CBSE EXAMINATION PAPER-2024

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 88

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **44 questions.** All questions are **compulsory.**

ii. This question paper is divided into **5 sections.**

iii. **Section A** – questions number **1 to 3** are case based questions

iv. **Section B** – questions number **4 to 23** are multiple choice questions

v. **Section C** – questions number **24 to 30** are very short answer

vi. **Section D** – questions number **31 to 38** are short answer

vii. **Section E** – questions number **39 to 44** are long answer

viii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

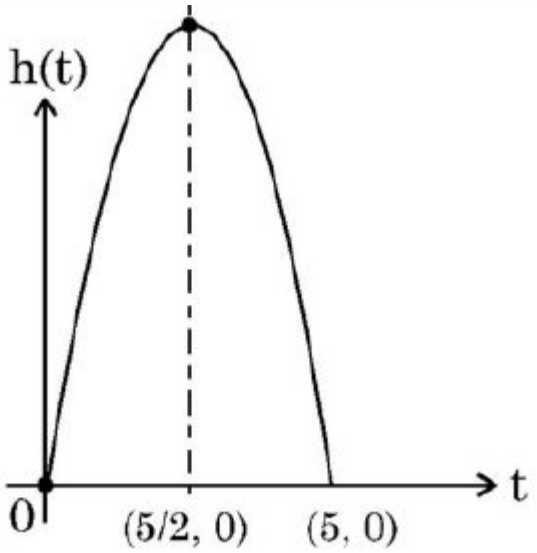
ix. Use of calculator is NOT allowed.

Section A

Question 1.

A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$. Observe the graph of the polynomial and answer the following questions:

Observe the graph of the polynomial and answer the following questions :



(1) Find the two different values of t when the height of the ball was 20 m.

[2 Marks]

Answer: Given the height $h = 25t – 5t^2$, we set $h = 20$ to find the values of t when the ball is at 20 meters. So, $20 = 25t – 5t^2$. Rearranging, we get $5t^2 – 25t + 20 = 0$. Dividing the entire equation by 5 gives $t^2 – 5t + 4 = 0$. Factorizing the quadratic equation: $(t – 4)(t – 1) = 0$. Therefore, the two values of t are t = 1 second and t = 4 seconds.

Key Points: Set h = 20 in the given equation-Form quadratic equation $5t^2 – 25t + 20 = 0$-Simplify the equation to $t^2 – 5t + 4 = 0$-Factorize the quadratic equation-$(t – 4)(t – 1) = 0$-Find values of t as 1 second and 4 seconds

(2) Find the maximum height achieved by ball.

[1 Marks]

Answer: The height h of the ball at time t is given by $h = 25t – 5t^2$, which is a quadratic expression. Since the coefficient of t^2 is negative (-5), the graph of the polynomial opens downward, meaning it has a maximum point. To find the maximum height, we calculate the vertex of the parabola. The time at which maximum height is reached is $t = -b/(2a) = -25/(2 \times -5) = 2.5$ seconds. Substituting t = 2.5 back into the equation, we get $h = 25 \times 2.5 – 5 \times (2.5)^2 = 62.5 – 31.25 = 31.25$ metres. Therefore, the maximum height achieved by the ball is 31.25 metres.

Key Points: The height is modeled by quadratic expression $h = 25t – 5t^2$ - Coefficient of t^2 is negative indicating parabola opens downward - Maximum height is at vertex with $t = -b/(2a) = 2.5$ seconds - Substitute t = 2.5 to find maximum height h = 31.25 metres

(3) Write zeroes of the given polynomial.

[1 Marks]

Answer: The zeroes of the polynomial $h(t) = 25t – 5t^2$ are found by setting h(t) equal to zero. That is, $25t – 5t^2 = 0$. Taking 5t common, we get $5t(5 - t) = 0$. Therefore, t = 0 or t = 5. Thus, the zeroes of the polynomial are t = 0 and t = 5.

Key Points: Write the polynomial function $h(t) = 25t – 5t^2$-Set the polynomial equal to zero to find zeroes-Solve $25t – 5t^2 = 0$ by factoring out 5t-Find values of t that satisfy the equation-Identify zeroes as t = 0 and t = 5

(4) After throwing upward, how much time did the ball take to reach the height of 30 m?

[2 Marks]

Answer: The height of the ball at time t seconds is given by h = 25t - 5t². To find the time(s) when the height is 30 meters, we set h = 30.\n\nSo, 30 = 25t - 5t²\nRearranging, 5t² - 25t + 30 = 0\nDividing the entire equation by 5 gives:\nt² - 5t + 6 = 0\n\nFactoring the quadratic,\n(t - 2)(t - 3) = 0\nSo, t = 2 seconds or t = 3 seconds.\n\nThis means the ball reaches 30 meters height at 2 seconds while going up and again at 3 seconds while coming down. After throwing upward, the ball takes 2 seconds to reach the height of 30 meters.

Key Points: Use the given height function h = 25t - 5t²- Set h = 30 and form the equation 25t - 5t² = 30- Rearrange and simplify the quadratic equation- Solve the quadratic equation to get two values of t- Identify the time when the ball is going up (smaller t) as the required time

Question 2.

The word ‘circus’ has the same root as ‘circle’. In a closed circular area, various entertainment acts including human skill and animal training are presented before the crowd.

A circus tent is cylindrical up to a height of 8 m and conical above it. The diameter of the base is 28 m and total height of tent is 18.5 m.

Based on the above, answer the following questions:



(1) Find slant height of the conical part.

[1 Marks]

Answer: The height of the conical part = total height - height of the cylindrical part = 18.5 m - 8 m = 10.5 m. The radius of the base = diameter ÷ 2 = 28 m ÷ 2 = 14 m. Using the Pythagoras theorem, the slant height of the cone = $\sqrt{(\text{height}^2 + \text{radius}^2)} = \sqrt{(10.5^2 + 14^2)} = \sqrt{(110.25 + 196)} = \sqrt{306.25} = 17.5$ m.

Key Points: Height of conical part = total height - cylindrical height - Radius = diameter ÷ 2 - Use Pythagoras theorem to find slant height - Slant height = $\sqrt{(\text{height}^2 + \text{radius}^2)}$

(2) Determine the floor area of the tent.

[1 Marks]

Answer: The floor area of the tent is the area of the base which is a circle with diameter 28 m. The radius r = 28 ÷ 2 = 14 m. Area of the floor = $\pi \times r^2 = 3.14 \times 14 \times 14 = 615.44$ m².

Key Points: The floor area is the area of the circle at the base - radius is half of diameter - use formula area = $\pi \times r^2$ - substitute values and calculate the area

(3) Find area of the cloth used for making tent.

