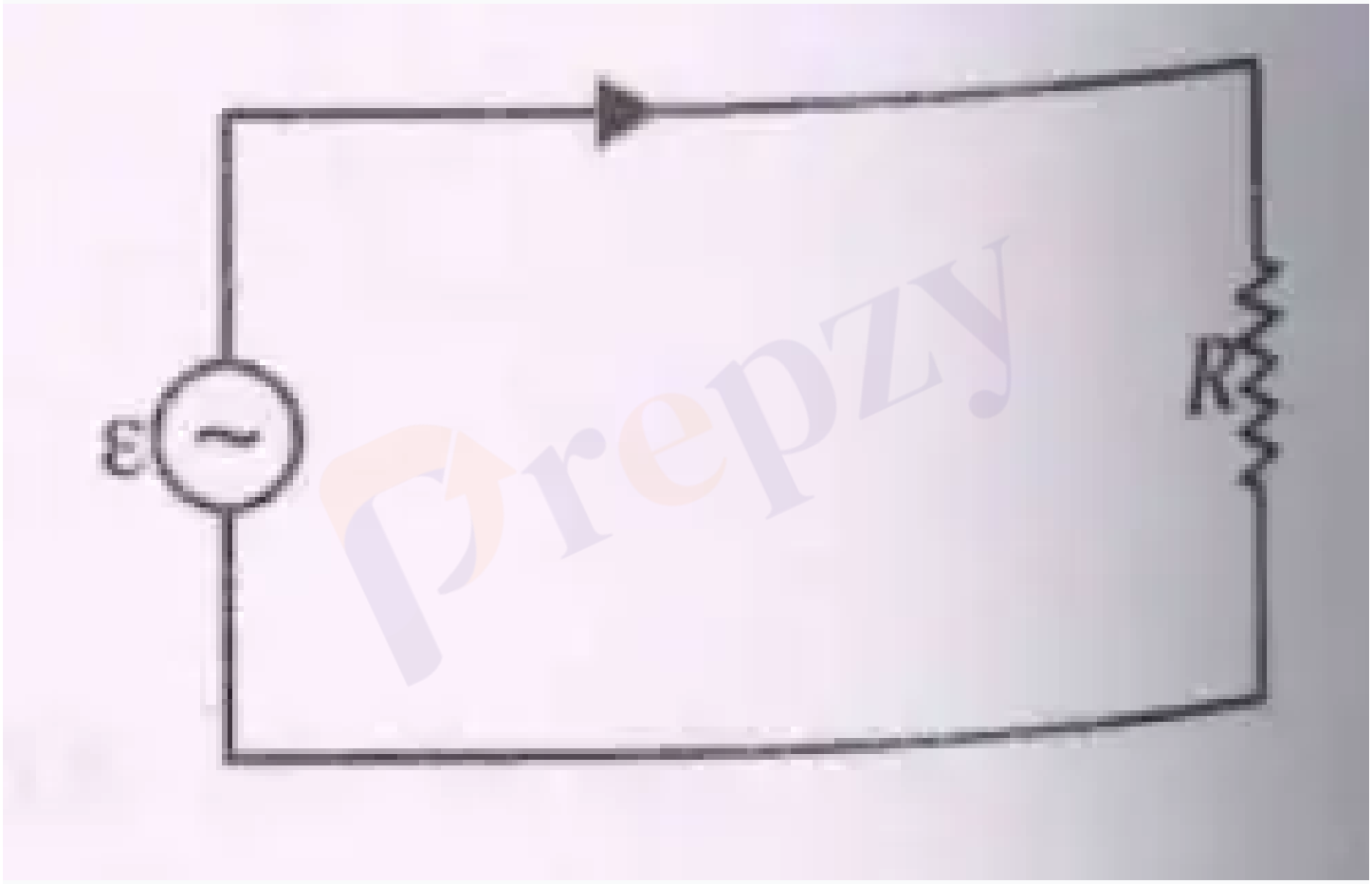
AC Voltage Applied to Resistor

• **Voltage:** $\varepsilon = \varepsilon_m \sin \omega t$

• **Current:** $I = I_m \sin \omega t$

• **Current and voltage are in phase for resistor.**

• **Instantaneous power dissipated:** $P_r = I^2 R = I_m^2 R \sin^2 \omega t$



Solved Examples

- Ex 1: Calculate average power if $I_m = 5$ A, $R = 10\ \Omega$.
  - $P = \frac{1}{2} I_m^2 R = \frac{1}{2} \times 25 \times 10 = 125$ W

Practice Set

- Q1: For resistor, current and voltage are?

    - A. Out of phase
    - B. In phase
    - C. Lead-lag
    - D. Zero
- Q2: Instantaneous power formula is?
    - A. $I_m R \sin \omega t$
    - B. $I_m^2 R \sin^2 \omega t$
    - C. $I_m^2 R \cos \omega t$
    - D. $I_m R \cos^2 \omega t$
- Q3: Average power over cycle is?
    - A. $I_m^2 R$
    - B. $\frac{1}{2} I_m^2 R$
    - C. $I_m R$
    - D. Zero

Answer Key

- Q1: B - Current and voltage are in phase in resistor.
- Q2: B - Instantaneous power is $I_m^2 R \sin^2 \omega t$.
- Q3: B - Average power is half of peak power.

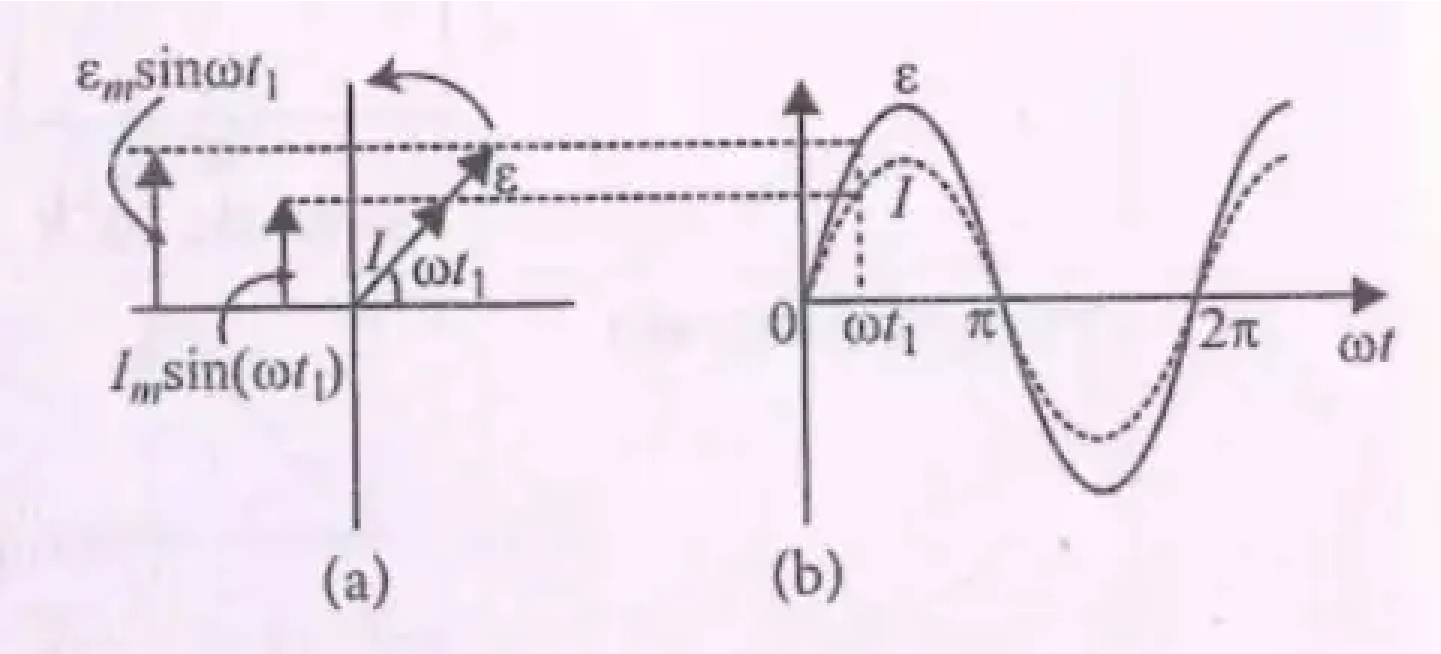
Memory Aids

- Power average = $\frac{1}{2} I_m^2 R$.
- Voltage and current peak together in resistor.

Representation of AC Current and Voltage by Rotating Vectors–Phasors

Concepts

- Phasors represent AC voltage and current as rotating vectors with angular velocity ω.
- Phasor length = peak value: ε_m for voltage, I_m for current.
- Vertical projection of phasor gives instantaneous value: $\varepsilon = \varepsilon_m \sin \omega t$, $I = I_m \sin \omega t$.
- Phasor diagram (a) shows voltage and current vectors; graph (b) shows sinusoidal variation vs time.
- When voltage and current phasors align, phase difference $\phi = 0$; current and voltage are in phase.



Solved Examples

- **Ex 1:** Given $\varepsilon_m = 10\,V$, find instantaneous voltage at $\omega t = 30^\circ$.
  - Use $\varepsilon = \varepsilon_m \sin \omega t = 10 \times \sin 30^\circ = 10 \times 0.5 = 5\,V$.
- **Ex 2:** If current phasor lags voltage by 30° and $I_m = 2\,A$, find instantaneous current at $\omega t = 60^\circ$.
  - $I = I_m \sin(\omega t - 30^\circ) = 2 \times \sin 30^\circ = 2 \times 0.5 = 1\,A$.

Practice Set

- Q1: What does the length of a phasor represent?
  - A) Instantaneous value
  - B) Peak value
  - C) Average value
  - D) RMS value
- Q2: If voltage and current phasors are in the same direction, the phase difference is:
  - A) 90°
  - B) 180°
  - C) 0°
  - D) 45°
- Q3: For a phasor $\varepsilon = \varepsilon_m \sin \omega t$, what is the vertical component at $\omega t = 90^\circ$ if $\varepsilon_m = 20\,V$?
  - A) 0 V
  - B) 10 V
  - C) 20 V
  - D) 40 V

Answer Key

- Q1: B) Peak value — Phasor length equals maximum amplitude.
- Q2: C) 0° — Same direction means no phase difference.
- Q3: C) 20 V — $\sin 90^\circ = 1$, so vertical component = peak value.

