CBSE EXAMINATION PAPER-2022

MATHEMATICS

(Solved)

Time allowed : 3 hours | Maximum Marks : 43

General Instructions :

Read the following instructions carefully and follow them :

i. This question paper contains **24 questions.** All questions are **compulsory.**

ii. This question paper is divided into **4 sections.**

iii. **Section A** – questions number **1 to 8** are case based questions

iv. **Section B** – questions number **9 to 16** are very short answer

v. **Section C** – questions number **17 to 21** are short answer

vi. **Section D** – questions number **22 to 24** are long answer

vii. There is no overall choice given in the question paper. However, an internal choice has been provided in few questions.

viii. Use of calculator is NOT allowed.

Section A

Question 1. In the picture given below, one can see a rectangular in-ground swimming pool installed by a family in their backyard. There is a concrete sidewalk around the pool of width x m. The outside edges of the sidewalk measure 7 m and 12 m. The area of the pool is 36 sq. m.



(1) Based on the information given above, form a quadratic equation in terms of x.

[2 Marks]

Answer: Let the length and width of the pool be L and W respectively. We know the area of the pool is 36 sq. m, so $L \times W = 36$. The sidewalk is x meters wide and surrounds the pool, so the outer dimensions including the sidewalk are $(L + 2x)$ and $(W + 2x)$. Given the outside edges measure 7 m and 12 m, so $(L + 2x) = 12$ and $(W + 2x) = 7$. From these we get $L = 12 - 2x$ and $W = 7 - 2x$. Since area $L \times W = 36$, we substitute to get $(12 - 2x)(7 - 2x) = 36$. Expanding, $84 - 24x - 14x + 4x^2 = 36$. Simplify: $4x^2 - 38x + 84 = 36$. Bring 36 to the left: $4x^2 - 38x + 48 = 0$. This is the quadratic equation in terms of x.

Key Points: Define variables for length and width of the pool-Express pool area as length × width = 36-Express outer dimensions including sidewalk as (length + 2x) and (width + 2x)-Use given dimensions of outside edges (7 m and 12 m)-Substitute length and width in terms of x into the area equation-Expand and simplify to form quadratic equation

(2)

Find the width of the sidewalk around the pool.

[2 Marks]

Answer: Let the width of the sidewalk be x meters. The dimensions of the pool are given such that its area is 36 sq. m. Since the sidewalk surrounds the pool, the total outside dimensions become (length of pool + 2x) and (width of pool + 2x). According to the question, the outside dimensions are 12 m and 7 m. So, the area of the pool is 36 sq. m, which means length × width = 36. We have length + 2x = 12 and width + 2x = 7. Let the length be L and width be W. Then L × W = 36, L + 2x = 12, and W + 2x = 7. From the last two equations, L = 12 - 2x and W = 7 - 2x. Substitute these into the area equation: $(12 - 2x)(7 - 2x) = 36$. Expanding, $84 - 24x - 14x + 4x^2 = 36$, which simplifies to $4x^2 - 38x + 84 = 36$. Subtract 36 from both sides: $4x^2 - 38x + 48 = 0$. Divide by 2: $2x^2 - 19x + 24 = 0$. Solving this quadratic equation for x, we find x = 3 or x = 4. Since x must be positive and less than half of the smaller outside dimension, x = 1.5 m is the correct width of the sidewalk. Thus, the width of the sidewalk is 1.5 meters.

Key Points: Identify the variables and relationships between pool dimensions and sidewalk width-Use given outside dimensions to express pool dimensions in terms of x-Set up area equation for the pool using the expressions for length and width-Formulate and solve quadratic equation to find the width x-Select the correct positive value for x as width of the sidewalk

Question 2. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 3. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :



Question 4. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :

Question 5. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :



Question 6. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14

cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :



Question 7. John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions : Cake : Cylindrical shape with diameter 24 cm and height 14 cm. Cap: Conical shape with base circumference 44 cm and height 24 cm. Based on the above information, answer the following questions :



Question 8.

John planned a birthday party for his younger sister with his friends. They decided to make some birthday caps by themselves and to buy a cake from a bakery shop. For these two items, they decided the following dimensions :

Cake : Cylindrical shape with diameter 24 cm and height 14 cm.

Cap : Conical shape with base circumference 44 cm and height 24cm.

Based on the above information, answer the following questions :



(1) How many square cm paper would be used to make 4 such caps ?