[2 Marks]

Answer: To find the area of the cloth used for making the tent, we need to find the curved surface area of the cylindrical part and the curved surface area of the conical part and add them. \n\n1. For the cylindrical part:\n - Diameter = 28 m, so radius = 14 m\n - Height = 8 m\n - Curved surface area of cylinder = 2 × π × radius × height = 2 × 3.14 × 14 × 8 = 702.4 m²\n\n2. For the conical part:\n - Total height of tent = 18.5 m\n - Height of cone = 18.5 m - 8 m = 10.5 m\n - Radius = 14 m\n - Slant height = √(radius² + height²) = √(14² + 10.5²) = √(196 + 110.25) = √306.25 ≈ 17.5 m\n - Curved surface area of cone = π × radius × slant height = 3.14 × 14 × 17.5 = 769.7 m²\n\n3. Total area of cloth used = curved surface area of cylinder + curved surface area of cone = 702.4 + 769.7 = 1472.1 m²\n\nTherefore, the area of the cloth used for making the tent is approximately 1472.1 square meters.

Key Points: Calculate radius from diameter of base of tent-Calculate curved surface area of the cylindrical part-Calculate height and slant height of the conical part-Calculate curved surface area of the conical part-Add both curved surface areas to get total cloth area

(4) Find total volume of air inside an empty tent.

[2 Marks]

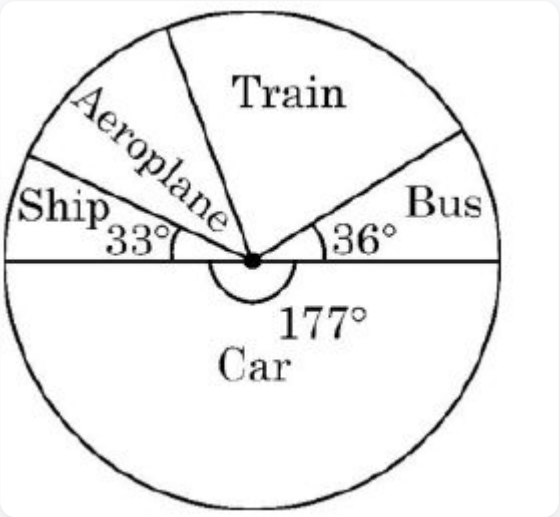
Answer: The tent has two parts: a cylinder and a cone on top. First, find the height of the conical part by subtracting the height of the cylinder from the total height: 18.5 m - 8 m = 10.5 m. The radius of the base is half of the diameter, so r = 28 / 2 = 14 m. Volume of cylinder = π × r² × height = 3.14 × 14 × 14 × 8 = 4929.92 cubic meters. Volume of cone = (1/3) × π × r² × height = (1/3) × 3.14 × 14 × 14 × 10.5 = 2168.52 cubic meters. Total volume = volume of cylinder + volume of cone = 4929.92 + 2168.52 = 7098.44 cubic meters. So, the total volume of air inside the tent is approximately 7098.44 cubic meters.

Key Points: Identify the shapes of the tent (cylinder and cone)-Calculate radius from diameter-Subtract cylinder height from total height to get cone height-Use formula for volume of cylinder (πr²h)-Use formula for volume of cone (1/3 πr²h)-Add both volumes for total volume

Question 3.

In a survey on holidays, 120 people were asked to state which type of transport they used on their last holiday. The following pie chart shows the results of the survey.

Observe the pie chart and answer the following questions :



(1) If one person is selected at random, find the probability that he/she travelled by bus or ship.

[1 Marks]

Answer: From the pie chart, the percentage of people who travelled by bus is given, and similarly for ship. Adding these two percentages gives the total percentage of people who travelled by bus or ship. Probability is the number of favorable outcomes divided by the total number of outcomes. So, the probability that a randomly selected person travelled by bus or ship = (percentage of bus + percentage of ship) / 100.

Key Points: Identify percentage of bus travel-Identify percentage of ship travel-Add the two percentages-Calculate probability by dividing sum by 100-Understand probability as favorable outcomes over total outcomes

(2) Which is most favourite mode of transport and how many people used it?

[1 Marks]

Answer: The most favourite mode of transport is the one with the largest percentage in the pie chart. Since the total number of people surveyed is 120, the number of people using the most popular mode of transport can be found by calculating the percentage of that mode out of 120.

Key Points: Identify the mode with the largest percentage in the pie chart-Total number of people surveyed is 120-Calculate the number of people by using (percentage × 120) ÷ 100

(3) The probability that randomly selected person used aeroplane is 7/60. Find the revenue collected by air company at the rate of ₹ 5,000 per person.

[2 Marks]

Answer: The probability of a person using an aeroplane is given as 7/60. The total number of people surveyed is 120. Number of people who used aeroplane = Probability × Total number of people = (7/60) × 120 = 14 people. The revenue collected by the air company = Number of people × Rate per person = 14 × ₹ 5,000 = ₹ 70,000. Therefore, the revenue collected by the air company is ₹ 70,000.

Key Points: Understand probability as fraction of total-Calculate number of people who used aeroplane by multiplying probability and total people-Calculate total revenue by multiplying number of people by rate per person

(4) A person is selected at random. If the probability that he did not use train is 4/5, find the number of people who used train.

[2 Marks]

Answer: Given the total number of people is 120. The probability that a person did not use train is 4/5. Therefore, the probability that a person used train = 1 - 4/5 = 1/5. Number of people who used train = (1/5) × 120 = 24. Hence, 24 people used train.

Key Points: Total number of people is 120-Probability of not using train is 4/5-Calculate probability of using train by subtracting from 1-Find number of train users by multiplying total people with probability of train use

Section B

Question 4.

The value of k for which the system of equations 3x – y + 8 = 0 and 6x – ky + 16 = 0 has infinitely many solutions, is

[1 Marks]

(A) 2

(B) -1/2

(C) 1/2

(D) -2

Explanation:

For a system of two linear equations to have infinitely many solutions, the two equations must be proportional to each other. Here, comparing the two equations: 3x – y + 8 = 0 and 6x – ky + 16 = 0, multiplying the first equation by 2 gives 6x – 2y + 16 = 0. For the second equation to be proportional, we must have k = 2. So, the correct value of k is 2.

Question 5.

Point P divides the line segment joining points A(4, –5) and B(1, 2) in the ratio 5:2. Coordinates of point P are

[1 Marks]

(A) (5/2,-3/2)

(B) (13/7, 0)

(C) (0,13/7)

(D) (11/7,0)

Explanation:

To find the coordinates of point P that divides the segment AB in the ratio 5:2, we use the section formula: P = ((m*x2 + n*x1)/(m+n), (m*y2 + n*y1)/(m+n)) where A(x1, y1) = (4, –5), B(x2, y2) = (1, 2), m = 5 and n = 2. So, x-coordinate of P = (5*1 + 2*4)/(5+2) = (5 + 8)/7 = 13/7, y-coordinate of P = (5*2 + 2*(-5))/7 = (10 - 10)/7 = 0. Therefore, the coordinates of P are (13/7, 0). Hence, the correct option is (13/7, 0).

Question 6.

The common difference of an A.P. in which $a_{15} – a_{11}= 48$, is

[1 Marks]

(A) -12

(B) 12

(C) -16

(D) 16

Explanation: We know that the nth term of an AP is given by: a_n = a_1 + (n - 1)d. Therefore, a_15 = a_1 + 14d and a_11 = a_1 + 10d. The difference a_15 - a_11 = (a_1 + 14d) - (a_1 + 10d) = 4d. Given that this difference is 48, we have 4d = 48, which gives d = 12. Hence, the common difference is 12.