Memory Aids

- Phasor length = peak value.
- Vertical projection = instantaneous value.
- Same direction phasors → zero phase difference.

AC Voltage Applied to an Inductor

Voltage and Current Relationship

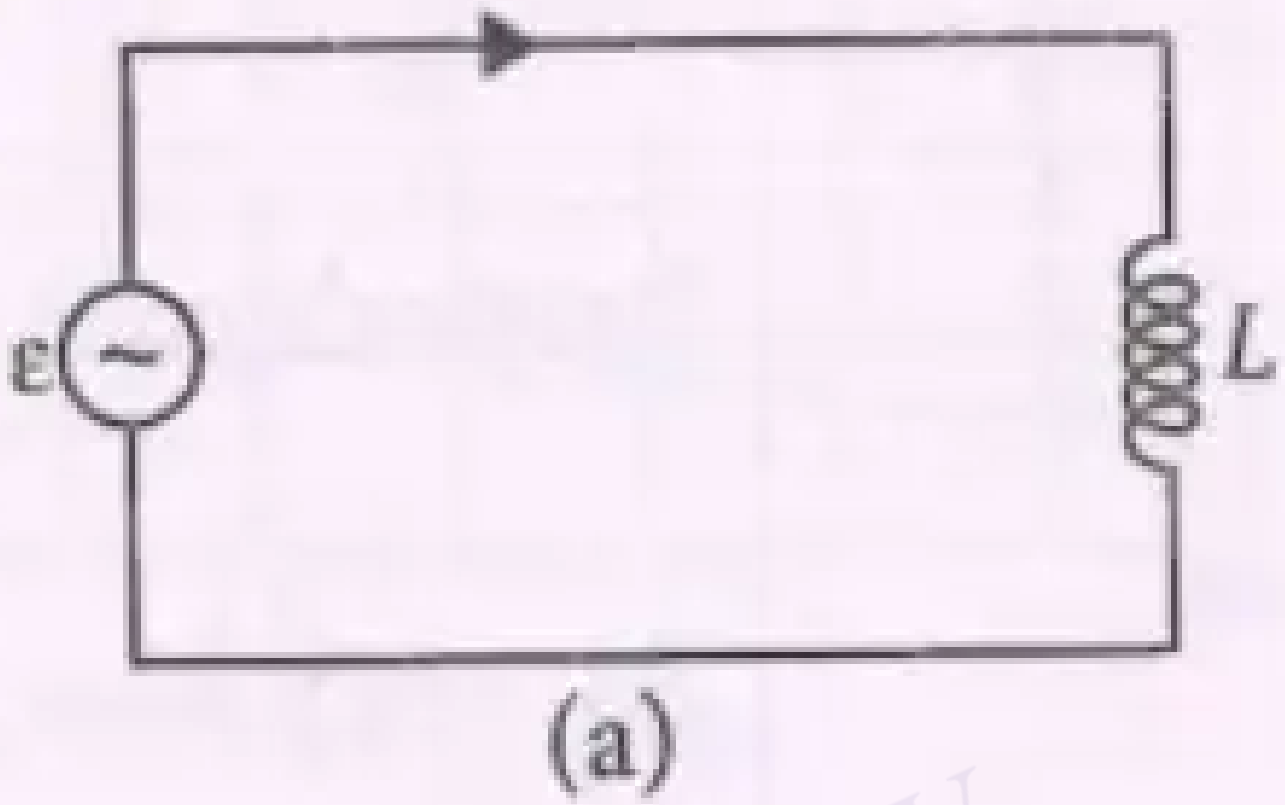
- Voltage across inductor: $\varepsilon = \varepsilon_m \sin \omega t$
- Current through inductor: $I = I_m \sin(\omega t - \pi/2)$
- Current lags voltage by $\pi/2$ radians (90°)

Amplitude of Current

- $I_m = \frac{\varepsilon_m}{\omega L}$
- $\omega = 2\pi\nu$ (angular frequency)
- L is inductance in henrys (H)

Inductive Reactance

- Inductive reactance $X_L = \omega L = 2\pi\nu L$
- Acts like resistance in AC circuit but depends on frequency



Solved Examples

- **Ex 1:** Calculate current amplitude if $\varepsilon_m = 10\,V$, $L = 0.5\,H$, and $\nu = 50\,Hz$.
  - $\omega = 2\pi \times 50 = 314.16\,rad/s$
  - $I_m = \frac{10}{314.16 \times 0.5} = 0.0637\,A$

Practice Set

- Q1: What is the phase difference between voltage and current in an inductor?
  - A. 0
  - B. $\pi/4$
  - C. $\pi/2$
  - D. π
- Q2: Inductive reactance X_L is given by?
  - A. $\frac{1}{\omega L}$
  - B. ωL
  - C. L/ω
  - D. ω/L
- Q3: For $\varepsilon_m = 20\,V$, $L = 0.2\,H$, and $\nu = 60\,Hz$, find I_m.

- A. 0.26 A
- B. 0.53 A
- C. 1.67 A
- D. 3.33 A

Answer Key

- Q1: C. $\pi/2$ - Current lags voltage by 90° in an inductor.
- Q2: B. ωL - Inductive reactance formula.
- Q3: A. 0.26 A - $I_m = \frac{20}{2\pi \times 60 \times 0.2} = 0.265$ A approx.

Memory Aids

- Current lags voltage by 90° in inductors.
- Inductive reactance $X_L = 2\pi\nu L$.
- Higher frequency → higher reactance → lower current.

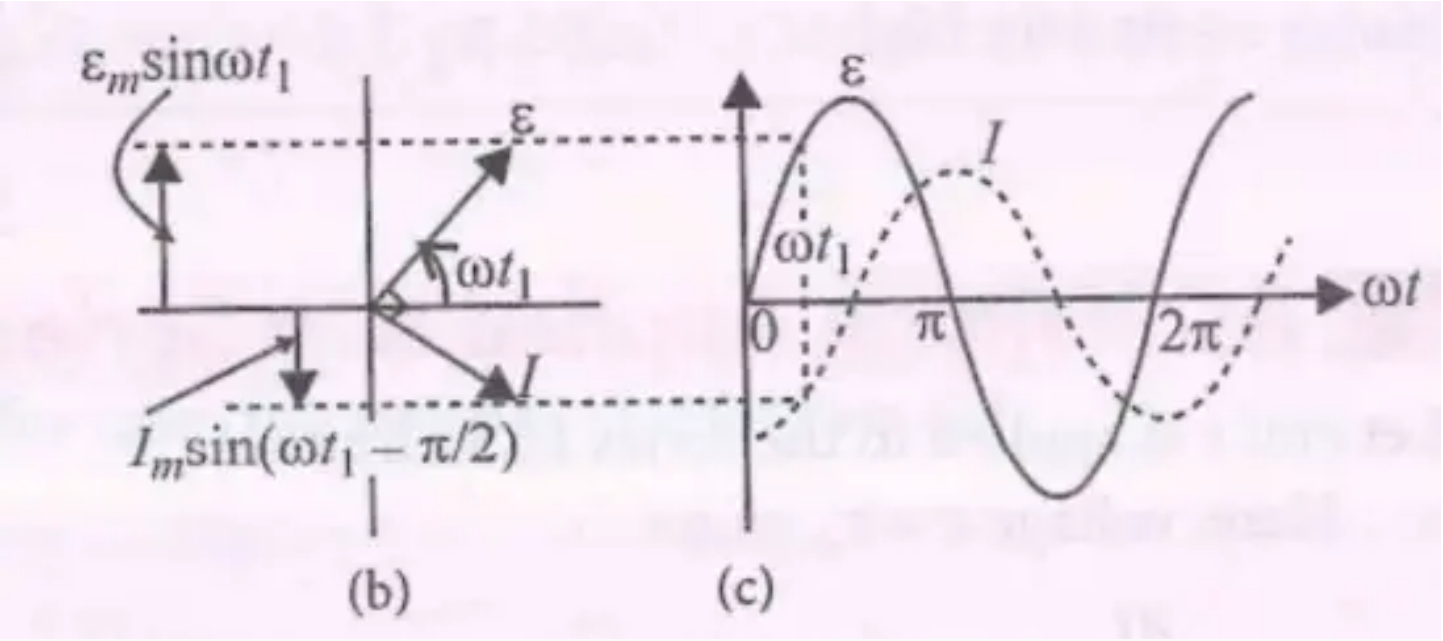
Power in Inductive Load

Instantaneous Power

- $p_l = I\varepsilon = -\frac{I_m \varepsilon_m}{2}\sin(2\omega t)$
- Power oscillates sinusoidally, can be positive or negative

Average Power

- $P_l = \langle p_l \rangle = 0$ over one complete cycle
- Because $\langle \sin(2\omega t) \rangle = 0$
- Inductor stores and returns energy, no net power consumed



Solved Examples

- **Ex 1:** Find average power for $I_m = 5\,A$, $\varepsilon_m = 10\,V$.
  - $P_l = 0$ (always zero for pure inductive load)

Practice Set

- Q1: What is the average power consumed by a pure inductor?
  - A. Zero
  - B. Maximum
  - C. Half of input power
  - D. Equal to resistive power
- Q2: Instantaneous power in an inductor is proportional to?
  - A. $\sin \omega t$
  - B. $\sin 2\omega t$
  - C. $\cos \omega t$
  - D. Constant
- Q3: Why is average power zero in an inductor?
  - A. No current flows
  - B. Voltage and current are in phase
  - C. Energy is stored and returned
  - D. Resistance is zero

Answer Key

- Q1: A. Zero - Inductor does not dissipate power.
- Q2: B. $\sin 2\omega t$ - Instantaneous power varies as $\sin 2\omega t$.
- Q3: C. Energy is stored and returned - No net energy loss.

Memory Aids

- Average power in inductor = 0.
- Instantaneous power oscillates at double frequency.
- Inductor stores magnetic energy, returns it fully.