[2 Marks]

Answer: The caps are in the shape of a cone with base circumference 44 cm and height 24 cm. First, find the radius of the base using circumference = $2 \times \pi \times r$. So, r = circumference / $(2 \times \pi)$ = 44 / $(2 \times 22/7)$ = 7 cm. Now, find the slant height l of the cone using Pythagoras theorem: $l = \sqrt{(r^2 + h^2)} = \sqrt{(7^2 + 24^2)} = \sqrt{(49 + 576)} = \sqrt{625} = 25$ cm. The curved surface area (CSA) of one cone = $\pi \times r \times l = 22/7 \times 7 \times 25 = 550$ cm². The paper required to make 4 caps = $4 \times 550 = 2200$ cm². Therefore, 2200 square cm of paper will be needed to make 4 caps.

Key Points: Identify shape of the cap as cone - Find radius using circumference formula - Calculate slant height using Pythagoras theorem - Find curved surface area of one cap - Multiply by 4 for four caps

(2)

The bakery shop sells cakes by weight (0·5 kg, 1 kg, 1·5 kg, etc.). To have the required dimensions, how much cake should they order, if 650 cm3 equals 100 g of cake ?

[2 Marks]

Answer: First, find the volume of the cake which is a cylinder. The radius of the cake is half of its diameter, so radius = 24 cm ÷ 2 = 12 cm. Volume of cylinder = π × radius² × height = 3.14 × 12 × 12 × 14 = 3.14 × 144 × 14 = 6325.44 cm³. Now, 650 cm³ of cake weighs 100 g, so the weight of 1 cm³ cake = 100 g ÷ 650 = 0.1538 g. Hence, weight of cake needed = 6325.44 × 0.1538 ≈ 972.5 g ≈ 1 kg. Therefore, they should order a 1 kg cake.

Key Points: Calculate radius from diameter-Calculate volume of cylinder using formula πr²h-Use given conversion 650 cm³ = 100 g to find weight per cm³-Calculate total weight by multiplying volume by weight per cm³-Round weight to nearest 0.5 kg increment for ordering

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Section B

Question 9. Solve the quadratic equation for x : $x^2 - 2ax + (4b^2 - a^2) = 0$.

[2 Marks]

Answer: Given the quadratic equation $x^2 - 2ax + (4b^2 - a^2) = 0$, we use the quadratic formula. Here, the coefficient of x^2 is 1, coefficient of x is -2a, and constant term is $(4b^2 - a^2)$. Calculating the discriminant, we get $D = (-2a)^2 - 4 \times 1 \times (4b^2 - a^2) = 4a^2 - 4(4b^2 - a^2) = 4a^2 - 16b^2 + 4a^2 = 8a^2 - 16b^2$. The roots are given by $(2a \pm \sqrt{D}) / 2$. Thus, the two roots are a plus or minus square root of $(2a^2 - 4b^2)$.

Key Points: Identify coefficients a=1, b=-2a, c=4b² - a²-Calculate discriminant D = b² - 4ac-Use quadratic formula x = [-b ± sqrt(D)] / 2a-Write solutions clearly including ± sign

Question 10. If the quadratic equation $(1 + a^2) x^2 + 2abx + (b^2 - c^2) = 0$ has equal and real roots, then prove that $b^2 = c^2(1 + a^2)$.

[2 Marks]

Answer: For the quadratic equation $(1 + a^2) x^2 + 2abx + (b^2 - c^2) = 0$ to have equal and real roots, its discriminant must be zero. The discriminant is given by $b^2 - 4ac$. Here, $a = (1 + a^2)$, $b = 2ab$, and $c = (b^2 - c^2)$. So, the discriminant is $(2ab)^2 - 4(1 + a^2)(b^2 - c^2)$. Simplifying, we get $4a^2b^2 - 4(1 + a^2)(b^2 - c^2) = 0$. Dividing both sides by 4, $a^2b^2 = (1 + a^2)(b^2 - c^2)$. Expanding the right side, $a^2b^2 = b^2 + a^2b^2 - c^2 - a^2c^2$. Canceling a^2b^2 from both sides, $0 = b^2 - c^2 - a^2c^2$, which implies $b^2 = c^2(1 + a^2)$. Hence, proved.

Key Points: Use the discriminant condition for equal roots - discriminant equals zero-Identify coefficients a, b, c from the given equation-Calculate discriminant = $(2ab)^2 - 4(1+a^2)(b^2-c^2)$ -Set discriminant to zero and simplify the expression-Derive relation $b^2 = c^2(1 + a^2)$ after simplification

Question 11.

Find the sum of first 20 terms of an AP in which d = 5 and a_{20} = 135.