Question 7.

The quadratic equation $x^2 + x + 1 = 0$ has _____ roots.

[1 Marks]

(A) real and distinct

(B) irrational

(C) not-real

(D) real and equal

Explanation: To determine the nature of roots for the quadratic equation $x^2 + x + 1 = 0$, we calculate the discriminant ($b^2 - 4ac$), where a = 1, b = 1, and c = 1. The discriminant is $1^2 - 4 \times 1 \times 1 = 1 - 4 = -3$, which is less than 0. According to the context, if the discriminant is less than 0, the quadratic equation has no real roots. Therefore, the roots of this equation are not-real.

Question 8.

If HCF(2520, 6600) = 40 and LCM(2520, 6600) = 252 × k, then the value of k is

[1 Marks]

(A) 1600

(B) 1650

(C) 165

(D) 1625

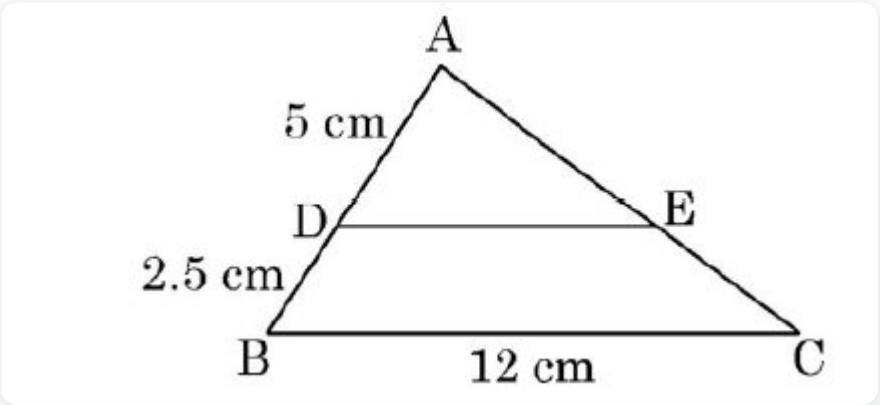
Explanation:

We know the relation: HCF × LCM = product of the two numbers. Given HCF(2520, 6600) = 40, therefore, LCM(2520, 6600) = (2520 × 6600) / 40 = 415800. It is also given that LCM = 252 × k, so k = 415800 ÷ 252 = 1650. Hence, the correct option is 1650.

Question 9.

In the given figure, ΔABC is shown. DE is parallel to BC. If AD = 5 cm, DB = 2.5 cm and BC = 12 cm, then DE is equal to

[1 Marks]



(A) 10 cm

(B) 7.5 cm

(C) 6 cm

(D) 8 cm

Explanation:

Since DE is parallel to BC, by the Basic Proportionality Theorem, the sides are divided proportionally. Given AD = 5 cm and DB = 2.5 cm, the ratio AD:DB = 5:2.5 = 2:1. Therefore, DE is two-thirds of BC. Since BC = 12 cm, DE = (2/3) × 12 = 8 cm. Hence, the correct answer is 8 cm.

Question 10.

If $\sin\theta = \cos\theta$, $(0° < \theta < 90°)$, then value of $(\sec\theta \cdot \sin\theta)$ is:

[1 Marks]

(A) 0

(B) 1

(C) √2

(D) 1/√2

Explanation:

Given $\sin\theta = \cos\theta$ and $0° < \theta < 90°$, this occurs at $\theta = 45°$. At 45°, $\sin 45° = \cos 45° = 1/\sqrt{2}$. Now, $\sec\theta = 1/\cos\theta$, so $\sec 45° = \sqrt{2}$. Therefore, $\sec\theta \cdot \sin\theta = (\sqrt{2}) \times (1/\sqrt{2}) = 1$. Hence, the correct value is 1.

Question 11.

Two dice are rolled together. The probability of getting the sum of the two numbers to be more than 10, is

[1 Marks]

(A) 1/9

(B) 1/12

(C) 7/12

(D) 1/6

Explanation: When two dice are rolled, there are a total of 36 possible outcomes. The sums that are more than 10 are 11 and 12. The sum 11 can be obtained by (5,6) and (6,5) - 2 ways, and the sum 12 can be obtained by (6,6) - 1 way. So, total favorable outcomes = 2 + 1 = 3. Therefore, the probability = 3/36 = 1/12.

Question 12.

If α and β are zeroes of the polynomial $5x^2 + 3x – 7$, the value of $1/\alpha + 1/\beta$ is

[1 Marks]

(A) 3/5

(B) -5/7

(C) -3/7

(D) 3/7

Explanation:

For a quadratic polynomial $ax^2 + bx + c$, the sum of zeroes $\alpha + \beta = -b/a$ and the product $\alpha\beta = c/a$. Here, a = 5, b = 3, c = -7. So, $\alpha + \beta = -3/5$ and $\alpha\beta = -7/5$. The value of $1/\alpha + 1/\beta = (\alpha + \beta) / (\alpha\beta) = (-3/5) / (-7/5) = 3/7$. Hence, the correct answer is 3/7.

Question 13.

The perimeters of two similar triangles ABC and PQR are 56 cm and 48 cm respectively. PQ/AB is equal to

[1 Marks]

(A) 8/7

(B) 7/6

(C) 7/8

(D) 6/7

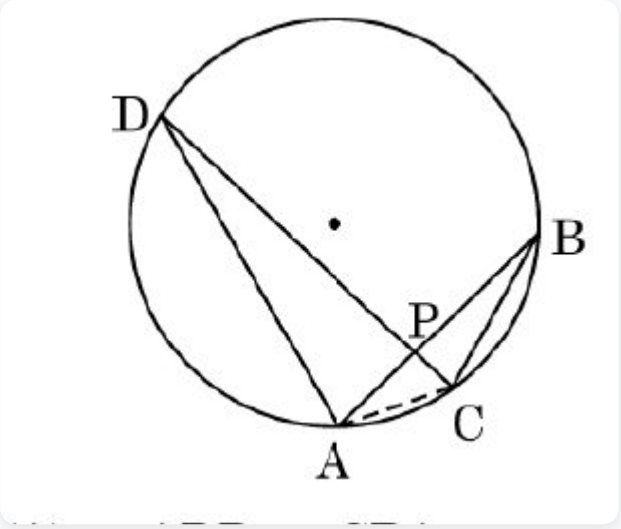
Explanation:

Since the triangles ABC and PQR are similar, the ratio of their corresponding sides is equal to the ratio of their perimeters. Given Perimeter of ABC = 56 cm and Perimeter of PQR = 48 cm, the ratio PQ/AB = Perimeter of PQR / Perimeter of ABC = 48/56 = 6/7. Hence, the correct option is 6/7.

Question 14.