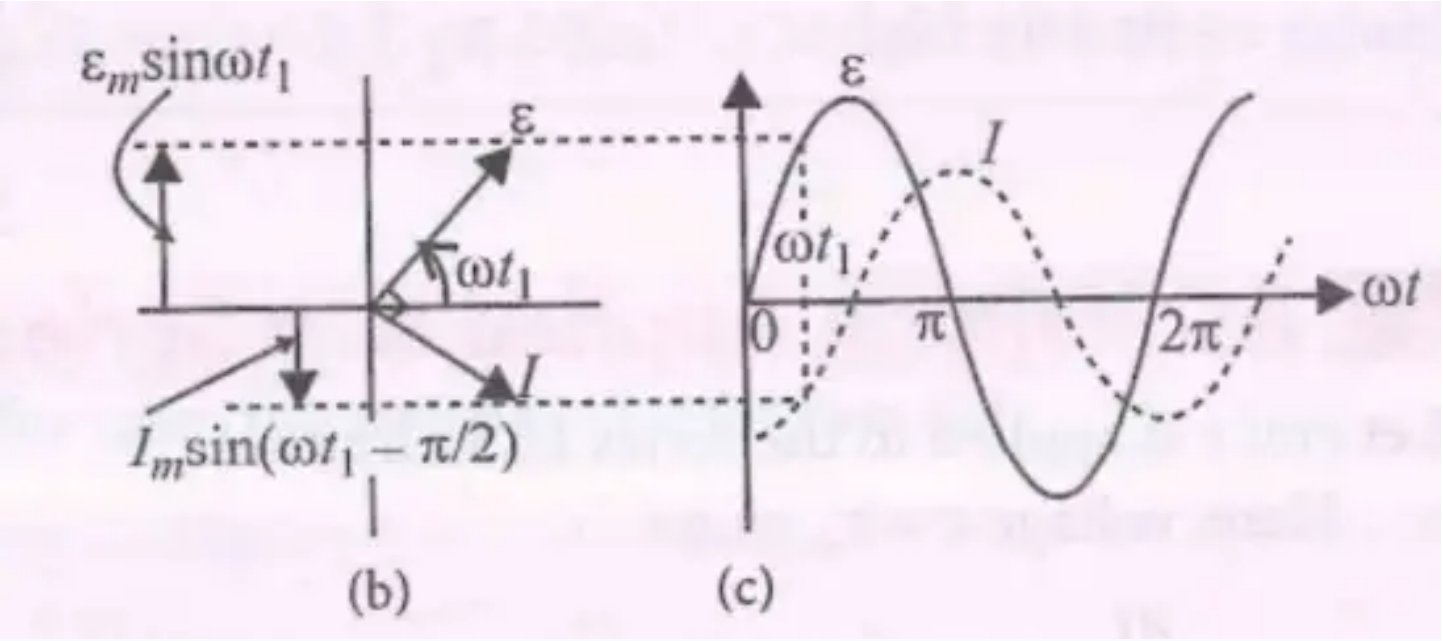
Phasor and Waveform Representation

Phasor Diagram

- Voltage vector leads current vector by $\pi/2$ radians
- Voltage represented as $E_m \sin(\omega t)$
- Current represented as $I_m \sin(\omega t - \pi/2)$

Waveform Graphs

- Voltage and current are sinusoidal waves with phase difference
- Voltage wave peaks before current wave by 90°
- Graph plots ε and I versus ωt



Solved Examples

- **Ex 1:** Identify phase difference from phasor diagram.
  - Voltage leads current by $\pi/2$.

Practice Set

- Q1: In phasor diagram, current lags voltage by?
  - A. 0
  - B. $\pi/4$
  - C. $\pi/2$
  - D. π
- Q2: Voltage and current waveforms differ by?
  - A. 0°
  - B. 45°
  - C. 90°
  - D. 180°
- Q3: Which vector leads in an inductive circuit?
  - A. Current
  - B. Voltage
  - C. Both same
  - D. None

Answer Key

- Q1: C. $\pi/2$ - Current lags voltage by 90°.
- Q2: C. 90° - Phase difference in inductive circuit.
- Q3: B. Voltage - Voltage leads current in inductors.

Memory Aids

- Voltage vector ahead of current by 90° in inductors.
- Phasor diagram shows phase relations clearly.
- Waveform peaks confirm phase lag of current.

AC Voltage Applied to a Series LCR Circuit

Concepts and Formulas

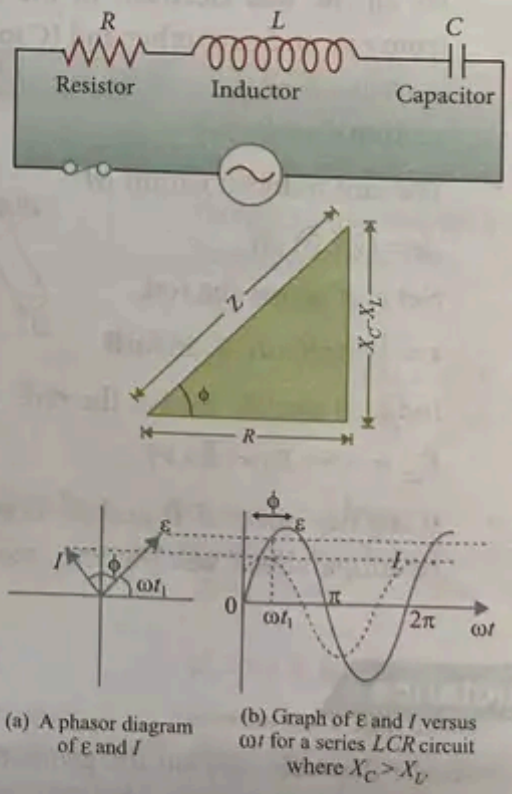
- Voltage applied: $\varepsilon = \varepsilon_m \sin \omega t$
- Voltage equation: $\varepsilon = L\frac{dI}{dt} + IR + \frac{q}{C} = \varepsilon_m \sin \omega t$
- Voltages across capacitor V_C and inductor V_L are opposite in direction.
- Total voltage squared: $\varepsilon_m^2 = I_m^2\left[R^2 + (X_C - X_L)^2\right]$
- Impedance: $Z = \sqrt{R^2 + (X_C - X_L)^2}$
- Current amplitude: $I_m = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2+(X_C-X_L)^2}}$
- Phase angle: $\tan\phi = \frac{X_C - X_L}{R}$
- If $X_C > X_L$, circuit is capacitive; current leads voltage.
- If $X_C < X_L$, circuit is inductive; current lags voltage.

- Voltage $\varepsilon = \varepsilon_m \sin\omega t$
  V_C and V_L are always along the same line but in opposite directions.

  So, $\varepsilon_m^2 = I_m^2[R^2 + (X_C - X_L)^2]$
- Impedance,

  $$Z = \sqrt{R^2 + (X_C - X_L)^2} \; ; \; I_m = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$
- Phase (ϕ) between voltage and current is given by

  $$\tan\phi = \frac{V_{Cm} - V_{Lm}}{V_{Rm}} = \frac{X_C - X_L}{R}$$
- If $X_C > X_L$, ϕ is positive and circuit is capacitive. Consequently, the current in the circuit leads the source voltage.
- If $X_C < X_L$, ϕ is negative and circuit is inductive. Consequently, the current in the circuit lags the source voltage.



(a) A phasor diagram of ε and I

(b) Graph of ε and I versus ωt for a series LCR circuit where $X_C > X_L$

Solved Examples

- **Ex 1:** Calculate impedance Z if $R = 10\ \Omega$, $X_C = 5\ \Omega$, $X_L = 3\ \Omega$.
  - $Z = \sqrt{10^2 + (5-3)^2} = \sqrt{100 + 4} = \sqrt{104} = 10.2\ \Omega$
- **Ex 2:** Find current amplitude I_m if $\varepsilon_m = 20\ V$, $Z = 10\ \Omega$.
  - $I_m = \frac{20}{10} = 2\ A$

Practice Set

- Q1: What is the expression for impedance Z in a series LCR circuit?
  - A) $\sqrt{R^2 + (X_L + X_C)^2}$
  - B) $\sqrt{R^2 + (X_C - X_L)^2}$
  - C) $R + X_L + X_C$
  - D) $\frac{R}{X_C - X_L}$
- Q2: If $X_C > X_L$, what is the phase relationship between current and voltage?
  - A) Current lags voltage
  - B) Current leads voltage
  - C) Current and voltage are in phase
  - D) No current flows
- Q3: For $R = 5\ \Omega$, $X_C = 8\ \Omega$, $X_L = 3\ \Omega$, and $\varepsilon_m = 30\ V$, find I_m.

- A) 3 A
- B) 4 A
- C) 5 A
- D) 6 A

Answer Key

- Q1: B - Impedance is $Z = \sqrt{R^2 + (X_C - X_L)^2}$ in series LCR circuit.
- Q2: B - If $X_C > X_L$, current leads voltage (capacitive circuit).
- Q3: A - $Z = \sqrt{5^2 + (8-3)^2} = \sqrt{25+25} = \sqrt{50} = 7.07$, $I_m = 30/7.07 = 4.24\ A$ approx 4 A (closest option is 4 A).