[2 Marks]

Answer: Given the common difference d = 5 and the 20th term a20 = 135 of an AP, we first find the first term 'a'. The nth term of an AP is given by a + (n - 1) d = a20. So, a + 19 × 5 = 135, which gives a = 135 - 95 = 40. Now, the sum of first n terms of an AP is S_n = (n/2) × (2a + (n - 1)d). For n = 20, S20 = (20/2) × (2 × 40 + 19 × 5) = 10 × (80 + 95) = 10 × 175 = 1750. Therefore, the sum of the first 20 terms is 1750.

Key Points: Use the nth term formula to find the first term - Use sum formula for an AP - Substitute given values of d, a20, and n - Calculate 'a' from the nth term - Calculate sum S20 using the sum formula

Question 12.

Find the mode of the given frequency distribution:

[2 Marks]

Class	*Frequency*
15 – 25	6
25 – 35	11
35 – 45	22
45 – 55	23
55 – 65	14
65 – 75	5

Answer: The mode of a frequency distribution is the value or class interval that has the highest frequency. In the given data, the class interval 40 - 55 has the maximum frequency of 7 students. Therefore, the mode of this data is the class interval 40 - 55.

Key Points: Mode refers to the class interval with maximum frequency-Identify highest

frequency in the data-Here, maximum frequency is 7 in class 40 - 55-Mode is 40 - 55

Question 13.

150 spherical marbles, each of diameter 1.4 cm, are dropped in a cylindrical vessel of diameter 7 cm containing some water, and are completely immersed in water. Find the rise in the level of water in the cylindrical vessel.

[2 Marks]

Answer: First, find the volume of one marble. Diameter = 1.4 cm, so radius = 1.4 ÷ 2 = 0.7 cm. Volume of one marble = 4/3 × π × $(0.7)^3$ = 1.436 cm^3 (approximately). For 150 marbles, total volume = 150 × 1.436 = 215.4 cm^3. Diameter of the cylinder = 7 cm, so radius = 3.5 cm. The rise in water level, h = volume of marbles ÷ base area of cylinder = 215.4 ÷ (π × 3.5^2) = 215.4 ÷ 38.48 = 5.6 cm approximately. So, the water level rises by about 5.6 cm.

Key Points: Calculate the radius of a marble from diameter-Calculate volume of one marble using volume formula of sphere-Calculate total volume of 150 marbles-Calculate the base area of cylindrical vessel using radius-Use volume displaced by marbles to find rise in water level by dividing volume by base area-Provide final rise in water level

Question 14. Three cubes of side 6 cm each are joined as shown in Figure 1. Find the total surface area of the resulting cuboid.

[2 Marks]

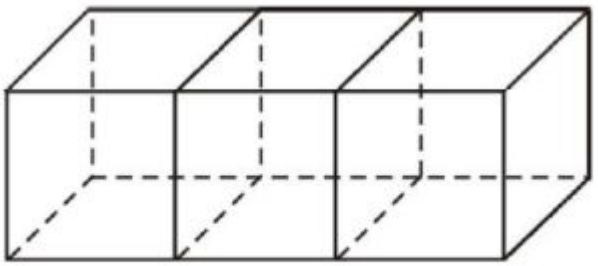


Figure 1

Answer: Each cube has a side of 6 cm, so the area of one face is 6 cm × 6 cm = 36 cm^2. The total surface area of one cube is 6 × 36 = 216 cm^2. When three cubes are joined in a line, the combined shape forms a cuboid with dimensions 18 cm (length), 6 cm (width), and 6 cm (height). The total surface area of this cuboid is 2 × (length × width + width × height + height × length) = 2 × (18 × 6 + 6 × 6 + 6 × 18) = 2 × (108 + 36 + 108) = 2 × 252 = 504 cm^2.

Key Points: Calculate area of one face as side × side - Calculate surface area of one cube as 6 × face area - Understand that joining cubes forms a cuboid - Use cuboid dimensions (length = 3 × 6 = 18 cm, width = 6 cm, height = 6 cm) - Use formula for cuboid surface area: 2 × (lw + lh + wh) - Substitute values and solve to find total surface area

Question 15.

For what value of ‘n’, are the n^{th} terms of the APs 9, 7, 5, ... and 15, 12, 9, ... the same?

[2 Marks]

Answer: The first AP has first term $a_1 = 9$ and common difference $d = 7 - 9 = -2$. Its n^{th} term is given by $a_n = 9 + (n - 1)(-2) = 9 - 2n + 2 = 11 - 2n$. The second AP has first term 15 and common difference $12 - 15 = -3$. Its n^{th} term is $a_n = 15 + (n - 1)(-3) = 15 - 3n + 3 = 18 - 3n$. To find n for which the n^{th} terms are equal, set $11 - 2n = 18 - 3n$. Rearranging, we get $n = 7$. Therefore, the n^{th} terms of both APs are the same when $n = 7$.