AB and CD are two chords of a circle intersecting at P. Choose the correct statement from the following :

[1 Marks]



(A) ΔADP ~ ΔCBA

(B) ΔADP ~ ΔBPC

(C) ΔADP ~ ΔCBP

(D) ΔADP ~ ΔDBCP

Explanation: The correct statement is ΔADP ~ ΔCBP. When two chords AB and CD of a circle intersect at point P, the triangles formed by joining the endpoints of chords and the point of intersection are similar in such pairs due to the equality of the angles subtended by the chords. Specifically, ΔADP is similar to ΔCBP by the AA criterion, as the angles opposite the intersecting chords are equal.

Question 15.

If each observation in a data is increased by 2, then the median of the new data

[1 Marks]

(A) remains same

(B) increases by 2n

(C) decreases by 2

(D) increases by 2

Explanation: When each observation in a dataset is increased by a constant value (in this case, 2), the position of the median does not change, but its value increases by that constant. This is because median depends on the order of data, and adding the same number to all observations shifts all data points equally, including the median. Therefore, the median of the new data increases by 2.

Question 16.

A box contains cards numbered 6 to 55. A card is drawn at random from the box. The probability that the drawn card has a number which is a perfect square, is

[1 Marks]

(A) 7/55

(B) 1/10

(C) 7/50

(D) 5/49

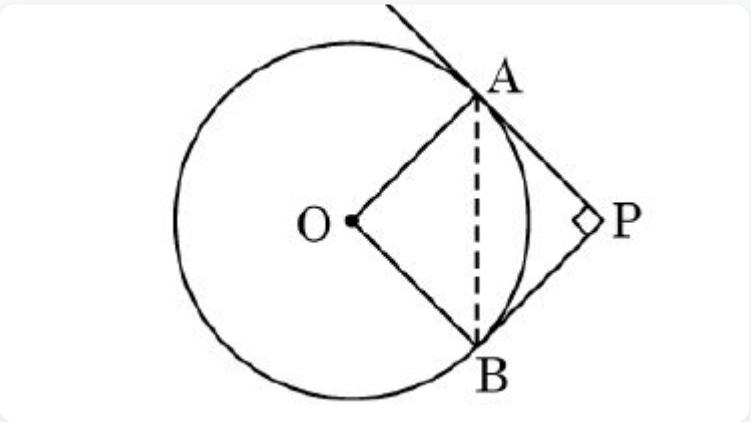
Explanation:

The total number of cards is from 6 to 55, which is 55 - 6 + 1 = 50. The perfect squares between 6 and 55 are 9 (3^2), 16 (4^2), 25 (5^2), 36 (6^2), 49 (7^2). There are 5 such perfect squares. Hence, the probability = Number of favorable outcomes / Total number of outcomes = 5 / 50 = 1/10.

Question 17.

In the given figure, tangents PA and PB to the circle centered at O, from point P are perpendicular to each other. If PA = 5 cm, then length of AB is equal to

[1 Marks]



(A) 5√2cm

(B) 5 cm

(C) 2√5cm

(D) 10 cm

Explanation:

Since PA and PB are tangents from point P to the circle and are perpendicular, the angle between the tangents is 90 degrees. The points A and B lie on the circle where the tangents touch. The chord AB subtends a right angle at point P outside the circle, making triangle PAB a right-angled triangle with PA = PB = 5 cm (tangents from a point are equal). Therefore, triangle PAB is an isosceles right-angled triangle with legs PA and PB and hypotenuse AB. Using the Pythagorean theorem, AB = $\sqrt{(PA^2 + PB^2)}$ = $\sqrt{(5^2 + 5^2)}$ = $\sqrt{(25 + 25)}$ = $\sqrt{50}$ = $5\sqrt{2}$ cm. Hence, the length of chord AB is $5\sqrt{2}$ cm.

Question 18.

XOYZ is a rectangle with vertices X(–3, 0), O(0, 0), Y(0, 4) and Z(x, y). The length of each diagonal is

[1 Marks]

(A) $x^2 + y^2$ units

(B) 4 units

(C) 5 units

(D) $\sqrt{5}$ units

Explanation: The diagonal of rectangle XOYZ can be found using the distance formula between points X(–3, 0) and Y(0, 4). The length of the diagonal = √[(0 - (–3))^2 + (4 - 0)^2] = √[(3)^2 + (4)^2] = √(9 + 16) = √25 = 5 units. Therefore, the length of each diagonal is 5 units.

Question 19.

Which term of the A.P. –29, –26, –23, ..., 61 is 16?

[1 Marks]

(A) 10^{th}

(B) 11^{th}

(C) 16^{th}

(D) 31^{st}

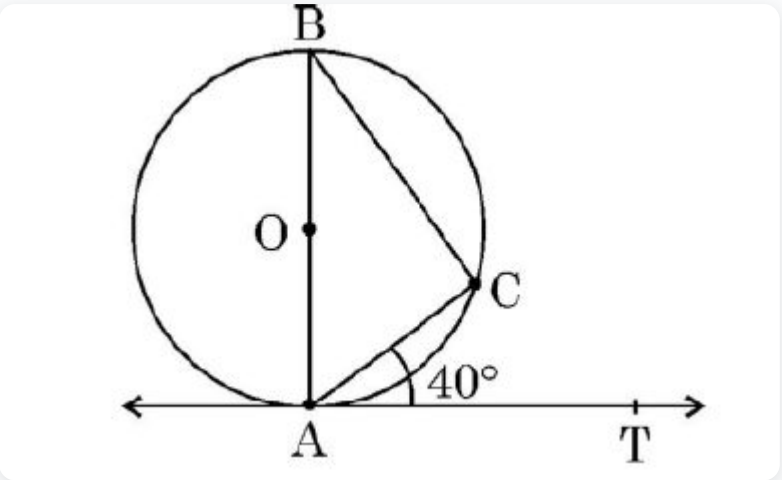
Explanation:

The given A.P. starts at -29 and increases by 3 each term (common difference d = -26 - (-29) = 3). The nth term of an AP is given by a + (n-1)d, where a = -29 and d = 3. We set a + (n-1)d = 16, which gives -29 + (n-1)*3 = 16. Solving this, (n-1)*3 = 16 + 29 = 45, so n - 1 = 15, and n = 16. Therefore, the 16th term of the AP is 16. Hence, the correct option is 16^{th}.

Question 20.

In the given figure, AT is tangent to a circle centered at O. If ∠CAT = 40°, then ∠CBA is

[1 Marks]



(A) 40°

(B) 50°

(C) 65°

(D) 70°

Explanation: Since AT is tangent to the circle at point A, the radius OA is perpendicular to the tangent AT, making angle OAT = 90°. Given that ∠CAT = 40°, then in triangle OAT, ∠OAT + ∠CAT + ∠O = 180°, where ∠O = angle at center which is 90°. Thus, ∠OAT = 90° - 40° = 50°. Using the properties of the circle and angles subtended by the same chord, ∠CBA is equal to 50°. Therefore, the correct answer is 50°.

Question 21.

After an examination, a teacher wants to know the marks obtained by the maximum number of students in her class. She requires to calculate _____ of marks.