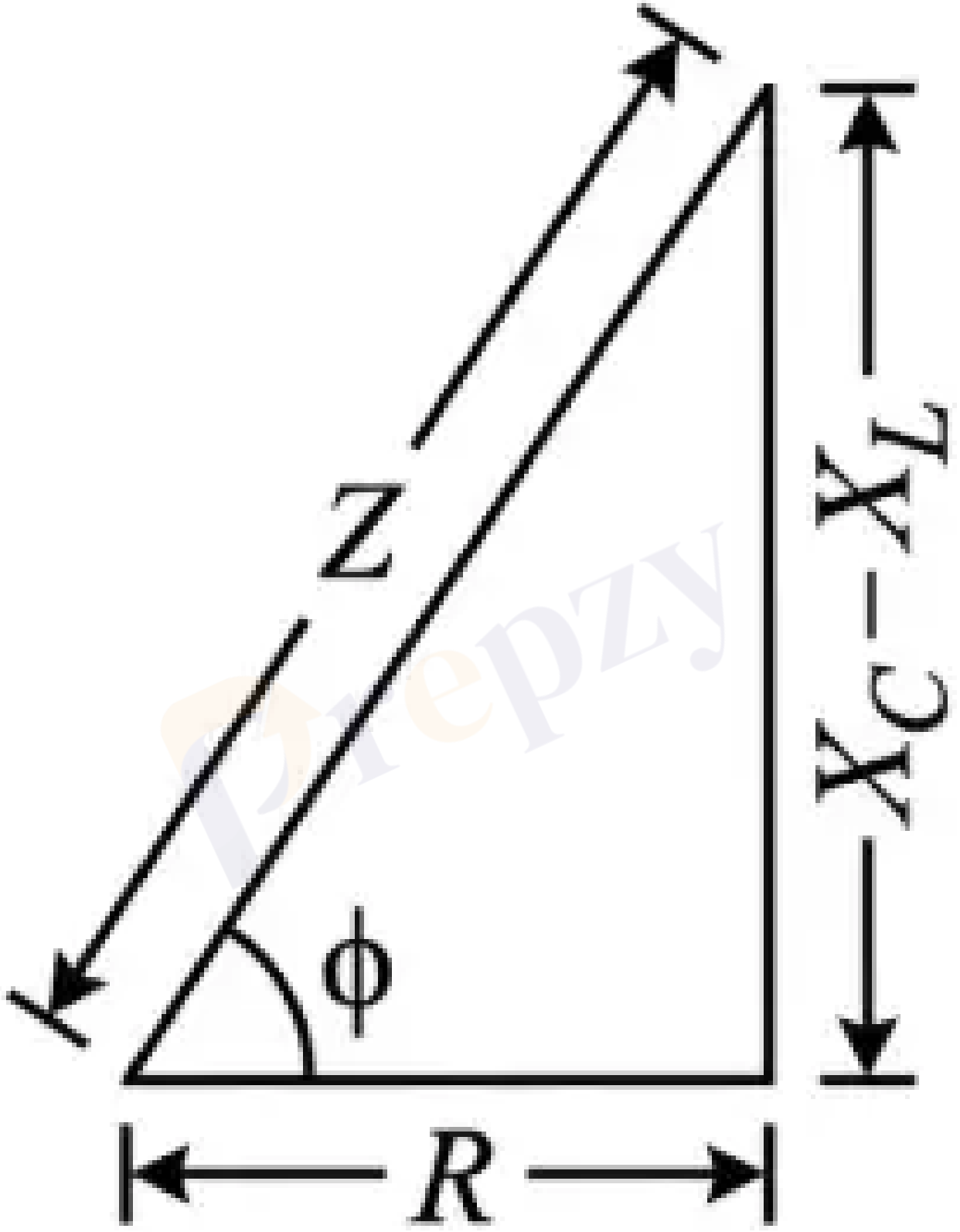
Memory Aids

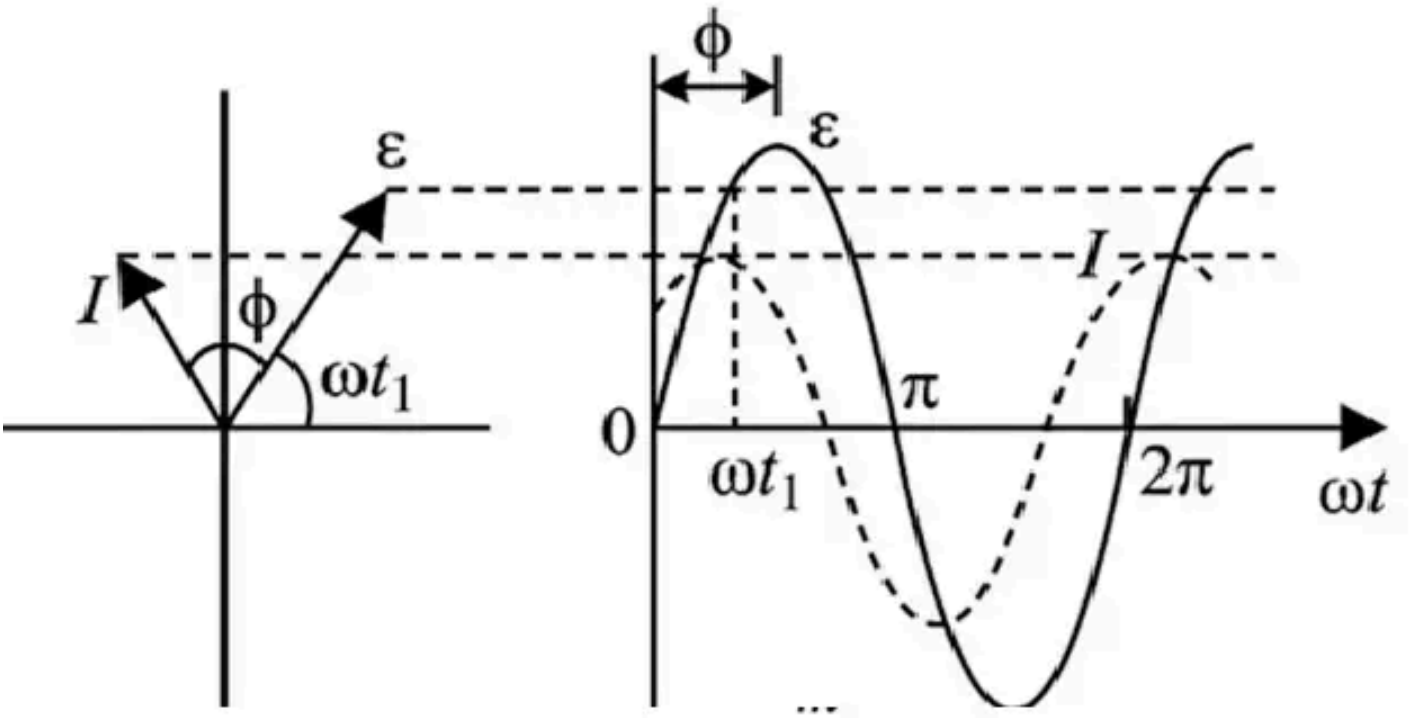
- Impedance formula: "R and difference of reactances squared under root"
- Phase angle: $\tan\phi = \frac{X_C - X_L}{R}$
- Current leads if capacitive, lags if inductive

Phasor and Phase Angle

Concepts and Formulas

- Phasor diagram shows voltage ε and current I vectors with angle ϕ.
- Right triangle with sides: R, $X_C - X_L$, hypotenuse Z.
- Phase angle: $\tan\phi = \frac{X_C - X_L}{R}$
- If $\phi > 0$, circuit is capacitive; current leads voltage.
- If $\phi < 0$, circuit is inductive; current lags voltage.


Z
$X_L - X_C$
ϕ
R



(a) A phasor diagram of ε and I

(b) Graph of ε and i versus ωt for a series LCR circuit where $X_C > X_L$

Solved Examples

- **Ex 1:** Find ϕ if $R = 4\ \Omega$, $X_C = 6\ \Omega$, $X_L = 2\ \Omega$.
  - $\tan\phi = \frac{6-2}{4} = 1$, $\phi = 45^\circ$
- **Ex 2:** Determine if current leads or lags voltage for $X_C = 3\ \Omega$, $X_L = 5\ \Omega$.
  - Since $X_C < X_L$, current lags voltage (inductive).

Practice Set

- Q1: What does a positive phase angle ϕ indicate?
  - A) Inductive circuit
  - B) Capacitive circuit
  - C) Resistive circuit
  - D) No current
- Q2: In the phasor diagram, what is the hypotenuse representing?
  - A) Resistance R
  - B) Capacitive reactance X_C
  - C) Inductive reactance X_L

    - D) Impedance Z
- Q3: If $\tan\phi = 0$, what is the circuit type?
    - A) Purely resistive
    - B) Purely inductive
    - C) Purely capacitive
    - D) Resonant

Answer Key

- Q1: B - Positive ϕ means capacitive circuit.
- Q2: D - Hypotenuse in impedance triangle is impedance Z.
- Q3: A - $\tan\phi = 0$ means no reactance, purely resistive.