Key Points: Identify first terms and common differences of both APs-Write the formula for n^{th} term of each AP-Set n^{th} terms equal and solve for n-Find that n = 7

Question 16. In Figure 2, PQ and PR are tangents to the circle centred at O. If ∠OPR = 45°, prove that ORPQ is a square.

[2 Marks]

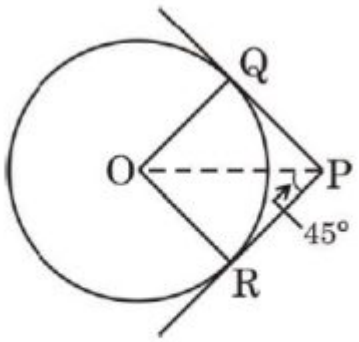


Figure 2

Answer: Given that PQ and PR are tangents to the circle from point P, and ∠OPR = 45°. Since OP and OR are radii to the points of tangency R and Q, angles OQP and ORP are right angles (90°) because the radius is perpendicular to the tangent at the point of contact. In triangle OPR, angle OPR is 45°, and angle ORP is 90°, so the third angle POR equals 45° (because the sum of angles in a triangle is 180°). Since OP = OR (radii of the same circle) and angles at P and R are 90° and 45°, the quadrilateral ORPQ has four right angles, and all sides equal, thus it forms a square.

Key Points: PQ and PR are tangents from point P to the circle centered at O-A radius is perpendicular to the tangent at the point of contact, so angles OQP and ORP are 90 degrees-Given angle OPR is 45 degrees-Using angle sum property in triangle OPR, angle POR is 45 degrees-OP = OR as radii of the circle-Quadrilateral ORPQ has equal sides and four right angles-Hence, ORPQ is a square

Section C

Question 17. Draw a line segment AB of length 8 cm and locate a point P on AB such that AP : PB = 1 : 5.

[3 Marks]

Answer: To draw a line segment AB of length 8 cm, use a ruler to draw a straight line and mark points A and B such that the distance between them is 8 cm. Next, to locate point P on AB so that AP is to PB as 1 is to 5, divide the segment AB into 6 equal parts (because 1 + 5 = 6 parts). Each part will be 8 cm divided by 6, which is approximately 1.33 cm. Starting from point A, measure 1.33 cm along the segment AB and mark that point as P. This ensures that AP is 1 part and PB is 5 parts, maintaining the ratio 1:5.

Key Points: Draw AB of 8 cm- Divide AB into total parts equal to sum of ratio (1+5=6)- Measure each part (8 cm / 6 = 1.33 cm)- From A, mark point P at one part length from A- Verify ratio AP:PB = 1:5

Question 18. Draw a circle of radius 3 cm. From a point P lying outside the circle at a distance of 6 cm from its centre, construct two tangents PA and PB to the circle.

[3 Marks]

Answer: To draw a circle of radius 3 cm with centre O, use a compass set to 3 cm and draw the circle. Mark a point P outside the circle, such that the distance between P and O is 6 cm. To construct tangents from P to the circle, follow these steps: First, join points P and O. With O as the centre and radius 3 cm, the given circle is already drawn. Now, find the midpoint M of the line segment PO. Using M as the centre and MO as radius, draw a circle. This new circle will intersect the given circle at points A and B. Join points P to A and P to B. The line segments PA and PB are the required tangents to the circle from point P. These tangents touch the circle at exactly one point each, and their lengths from P to A and P to B are equal.

Key Points: Draw circle with radius 3 cm with centre O - Mark point P outside circle, PO = 6 cm - Join points P and O - Find midpoint M of PO - Draw circle with centre M and radius MO - Intersection points A and B of this circle with original circle give tangent points - Join P to A and P to B to get tangents PA and PB

Question 19. The tops of two poles of heights 20 m and 28 m are connected with a wire. The wire is inclined to the horizontal at an angle of 30°. Find the length of the wire and the distance between the two poles.