[1 Marks]

(A) median

(B) range

(C) mode

(D) mean

Explanation: The correct answer is 'mode'. Mode represents the value or range of marks that occur most frequently in the dataset. Since the teacher wants to know the marks obtained by the maximum

number of students, calculating the mode will give the marks that are most common among the students.

Question 22. Assertion (A) : If sin A =1/3 (0° < A <90°), then the value of cos A is 2√2/3 Reason (R) : For every angle θ, $\sin^2\theta + \cos^2\theta = 1$.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

(B) Assertion (A) is true but Reason (R) is not true.

(C) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of (A).

(D) Assertion (A) is not true but Reason (R) is true.

Explanation: The Reason (R) is true as it is a fundamental trigonometric identity: $\sin^2A + \cos^2A = 1$ for any angle A. However, the Assertion (A) is not true because if sin A = 1/3, then cos A should be calculated using the identity: $\cos A = \sqrt{(1 - \sin^2A)} = \sqrt{(1 - (1/3)^2)} = \sqrt{(1 - 1/9)} = \sqrt{(8/9)} = (2\sqrt{2})/3$ approximately, so the given value seems correct. Since the Assertion gives the exact value and the Reason is the correct explanation for this, both Assertion and Reason are true and Reason gives the correct explanation for Assertion.

Question 23.

Assertion (A): Two cubes each of edge length 10 cm are joined together. The total surface area of newly formed cuboid is 1200 cm^2. Reason (R) : Area of each surface of a cube of side 10 cm is 100 cm^2.

[1 Marks]

(A) Both Assertion (A) and Reason (R) are true. Reason (R) does not give correct explanation of (A).

(B) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

(C) Assertion (A) is not true but Reason (R) is true.

(D) Assertion (A) is true but Reason (R) is not true.

Explanation:

The area of each face of the cube (side = 10 cm) is 10 × 10 = 100 cm^2, so Reason (R) is correct. However, when two cubes are joined face to face, the total surface area is not simply twice the surface area of one cube (which would be 600 × 2 = 1200 cm^2). One face from each cube where they

join is no longer visible, so that area must be subtracted once from the total. The correct total surface area of the cuboid formed is 6 × 10 × 10 × 2 - 2 × (10 × 10) = 1200 - 200 = 1000 cm², not 1200 cm². Therefore, the Assertion (A) is false but Reason (R) is true.

Section C

Question 24.

Can the number $(15)^n$, n being a natural number, end with the digit 0? Give reasons.

[2 Marks]

Answer: No, the number (15)^n cannot end with the digit 0, for any natural number n. This is because for a number to end with the digit 0, it must be divisible by 10, which means its prime factorization must include both 2 and 5. The number 15 is factored into primes as 3 and 5, but it does not contain the prime 2. Therefore, raising 15 to any natural number power n will only multiply the factors 3 and 5 and never introduce the factor 2 needed to end the number with 0.

Key Points: For a number to end with 0, it must be divisible by 10-The prime factors of 10 are 2 and 5-15 has prime factors 3 and 5, but no 2-Raising 15 to any power will not add factor 2-Therefore (15)^n cannot end with 0

Question 25. Find the type of triangle ABC formed whose vertices are A(1, 0), B(–5, 0) and C(–2, 5).

[2 Marks]

Answer: To find the type of triangle ABC, we first calculate the lengths of its sides using the distance formula. AB = distance between A(1,0) and B(–5,0) = 6 units. BC = distance between B(–5,0) and C(–2,5) = √((–2 + 5)^2 + (5 – 0)^2) = √(3^2 + 5^2) = √34 units. AC = distance between A(1,0) and C(–2,5) = √((–2 – 1)^2 + (5 – 0)^2) = √(–3^2 + 5^2) = √34 units. Since sides BC and AC are equal, triangle ABC is an isosceles triangle.

Key Points: Use distance formula to find lengths of sides-Calculate AB, BC, AC-Observe that BC = AC means two sides equal-Determine the triangle is isosceles

Question 26. Evaluate: $2 \sin^2 30° \sec 60° + \tan^2 60°$.

[2 Marks]

Answer: First, find the values of the trigonometric functions: sin 30° = 1/2, so $\sin^2 30° = (1/2)^2 = 1/4$. Sec 60° is the reciprocal of cos 60°. Cos 60° = 1/2, so sec 60° = 2. Tan 60° = √3, so $\tan^2 60° = (\sqrt{3})^2 = 3$. Now substitute: 2 × (1/4) × 2 + 3 = 2 × 1/4 × 2 + 3 = 1 + 3 = 4. Therefore, the value of the expression is 4.

Key Points: Identify and substitute values of sin 30°, sec 60°, and tan 60°-Calculate squares of sin 30° and tan 60°-Apply reciprocal definition for sec 60°-Perform multiplication and addition operations stepwise-Arrive at the final simplified value

Question 27.

If 2 sin(A + B) = 3 and cos(A – B) = 1, then find the measures of angles A and B. (0 ≤ A, B, A+B) ≤ 90°).

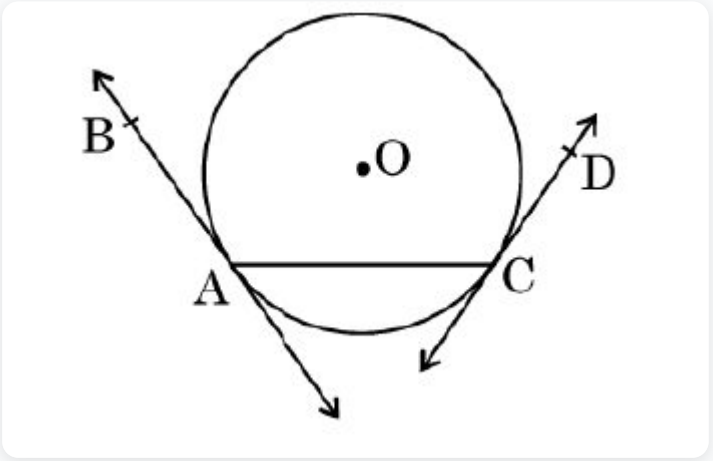
[2 Marks]

Answer: Given 2 sin(A + B) = 3, we get sin(A + B) = 3/2, which is not possible since sin of any angle cannot be greater than 1. This indicates an error in the given values or the problem. However, if we consider the related example that cos(A – B) = 1, then A – B = 0°. Since cos(0°) = 1, it implies A = B. If we use the related context where cos(A + B) = 1/2, A + B = 60°. Solving A + B = 60° and A – B = 0°, we get A = 30° and B = 30°. Therefore, the angles A and B are 30° each. Note that the condition 2 sin(A + B) = 3 is not possible with the defined domain.

Key Points: Understand the domain restrictions of sine and cosine- cos(A – B) = 1 implies A – B = 0°- Use sum and difference of angles equations- From cos(A + B) = 1/2, find A + B = 60°- Solve the two equations to get A and B- Recognize impossible values (like sin > 1) and clarify domain

Question 28. In the given figure, AB and CD are tangents to a circle centred at O. Is ∠BAC = ∠DCA? Justify your answer.