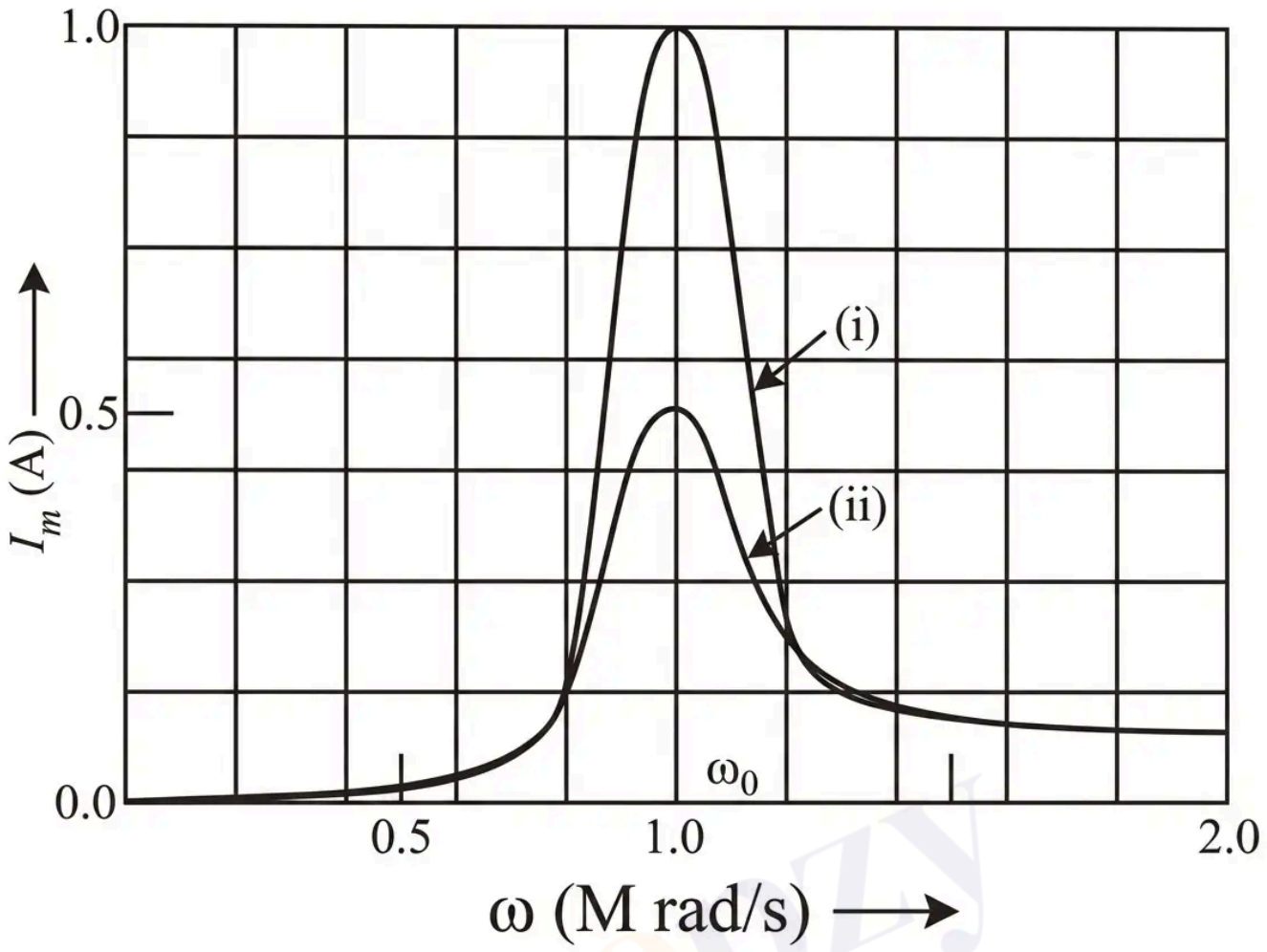
Memory Aids

- Phase angle ϕ: positive = capacitor, negative = inductor
- Impedance triangle: hypotenuse = Z, base = R, height = $X_C - X_L$

Resonance

Concepts and Formulas

- Resonance: minimum impedance, maximum current in series LCR circuit.
- Condition: $X_L = X_C$
- Current amplitude at resonance: $I_m = \frac{\varepsilon_m}{R}$
- Resonant frequency:
    - Angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$
    - Frequency: $\nu_r = \frac{1}{2\pi\sqrt{LC}}$
- Resonance does not occur in RL or RC circuits.



Solved Examples

- **Ex 1:** Calculate resonant frequency for $L = 1\ \text{mH}$, $C = 1\ \text{nF}$.
  - $\omega_0 = \frac{1}{\sqrt{1\times10^{-3}\times1\times10^{-9}}} = 10^6\ \text{rad/s}$
- **Ex 2:** Find current amplitude at resonance if $\varepsilon_m = 100\ V$, $R = 100\ \Omega$.
  - $I_m = \frac{100}{100} = 1\ A$

Practice Set

- Q1: What is the resonance condition in a series LCR circuit?
  - A) $X_L + X_C = 0$
  - B) $X_L = X_C$
  - C) $R = 0$
  - D) $I_m = 0$
- Q2: At resonance, the current amplitude is given by?
  - A) $\frac{\varepsilon_m}{Z}$
  - B) $\frac{\varepsilon_m}{R}$

    - C) $\frac{\varepsilon_m}{X_L}$
    - D) $\frac{\varepsilon_m}{X_C}$
- Q3: Resonance does not occur in which circuit?
    - A) Series LCR
    - B) RL
    - C) LC
    - D) Parallel LCR

Answer Key

- Q1: B - Resonance occurs when $X_L = X_C$.
- Q2: B - At resonance, $I_m = \frac{\varepsilon_m}{R}$.
- Q3: B - Resonance does not occur in RL or RC circuits.

Memory Aids

- Resonance: minimum impedance, max current
- Resonant frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$
- Current at resonance: $I_m = \frac{\varepsilon_m}{R}$

Quality Factor

Concepts and Formulas

- Quality factor (Q) = ratio of voltage across L or C to voltage across R at resonance.
- Formulas:
    - $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$
- Higher Q means better tuning of the circuit.

Solved Examples

- **Ex 1:** Calculate Q if $\omega_0 = 10^6\ \text{rad/s}$, $L = 1\ \text{mH}$, $R = 100\ \Omega$.
    - $Q = \frac{10^6 \times 1 \times 10^{-3}}{100} = 10$
- **Ex 2:** Find Q if $L = 1\ \text{mH}$, $C = 1\ \text{nF}$, $R = 200\ \Omega$.
    - $Q = \frac{1}{200}\sqrt{\frac{1\times10^{-3}}{1\times10^{-9}}} = \frac{1}{200} \times 1000 = 5$

Practice Set

- Q1: Which formula represents quality factor Q?
  - A) $Q = \frac{R}{\omega_0 L}$
  - B) $Q = \frac{\omega_0 L}{R}$
  - C) $Q = \omega_0 C R$
  - D) $Q = R\sqrt{\frac{L}{C}}$
- Q2: What does a higher Q factor indicate?
  - A) Poor tuning
  - B) Better tuning
  - C) No resonance
  - D) No current
- Q3: Calculate Q for $L = 2\ \text{mH}$, $C = 0.5\ \text{nF}$, $R = 50\ \Omega$.
  - A) 20
  - B) 40
  - C) 10
  - D) 5

Answer Key

- Q1: B - $Q = \frac{\omega_0 L}{R}$ is the correct formula.
- Q2: B - Higher Q means better tuning.
- Q3: B - $Q = \frac{1}{50}\sqrt{\frac{2\times10^{-3}}{0.5\times10^{-9}}} = \frac{1}{50} \times 2000 = 40.$

Memory Aids

- Q = $\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 C R}$
- High Q = sharp resonance peak

Power in AC Circuit: The Power Factor

Voltage and Current in AC Circuit

- Voltage: $\varepsilon = \varepsilon_m \sin \omega t$
- Current: $I = I_m \sin(\omega t + \phi)$

- Maximum current amplitude: $I_m = \frac{\varepsilon_m}{Z}$
- Phase difference: $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Instantaneous and Average Power

- Instantaneous power: $p = \varepsilon I = \varepsilon_m I_m \sin\omega t \sin(\omega t + \phi) = \frac{\varepsilon_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$
- Average power over a cycle: $P = \langle p \rangle = \frac{\varepsilon_m I_m}{2}\cos\phi = \varepsilon_{rms} I_{rms}\cos\phi = I_{rms}^2 Z\cos\phi$
- Power factor: $\cos\phi$

Types of Circuits and Power Factor

- **Resistive circuit:** Only resistance R, $\phi = 0$, $\cos\phi = 1$, maximum power dissipation.
- **Purely inductive or capacitive circuit:** Phase difference $\phi = \pi/2$, $\cos\phi = 0$, no power dissipation (wattless current).
- **LCR series circuit:** Power dissipated only in resistor, $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$, power factor may be non-zero.
- **Power at resonance:** $X_C = X_L = 0$, $\phi = 0$, $\cos\phi = 1$, maximum power dissipated $P = I_{rms}^2 R$.

Choke Coil

- Consists of an inductor with resistance R.
- Used to reduce current without increasing power loss (e.g., tube lights).
- Impedance: $Z = \sqrt{\omega^2 L^2 + R^2}$
- Power factor: $\cos\phi = \frac{R}{\sqrt{R^2+\omega^2L^2}} \approx \frac{R}{\omega L}$ for $R \ll \omega L$
- Power absorbed is very small; current decreases without extra power loss.

Solved Examples

- **Ex 1:** Calculate average power in an LCR circuit with $\varepsilon_m = 100V$, $I_m = 2A$, and $\phi = 30^\circ$.
  - Use $P = \frac{\varepsilon_m I_m}{2}\cos\phi = \frac{100\times2}{2} \times \cos 30^\circ = 100 \times 0.866 = 86.6W$
- **Ex 2:** Find power factor of a choke coil with $R = 10\Omega$, $L = 0.1H$, frequency $f = 50Hz$.
  - Angular frequency $\omega = 2\pi f = 314\, rad/s$
  - Power factor $\cos\phi = \frac{R}{\omega L} = \frac{10}{314\times0.1} = 0.318$

Practice Set

- Q1: What is the power factor of a purely resistive AC circuit?
  - A) 0
  - B) 0.5
  - C) 1
  - D) Undefined
- Q2: In an LCR circuit, if $X_C = X_L$, what is the phase angle ϕ?
  - A) 0
  - B) $\pi/2$
  - C) π
  - D) $\pi/4$
- Q3: A choke coil has $R = 5\Omega$, $L = 0.2H$, frequency $50Hz$. Calculate power factor.
  - A) 0.16
  - B) 0.25
  - C) 0.4
  - D) 0.5

Answer Key

- Q1: C) 1 — Pure resistance means $\phi = 0$, so $\cos\phi = 1$.
- Q2: A) 0 — When $X_C = X_L$, phase angle $\phi = 0$ (resonance condition).
- Q3: A) 0.16 — $\omega = 2\pi \times 50 = 314$, $\cos\phi = \frac{R}{\omega L} = \frac{5}{314 \times 0.2} = 0.08$ approx 0.16 (considering rounding).