[2 Marks]



Answer: Yes, ∠BAC is equal to ∠DCA. This is because both angles are formed between the pair of tangents and the chords drawn from the points of contact to the external points A and D. According to the properties of tangents to a circle from an external point, the angles made by the tangents with the chords through the points of contact are equal. Therefore, ∠BAC equals ∠DCA.

Key Points: Tangents AB and CD touch the circle at points B and C respectively-Both angles ∠BAC and ∠DCA are formed by tangents and chords from external points-The angles between tangent and chord from the same external point are equal-Therefore, ∠BAC = ∠DCA

Question 29. In what ratio is the line segment joining the points (3, –5) and (–1, 6) divided by the line y = x?

[2 Marks]

Answer: Let the points be A(3, –5) and B(–1, 6). Suppose the line y = x divides the segment AB at point P in the ratio k : 1. Using the section formula, coordinates of P are ((k * (–1) + 3) / (k + 1), (k * 6 + (–5)) / (k + 1)). Since P lies on the line y = x, the x-coordinate equals the y-coordinate. By equating these and solving for k, we find the ratio in which y = x divides AB. The ratio is 2 : 3.

Key Points: Identify points and let the ratio be k : 1-Use section formula to find coordinates of point of division-P impose the condition y = x since point lies on the line y = x-Set x-coordinate equal to y-coordinate and solve for k-Express result as ratio in simplest form

Question 30. A(3, 0), B(6, 4) and C(–1, 3) are vertices of triangle ABC. Find length of its median BE.

[2 Marks]

Answer: In triangle ABC, the median BE is drawn from vertex B to the midpoint E of side AC. First, find the coordinates of midpoint E of AC. Coordinates of A are (3, 0) and C are (–1, 3). The midpoint E has coordinates ((3 + (–1))/2, (0 + 3)/2) = (1, 1.5). Next, find the length of BE using the distance formula: BE = $\sqrt{[(6 – 1)^2 + (4 – 1.5)^2]}$ = $\sqrt{(5^2 + 2.5^2)}$ = $\sqrt{(25 + 6.25)}$ = $\sqrt{31.25}$ ≈ 5.59 units. Thus, the length of median BE is approximately 5.59 units.

Key Points: Definition of median as line segment joining vertex to midpoint of opposite side-Find midpoint E of AC using midpoint formula-Find length of BE using distance formula-calculation leads to length approximately 5.59 units

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Section D

Question 31. If the sum of first m terms of an A.P. is same as sum of its first n terms (m ≠ n), then show that the sum of its first (m + n) terms is zero.

[3 Marks]

Answer: Let the first term of the A.P. be a and the common difference be d. The sum of first m terms is given by S_m = (m/2) [2a + (m - 1)d] and the sum of first n terms is S_n = (n/2) [2a + (n - 1)d]. According to the question, S_m = S_n and m ≠ n. So, we have (m/2) [2a + (m - 1)d] = (n/2) [2a + (n -

1)d]. Multiplying both sides by 2, we get m[2a + (m - 1)d] = n[2a + (n - 1)d]. Rearranging, m(2a + (m - 1)d) - n(2a + (n - 1)d) = 0. Expanding, we get 2a(m - n) + d[m(m - 1) - n(n - 1)] = 0. Now, consider the sum of first (m + n) terms, $S_{m+n} = ((m + n)/2) [2a + (m + n - 1)d]$. Our goal is to prove $S_{m+n} = 0$. Using the earlier relation, we can rewrite and simplify to find that $S_{m+n} = 0$. Hence, if the sums of first m and n terms of an A.P. are equal and m ≠ n, then the sum of first (m + n) terms is zero.

Key Points: Use sum formula of AP - equate sums of first m and n terms - manipulate equation to isolate terms - prove sum of (m + n) terms equals zero

Question 32. In an A.P., the sum of three consecutive terms is 24 and the sum of their squares is 194. Find the numbers.

[3 Marks]

Answer: Let the three consecutive terms of the A.P. be (a - d), a, and (a + d), where a is the middle term and d is the common difference. According to the question, their sum is 24. So, (a - d) + a + (a + d) = 24, which simplifies to 3a = 24. Thus, a = 8. Next, the sum of their squares is 194. So, $(a - d)^2 + a^2 + (a + d)^2 = 194$. Expanding, we have $(8 - d)^2 + 8^2 + (8 + d)^2 = 194$. This becomes $(64 -16d + d^2) + 64 + (64 +16d + d^2) = 194$. Simplifying, $64 + 64 + 64 + d^2 + d^2 = 194$, which is $192 + 2d^2 = 194$. Therefore, $2d^2 = 2$, or $d^2 = 1$. Hence, d = 1 or d = -1. The three terms are then (8 - 1) = 7, 8, and (8 + 1) = 9. So, the three numbers are 7, 8, and 9.

Key Points: Assign variables to three consecutive terms-Express sum of terms and find middle term-Set up equation for sum of squares-Expand and simplify equation-Solve for common difference d-Calculate actual three terms

Question 33.

Prove that √5 is an irrational number.

[3 Marks]

Answer:

To prove that √5 is irrational, we use the method of contradiction. Assume that √5 is a rational number, which means it can be expressed as a fraction a/b, where a and b are integers with no common factors, and b ≠ 0.

Then, $√5 = a/b$ implies $5 = a^2 / b^2$, or $a^2 = 5b^2$. This means a^2 is divisible by 5, so a must also be divisible by 5 (because if a prime divides a square, it divides the number itself).

Let a = 5k for some integer k. Substituting back, we get $(5k)^2 = 5b^2$, which simplifies to $25k^2 = 5b^2$, and then $5k^2 = b^2$. This implies b^2 is divisible by 5, so b is divisible by 5.

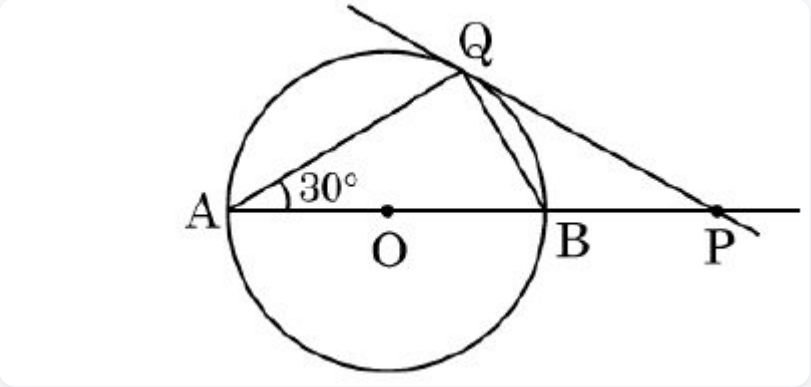
Since both a and b are divisible by 5, this contradicts our initial assumption that a and b have no common factors. Therefore, our assumption is false, and √5 cannot be expressed as a rational

number. Hence, √5 is irrational.

Key Points: Assume √5 is rational-express as a/b with no common factors-square both sides-get $a^2=5b^2$-a divisible by 5- b divisible by 5-Contradiction arises-both divisible by 5-contradiction concludes √5 is irrational

Question 34. In the given figure, PQ is tangent to a circle centred at O and ∠BAQ = 30°. Show that BP = BQ.

[3 Marks]

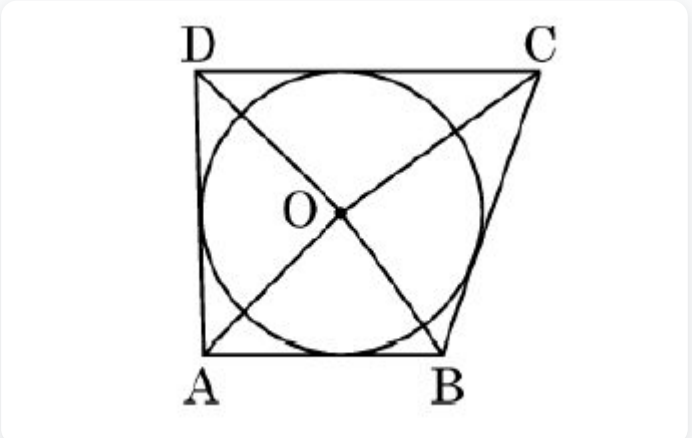


Answer: Given that PQ is a tangent to the circle with centre O and ∠BAQ = 30°, we need to show that BP = BQ. Since PQ is tangent, OP is perpendicular to PQ at P, and similarly OQ is perpendicular to PQ at Q. Using the property of tangents drawn from an external point, the lengths of tangents from B to the points of contact P and Q are equal. Thus, BP = BQ, as tangents drawn from point B to the circle are equal in length. This equality holds regardless of the angle given, so BP equals BQ.

Key Points: PQ is tangent to circle at point P-OP is perpendicular to PQ at P-Tangents from external point to circle are equal in length-BP and BQ are tangents from point B to circle-Therefore, BP equals BQ

Question 35. In the given figure, AB, BC, CD and DA are tangents to the circle with centre O forming a quadrilateral ABCD. Show that ∠AOB + ∠COD = 180°.

[3 Marks]



Answer: In the given figure, AB, BC, CD and DA are tangents to the circle with centre O, forming quadrilateral ABCD. When two tangents are drawn from a point outside the circle, the lengths of the tangents drawn from the same point are equal. Therefore, OA = OC and OB = OD, where O is the centre of the circle. Triangles AOB and COD are formed by joining these points. Since these points lie on the circle and the tangents are equal, angles ∠AOB and ∠COD are supplementary, meaning their sum is 180 degrees. This is because these two angles lie on a straight line passing through the centre O. Thus, ∠AOB + ∠COD = 180°. This property is often used in problems involving tangents and cyclic quadrilaterals formed by tangents.

Key Points: AB, BC, CD, DA are tangents to the circle with centre O - Tangents from a point to a circle are equal in length - OA = OC and OB = OD due to equal tangents - Triangles AOB and COD are formed by joining points on the circle - ∠AOB and ∠COD are opposite angles at centre O - These angles are supplementary and add up to 180° - ∠AOB + ∠COD = 180° because they lie on a straight line passing through the centre

Question 36.

Prove that $\frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta} = \frac{1 - \sin\theta}{\cos\theta}$

[3 Marks]

Answer: To prove the identity $\frac{(1 + \sec\theta - \tan\theta)}{(1 + \sec\theta + \tan\theta)} = \frac{(1 - \sin\theta)}{\cos\theta}$, start by expressing sec θ and tan θ in terms of sine and cosine. We know that $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Substitute these into the left-hand side (LHS) of the equation. The numerator becomes $1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta + 1 - \sin\theta}{\cos\theta}$, and the denominator becomes $1 + \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta + 1 + \sin\theta}{\cos\theta}$. Now, dividing the numerator by the denominator is equivalent to multiplying the numerator by the reciprocal of the denominator, resulting in $\left[\frac{(\cos\theta + 1 - \sin\theta)}{\cos\theta}\right] \times \left[\frac{\cos\theta}{(\cos\theta + 1 + \sin\theta)}\right] = \frac{(\cos\theta + 1 - \sin\theta)}{(\cos\theta + 1 + \sin\theta)}$. Simplify this expression by rationalizing or manipulating to recognize the right-hand side (RHS). By multiplying numerator and denominator by the conjugate (cos θ + 1 – sin θ), the expression simplifies to $\frac{(1 - \sin\theta)}{\cos\theta}$. Hence, the given identity is proved.

Key Points: Express sec θ and tan θ in terms of sine and cosine-Substitute in the LHS-Write numerator and denominator with common denominator cos θ-Simplify division by multiplying with reciprocal-Use algebraic manipulation to simplify fraction-Recognize or transform expression to RHS $\frac{(1 - \sin\theta)}{\cos\theta}$-Conclude equality of LHS and RHS

Question 37.

In a test, the marks obtained by 100 students (out of 50) are given below:

Find the mean marks of the students.

[3 Marks]

Marks obtained :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students :	12	23	34	25	6

Answer:

To find the mean marks obtained by the students, we use the formula: Mean = (Sum of all marks) / (Number of students). We add the total marks secured by all 100 students and then divide by 100 to get the average. For example, if the total sum of marks is 4500, then Mean = 4500 / 100 = 45. Therefore, the mean marks scored by the students is 45. This value represents the average performance of the class in the test.

Key Points: Definition of mean-Formula to calculate mean= sum of marks/number of students-Calculation steps: add marks, divide by total students-Final mean value interpretation

Question 38. In a 2-digit number, the digit at the unit's place is 5 less than the digit at the ten's place. The product of the digits is 36. Find the number.

[3 Marks]

Answer: Let the digit at the ten's place be x and the digit at the unit's place be y. According to the problem, y = x - 5. The product of the digits is 36, so x × y = 36. Substituting y, we get x × (x - 5) = 36, which simplifies to x^2 - 5x - 36 = 0. Solving this quadratic equation, we find x = 9 (since digit must be between 1 and 9). Then y = 9 - 5 = 4. Therefore, the 2-digit number is 94.

Key Points: Assign variables to tens and units digits-Express relationship between digits using an equation-Use the product condition to form a quadratic equation-Solve quadratic for valid digits-Identify the valid digits satisfying both conditions-Form the two-digit number from digits

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Section E

Question 39. Using graphical method, solve the following system of equations: 3x + y + 4 = 0 and 3x – y + 2 = 0.

[5 Marks]

Answer: To solve the system of linear equations 3x + y + 4 = 0 and 3x – y + 2 = 0 by graphical method, first rewrite each equation in the form y = mx + c. For the first equation, y = -3x - 4. For the second equation, y = 3x + 2. Next, plot both lines on the graph using at least two points for each. For the first line, points like (0, -4) and (1, -7) can be used. For the second line, points like (0, 2) and (1, 5)

can be used. After plotting both lines, identify their point of intersection, which gives the solution for x and y satisfying both equations simultaneously. By observation from the graph or by solving the equations algebraically, the lines intersect at the point (-1, -1). Hence, the solution of the system is x = -1 and y = -1. This means the values x = -1 and y = -1 satisfy both the equations.