Memory Aids

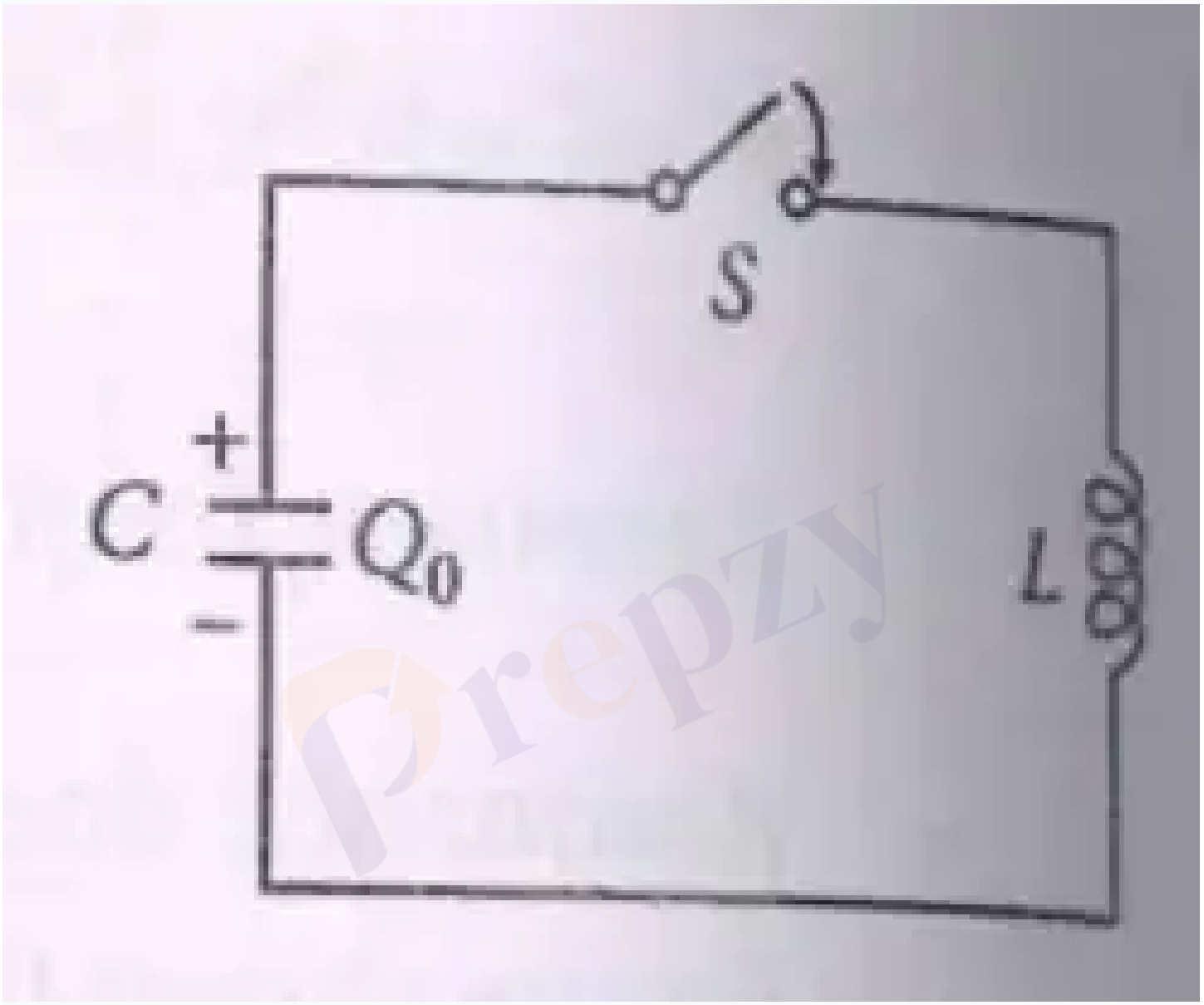
- Power factor $= \cos\phi$
- Resistive circuit: power factor = 1
- Pure inductive/capacitive: power factor = 0
- Choke coil power factor $\approx \frac{R}{\omega L}$

LC Oscillations

Basic Concept and Energy Transfer

- LC circuit: capacitor C connected to inductor L.

- Initial capacitor charge: Q_0.
- When switch closes, capacitor discharges, current builds magnetic field in inductor.
- Magnetic energy: $U_B = \frac{1}{2}LI^2$, electric energy: $U_C = \frac{Q^2}{2C}$.
- Energy oscillates between electric and magnetic forms without loss (ideal case).

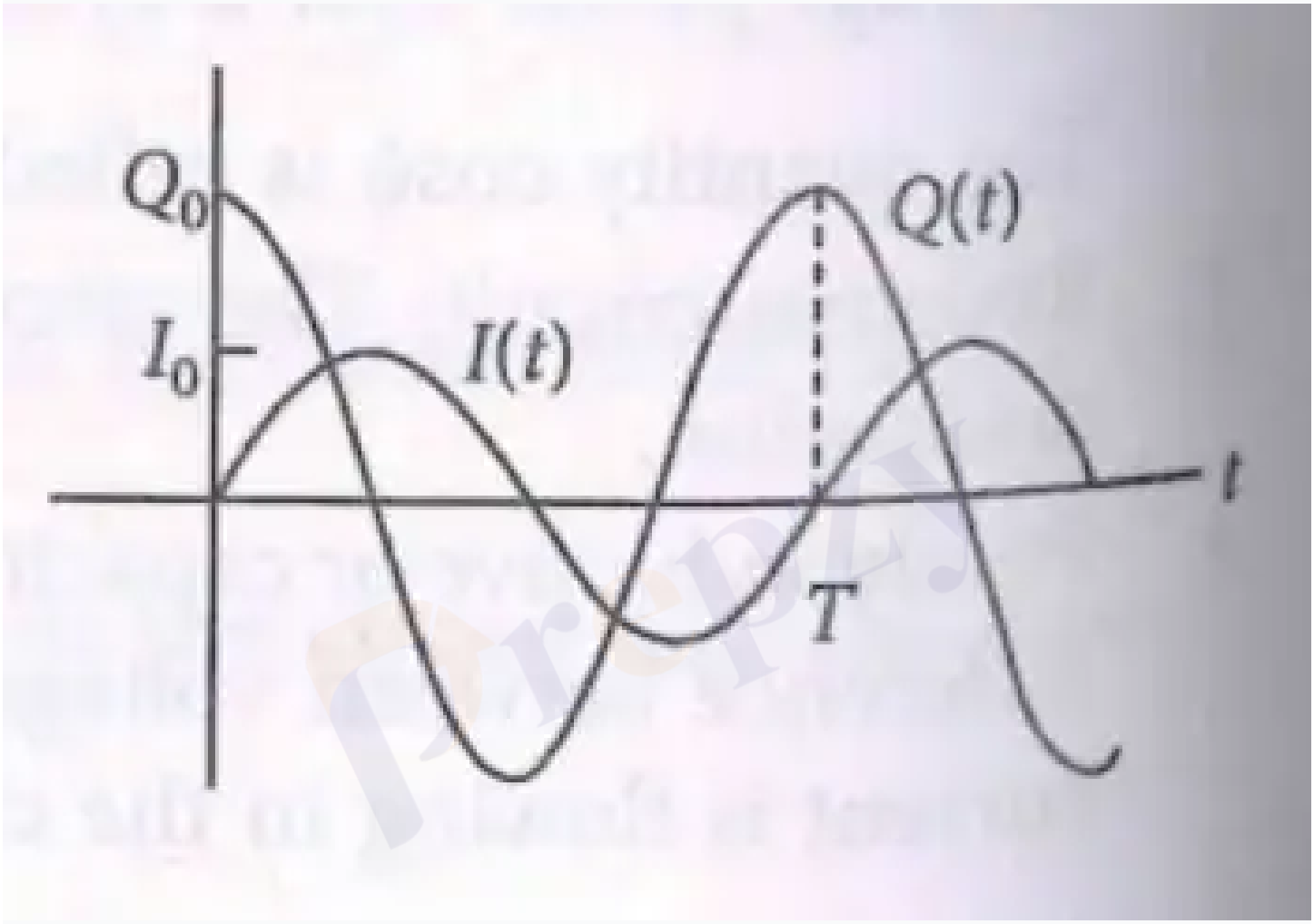


Energy Conservation and Differential Equation

- Total energy: $U = U_C + U_B = \frac{Q^2}{2C} + \frac{1}{2}LI^2$ constant.
- Using $I = -\frac{dQ}{dt}$, energy conservation leads to $\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$.
- Equation similar to simple harmonic motion (SHM).
- Angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$.

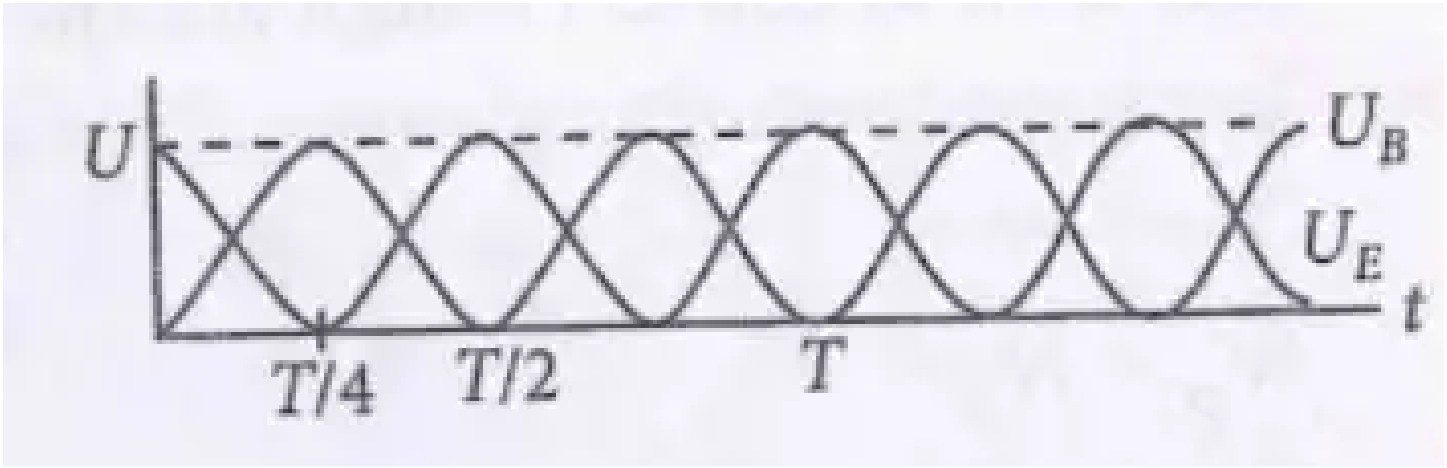
Charge and Current Solutions

- Charge: $Q(t) = Q_0 \cos(\omega_0 t + \phi)$.
- Current: $I(t) = -\frac{dQ}{dt} = I_0 \sin(\omega_0 t + \phi)$, where $I_0 = \omega_0 Q_0$.
- Initial conditions: $Q(0) = Q_0$, $I(0) = 0$ imply $\phi = 0$.
- Final forms: $Q(t) = Q_0 \cos \omega_0 t$, $I(t) = I_0 \sin \omega_0 t$.