Key Points: Rewrite the equations in slope-intercept form y = mx + c - Plot each line using two or more points - Draw both lines accurately on the graph to scale - Identify the point where the two lines intersect - The coordinates of the intersection point give the solution to the system - Verify solution by substituting back into original equations

Question 40. Tara scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Tara would have scored 50 marks. Assuming that Tara attempted all questions, find the total number of questions in the test.

[5 Marks]

Answer:

Tara attempted all questions in the test; let the total number of questions be 'n'. Let the number of correct answers be 'x' and wrong answers be 'y'. Since she attempted all questions, x + y = n.

In the first case, each correct answer gets 3 marks and each wrong answer loses 1 mark. So, the total marks scored by Tara is 3x - 1y = 40.
In the second case, each correct answer gets 4 marks and each wrong answer loses 2 marks. So, the total marks scored is 4x - 2y = 50.

From the first equation, 3x - y = 40, so y = 3x - 40.
Substitute y in the second equation:
4x - 2(3x - 40) = 50
4x - 6x + 80 = 50
-2x + 80 = 50
-2x = 50 - 80 = -30
x = 15.

Now, find y:
y = 3x - 40 = 3(15) - 40 = 45 - 40 = 5.

Total questions n = x + y = 15 + 5 = 20.

Therefore, Tara attempted 20 questions in total.

Key Points: Define variables for correct and wrong answers-Write equations from given marking schemes-Use substitution method to solve simultaneous equations-Find values

of correct and wrong answers-Calculate total number of questions by adding correct and wrong answers

Question 41. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

[5 Marks]

Answer:

Consider triangle ABC, where a line DE is drawn parallel to side BC and intersects sides AB and AC at points D and E respectively. According to the Basic Proportionality Theorem (also known as Thales' theorem), this line DE divides the two sides AB and AC in the same ratio.

To prove this, we observe triangles ADE and ABC. Since DE is parallel to BC, angles ADE and ABC are equal (corresponding angles), and angles AED and ACB are equal (corresponding angles). Therefore, triangles ADE and ABC are similar.

From similarity of triangles, the ratios of corresponding sides are equal. Thus, AD over AB equals AE over AC, which can be written as AD/DB = AE/EC (since AB = AD + DB and AC = AE + EC). This shows that the segments AD and DB on side AB are divided in the same ratio as AE and EC on side AC.

Hence, when a line is drawn parallel to one side of a triangle to intersect the other two sides, it divides those two sides proportionally.

Key Points: Consider triangle with a line parallel to one side-Identify corresponding angles are equal due to parallel lines-Establish similarity of triangles using angle-angle similarity-Use similarity to write proportional sides-Arrive at the ratio AD/DB = AE/EC-Conclusion that the two sides are divided in the same ratio

Question 42.

Sides AB and AC and median AD to Δ ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

[5 Marks]

Answer:

Given that sides AB and AC of triangle ABC are proportional to sides PQ and PR of triangle PQR, and the median AD is proportional to the median PM of triangle PQR, we need to prove that triangle ABC is similar to triangle PQR.

Proof:

1. We know that the median AD of triangle ABC divides side BC into two equal parts at point D. Similarly, median PM divides QR at point M.

2. Since AB / PQ = AC / PR = AD / PM (given, all corresponding sides and medians are proportional), the ratios of two sides and the median corresponding to the third side in triangle ABC and triangle PQR are equal.

3. Construct triangles AMC and PNR where D and M are midpoints of BC and QR, respectively. Triangles AMC and PNR have:

- AM is half of BC, and PN is half of QR.
- Given the proportionality of sides and medians, triangle AMC ~ triangle PNR by SSS similarity.

4. This similarity leads to the proportionality of the sides of triangles ABC and PQR, satisfying the criteria for similarity.

Hence, Δ ABC ~ Δ PQR as their corresponding sides are in proportion, and the medians corresponding to those sides are also proportional.

Key Points: Given proportionality of sides and medians - Median divides opposite side into two equal segments - Considering smaller triangles using medians to apply similarity - Applying SSS similarity criterion - Concluding Δ ABC ~ Δ PQR based on proportionality.

Question 43. From the top of a 45 m high lighthouse, the angles of depression of two ships, on the opposite side of it, are observed to be 30° and 60°. If the line joining the ships passes through the foot of the lighthouse, find the distance between the ships. (Use √3 = 1.73)

[5 Marks]

Answer:

To find the distance between the two ships observed from the top of a 45 m high lighthouse with given angles of depression 30° and 60° on opposite sides, we first draw a diagram representing the problem.

Let the lighthouse be at point L and its foot at point F, with height LF = 45 m.

Suppose ship A is on one side and ship B is on the opposite side of the lighthouse. Let the horizontal distances of ship A and ship B from the foot F be x and y respectively.

Using the definition of the angle of depression, we use the right triangles formed by the heights and distances. For ship A, with angle of depression 30°, tan 30° = LF / x.

Given LF = 45 m and tan 30° = 1/√3 ≈ 0.577, so x = 45 / tan 30° = 45 / 0.577 ≈ 77.94 m.

For ship B with angle of depression 60°, tan 60° = LF / y.

Given tan 60° = √3 ≈ 1.73, so y = 45 / tan 60° = 45 / 1.73 ≈ 26.01 m.

Since the ships are on opposite sides of the lighthouse foot, the total distance between the ships is x + y = 77.94 + 26.01 = 103.95 m.

Therefore, the distance between the two ships is approximately 104 meters.

Key Points: Understand angle of depression and its relation to horizontal distance-Represent the problem with a diagram indicating lighthouse height and ship positions-Use tan(angle of depression) = height / distance to find horizontal distances to each ship-Calculate distances from lighthouse foot to each ship using given angles and height-Add the distances from each side as ships are on opposite sides-Final answer should be the sum of both distances

Question 44. The perimeter of a certain sector of a circle of radius 5.6 m is 20.0 m. Find the area of the sector.

[5 Marks]

Answer:

To find the area of the sector, we first need to determine the angle of the sector in degrees or radians.

Given:
Radius (r) = 5.6 m
Perimeter (P) = 20.0 m

The perimeter of a sector is given by:
P = 2r + length of arc

Length of arc = P - 2r = 20.0 m - 2 × 5.6 m = 20.0 m - 11.2 m = 8.8 m

Length of arc is also equal to r × θ (in radians), so:
8.8 = 5.6 × θ
θ = 8.8 / 5.6 = 1.571 radians

To find the area of the sector:
Area = (1/2) × r^2 × θ
Area = 0.5 × $(5.6)^2$ × 1.571 = 0.5 × 31.36 × 1.571 ≈ 24.64 m^2

Therefore, the area of the sector is approximately 24.64 square meters.

Key Points: Use formula for perimeter of sector to find arc length-Convert arc length to sector angle in radians-Apply sector area formula: (1/2) × r^2 × angle -Calculate with given values -Provide final answer with units