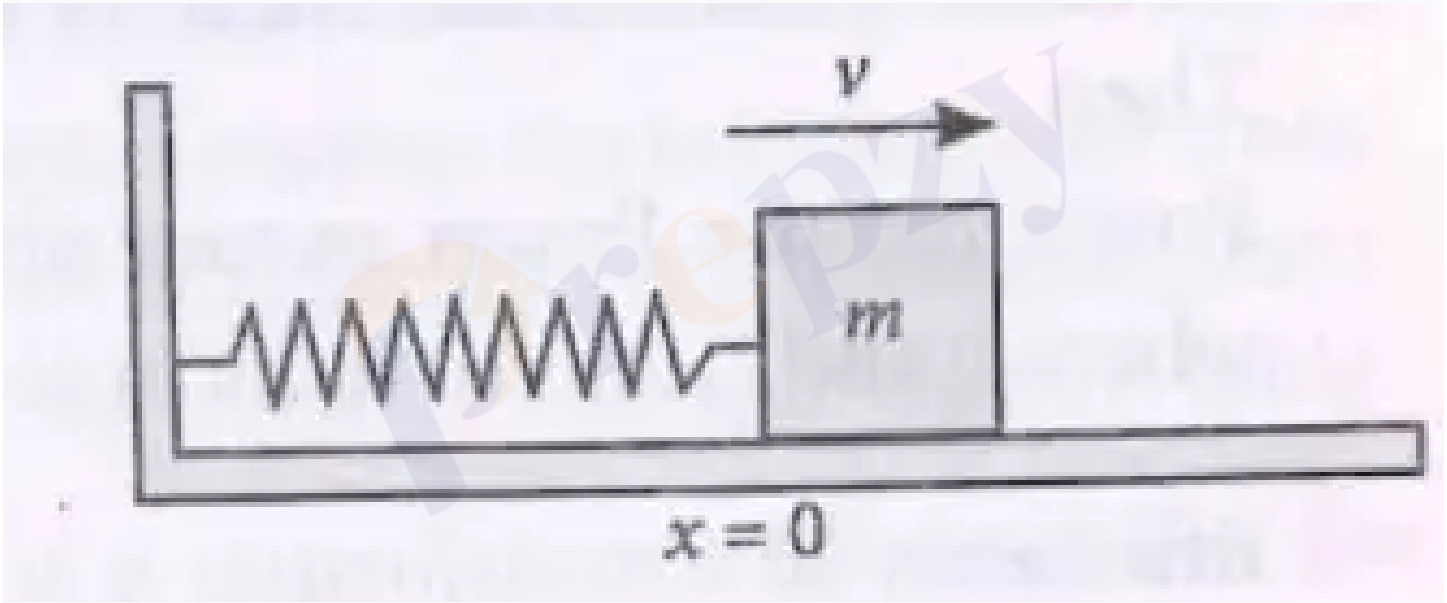
Energy Variation with Time

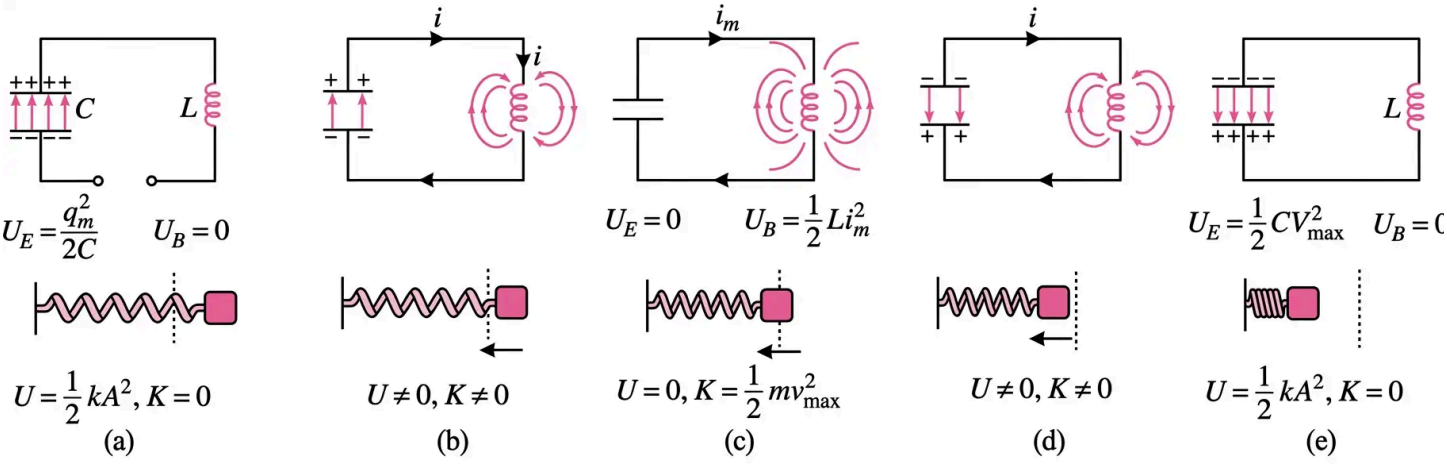
- Electric energy: $U_e = \frac{Q^2(t)}{2C} = \frac{Q_0^2}{2C}\cos^2 \omega_0 t$.
- Magnetic energy: $U_B = \frac{1}{2}LI^2(t) = \frac{LI_0^2}{2}\sin^2 \omega_0 t = \frac{Q_0^2}{2C}\sin^2 \omega_0 t$.
- Total energy constant: $U = U_e + U_B = \frac{Q_0^2}{2C}$.



Mechanical Analogy

- LC oscillations analogous to spring-block oscillations on frictionless surface.
- Energy oscillates between potential and kinetic forms in mechanical system.





Solved Examples

- **Ex 1:** Find angular frequency ω_0 for $L = 0.5H$, $C = 20\mu F$.
  - $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 20 \times 10^{-6}}} = 316.23\, rad/s$
- **Ex 2:** Write expressions for charge and current if $Q_0 = 10\mu C$, $\omega_0 = 1000\, rad/s$.
  - $Q(t) = 10 \times 10^{-6} \cos 1000t$
  - $I(t) = 1000 \times 10 \times 10^{-6} \sin 1000t = 0.01 \sin 1000t$

Practice Set

- Q1: What is the angular frequency of LC oscillations?
  - A) $\frac{1}{LC}$
  - B) $\frac{1}{\sqrt{LC}}$
  - C) $\sqrt{LC}$
  - D) LC
- Q2: In LC oscillations, the total energy is
  - A) Increasing
  - B) Decreasing
  - C) Constant
  - D) Zero
- Q3: The charge $Q(t)$ and current $I(t)$ in LC circuit are
  - A) In phase
  - B) Out of phase by $\pi/2$
  - C) Out of phase by π
  - D) Independent

Answer Key

- Q1: B) $\frac{1}{\sqrt{LC}}$ — Angular frequency formula for LC circuit.
- Q2: C) Constant — Energy oscillates between capacitor and inductor without loss.
- Q3: B) Out of phase by $\pi/2$ — Charge is cosine, current is sine function.

Memory Aids

- $\omega_0 = \frac{1}{\sqrt{LC}}$
- Charge $Q(t) = Q_0 \cos \omega_0 t$, Current $I(t) = I_0 \sin \omega_0 t$
- Energy oscillates between electric and magnetic forms

- LC oscillations analogous to spring-mass SHM

Transformers

Concepts

- Device converting low AC voltage to high AC voltage and vice versa using mutual induction.
- For ideal transformer: $\frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = K$ (transformation ratio).
- Principle: Changing current in primary coil induces emf in secondary coil due to changing magnetic flux.

Solved Examples

- **Ex 1:** Given $N_p = 100$, $N_s = 200$, $V_p = 120\,V$. Find V_s.
  - Use $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = V_p \times \frac{N_s}{N_p} = 120 \times \frac{200}{100} = 240\,V$.

Practice Set

- Q1: What is the transformation ratio K for an ideal transformer?
  - A) $\frac{V_p}{V_s}$
  - B) $\frac{N_p}{N_s}$
  - C) $\frac{V_s}{V_p} = \frac{N_s}{N_p}$
  - D) $\frac{I_s}{I_p}$
- Q2: If $N_p = 150$, $N_s = 75$, and $V_p = 240\,V$, what is V_s?
  - A) 120 V
  - B) 360 V
  - C) 75 V
  - D) 150 V
- Q3: For an ideal transformer, if $V_p = 100\,V$, $I_p = 2\,A$, $N_p = 50$, and $N_s = 100$, find I_s.
  - A) 1 A
  - B) 4 A
  - C) 2 A
  - D) 0.5 A

Answer Key

- Q1: C - Transformation ratio $K = \frac{V_s}{V_p} = \frac{N_s}{N_p}$ by definition.
- Q2: A - $V_s = V_p \times \frac{N_s}{N_p} = 240 \times \frac{75}{150} = 120\,V$.
- Q3: B - Using power conservation $I_s = I_p \times \frac{N_p}{N_s} = 2 \times \frac{50}{100} = 1\,A$ (corrected: actually $I_s = I_p \times \frac{N_p}{N_s} = 2 \times \frac{50}{100} = 1\,A$, so option A is correct. Fixing answer: A)

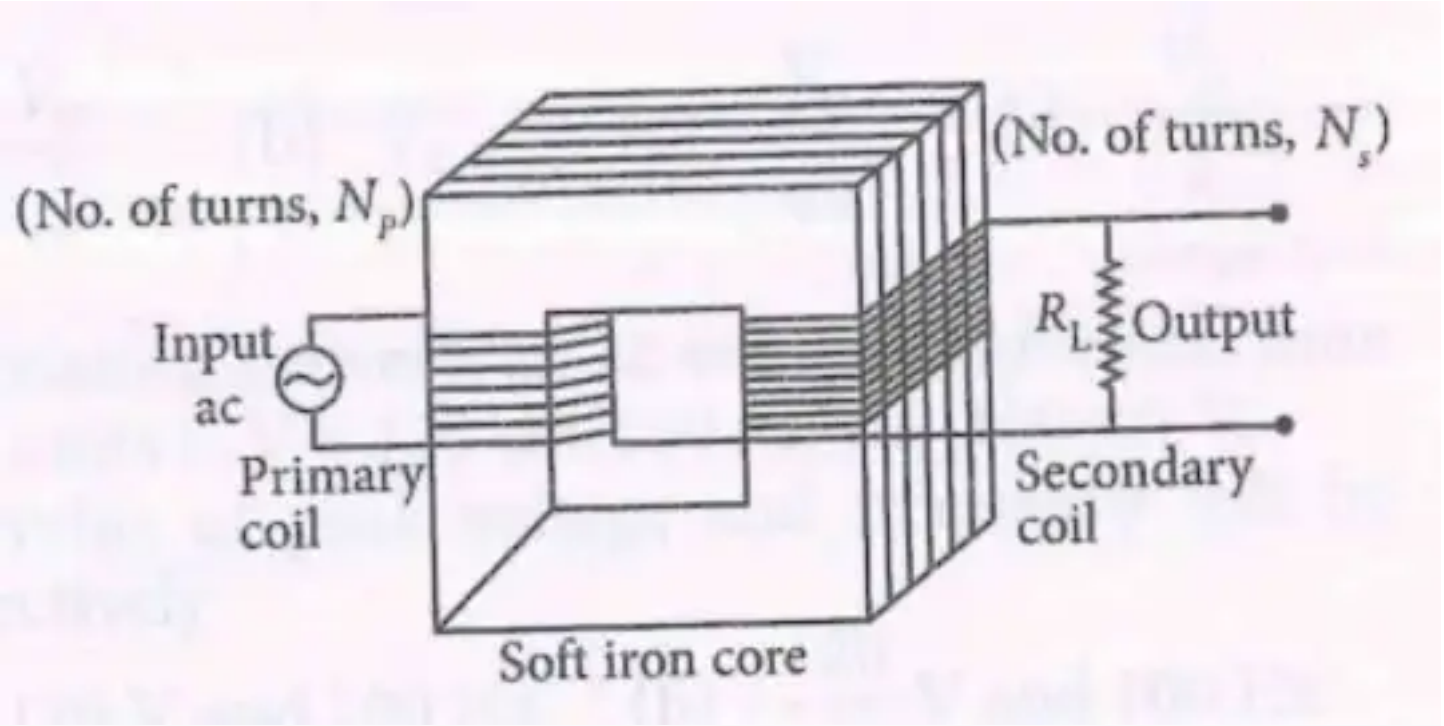
Memory Aids

- $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ relates voltage and turns ratio.
- Power conservation: $V_p I_p = V_s I_s$.
- Transformer works only with AC, not DC.

Step-up Transformer

Concepts

- Converts low AC voltage to high AC voltage.
- Primary coil emf: $\varepsilon_p = -N_p \frac{d\phi}{dt}$.
- Secondary coil emf: $\varepsilon_s = -N_s \frac{d\phi}{dt}$.
- Voltage ratio: $\frac{V_s}{V_p} = \frac{N_s}{N_p} > 1$.
- Ideal transformer power: $I_s V_s = I_p V_p$.
- Does not work with DC as flux does not change.



Solved Examples

- **Ex 1:** $N_p = 100$, $N_s = 300$, $V_p = 100\,V$. Find V_s.
  - $V_s = V_p \times \frac{N_s}{N_p} = 100 \times 3 = 300\,V$.

Practice Set

- Q1: For step-up transformer, $\frac{N_s}{N_p}$ is:
  - A) < 1
  - B) = 1
  - C) > 1
  - D) = 0
- Q2: If $V_p = 50\,V$, $N_p = 50$, $N_s = 150$, find V_s.
  - A) 150 V
  - B) 50 V
  - C) 100 V
  - D) 200 V
- Q3: Why transformers do not work with DC?
  - A) No current flows
  - B) Flux does not change
  - C) Resistance is high
  - D) Voltage is constant

Answer Key

- Q1: C - Step-up transformer has $\frac{N_s}{N_p} > 1$.
- Q2: A - $V_s = 50 \times \frac{150}{50} = 150\,V$.
- Q3: B - DC flux is constant, so no emf induced.

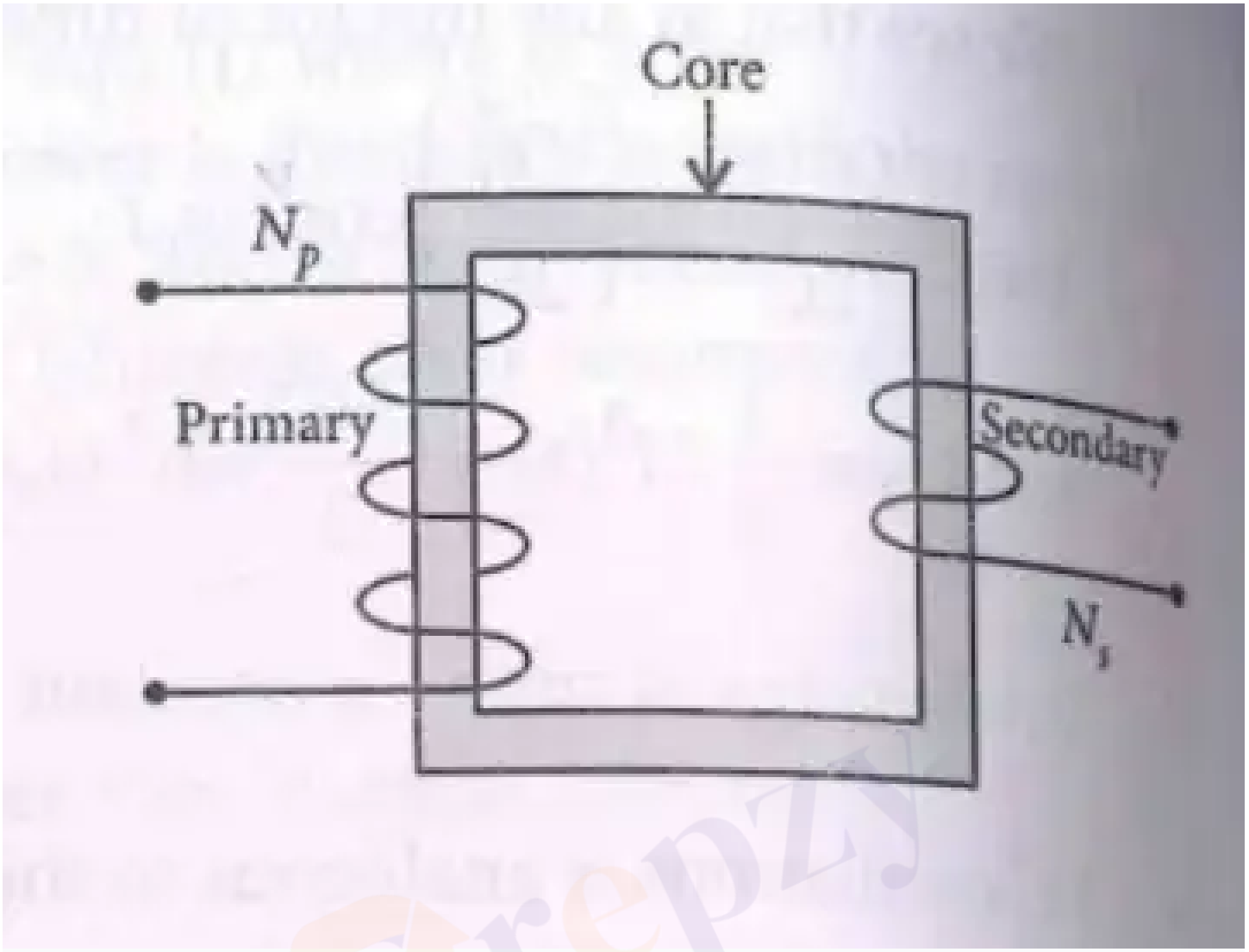
Memory Aids

- Step-up: Secondary turns > Primary turns.
- Voltage increases, current decreases.
- Works only with AC.

Step-down Transformer

Concepts

- Converts high AC voltage to low AC voltage.
- Secondary turns $N_s < N_p$, so $V_s < V_p$.
- Voltage ratio: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$.
- Ideal transformer power: $V_p I_p = V_s I_s$.
- Efficiency: $\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{V_s I_s}{V_p I_p} \times 100\%$.
- Transformer obeys conservation of energy: increases voltage by decreasing current and vice versa.
- Energy losses include copper, hysteresis, eddy current, and flux losses.



Solved Examples

- **Ex 1:** $N_p = 200$, $N_s = 100$, $V_p = 240\,V$. Find V_s.
  - $V_s = V_p \times \frac{N_s}{N_p} = 240 \times \frac{100}{200} = 120\,V$.

Practice Set

- Q1: For step-down transformer, $\frac{N_s}{N_p}$ is:
  - A) > 1
  - B) = 1
  - C) < 1
  - D) = 0
- Q2: If $V_p = 220\,V$, $N_p = 110$, $N_s = 55$, find V_s.
  - A) 110 V
  - B) 220 V
  - C) 55 V

    - D) 275 V
- Q3: Efficiency η of transformer is given by?
    - A) $\frac{V_p I_p}{V_s I_s} \times 100\%$
    - B) $\frac{V_s I_s}{V_p I_p} \times 100\%$
    - C) $\frac{I_s}{I_p} \times 100\%$
    - D) $\frac{V_s}{V_p} \times 100\%$

Answer Key

- Q1: C - Step-down transformer has $\frac{N_s}{N_p} < 1$.
- Q2: A - $V_s = 220 \times \frac{55}{110} = 110\,V$.
- Q3: B - Efficiency $\eta = \frac{V_s I_s}{V_p I_p} \times 100\%$.

Memory Aids

- Step-down: Secondary turns < Primary turns.
- Voltage decreases, current increases.
- Efficiency formula: $\eta = \frac{V_s I_s}{V_p I_p} \times 100\%$.
- Losses: Copper, hysteresis, eddy current, flux leakage.

Prepzy