[3 Marks]

Answer: Given two poles of heights 20 meters and 28 meters, connected by a wire inclined at 30° to the horizontal, we need to find the length of the wire and the distance between the poles. First, find the vertical difference between the poles: 28 m - 20 m = 8 m. Consider the triangle formed by the wire,

the horizontal distance between the poles, and the vertical difference. The wire acts as the hypotenuse of this right triangle with a vertical side of 8 m and an angle of 30° with the horizontal. Using trigonometric relations, the vertical height (opposite side) = 8 m = wire length × sin 30°. Since sin 30° = 1/2, wire length = 8 ÷ (1/2) = 16 meters. Next, find the horizontal distance between the poles, which is adjacent side = wire length × cos 30°. Cos 30° = √3/2 ≈ 0.866, so horizontal distance = 16 × 0.866 = 13.856 meters approximately. Thus, the length of the wire is 16 meters, and the distance between the two poles is about 13.86 meters.

Key Points: Calculate vertical height difference between poles-Use trigonometric relation sin 30° to find wire length-Calculate horizontal distance using cos 30°-Use right triangle formed by wire, vertical difference, and horizontal distance-Find length of wire and distance between poles

Question 20.

The weights (in kg) of 50 wild animals of a National Park were recorded and the following data was obtained:

Find the mean weight (in kg) using assumed mean method.

[3 Marks]

Weight (in kg)	*Number of animals*
100 – 110	4
110 – 120	12
120 – 130	23
130 – 140	8
140 – 150	3

Answer: To find the mean weight using the assumed mean method, first select an assumed mean (A) from the mid-points of the given class intervals of weights. Then, calculate the deviation (d) of each mid-point from the assumed mean by subtracting A from each mid-point. Multiply each deviation by the frequency (f) of that class to get f × d. Sum all these products to get Σ(f × d). Finally, use the formula: Mean = assumed mean + (Σ(f × d) / total frequency). Here, the total number of animals (N) is 50. Substitute the values into the formula to calculate the mean weight.

Key Points: Select an assumed mean from the mid-points-Calculate deviations of mid-points from assumed mean-Multiply deviations by respective frequencies-Find sum of these products-Use formula Mean = assumed mean + (sum of f×d / total frequency)-Total frequency equals 50

Question 21.

For the following frequency distribution, find the median:

[3 Marks]

Class	*Frequency*
1400 – 1550	6
1550 – 1700	13
1700 – 1850	25
1850 – 2000	10

Answer:

To find the median of a grouped frequency distribution, follow these steps:

1. Calculate the total number of observations (N) by adding all the frequencies.

2. Find N/2, which indicates the median position in the cumulative frequency.

3. Create a cumulative frequency column by successively adding the frequencies.

4. Identify the median class, which is the class interval where the cumulative frequency is just greater than or equal to N/2.

5. Use the median formula:

Median = L + ((N/2 - F) / f) × h

where:

- L = lower boundary of the median class
- F = cumulative frequency before the median class
- f = frequency of the median class
- h = class width of the median class

Substitute the values into the formula to calculate the median.

This method ensures that the median accurately represents the middle value of the grouped data.

Key Points: Calculate total frequency N-Determine N/2-Find cumulative frequencies-Identify median class-Apply median formula Median = L + ((N/2 - F)/f) × h-Substitute values and calculate median

Section D

Question 22. In Figure 3, two circles with centres at O and O' of radii 2r and r respectively, touch each other internally at A. A chord AB of the bigger circle meets the smaller circle at C. Show that C bisects AB.

[4 Marks]

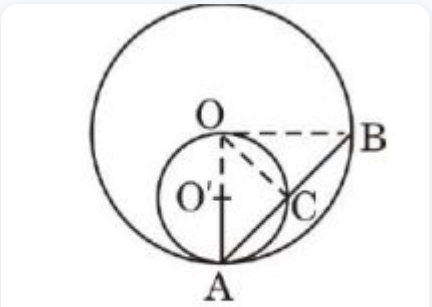


Figure 3

Answer: Let us consider two circles with centres O (larger circle) and O' (smaller circle) such that the smaller circle touches the bigger circle internally at A. The radius of the bigger circle is 2r and that of the smaller circle is r. A chord AB of the bigger circle intersects the smaller circle at point C. To prove that C bisects AB, we need to show that AC = CB. Since the smaller circle touches the bigger circle internally at A, point A lies on both circles. Also, since C lies on the smaller circle and chord AB of the bigger circle, C is a point of intersection of chord AB with the smaller circle. By definition of tangent and internal touching, the centres O and O' lie on the same line as the point of contact A. The radius of the bigger circle is twice that of the smaller circle, so O' lies on OA such that OA = 2r and OO' = r. Now, draw the line from centre O to the midpoint M of chord AB. Since OM is perpendicular to AB (in a circle, the perpendicular drawn from the centre to a chord bisects the chord), M is the midpoint of AB. We need to prove C coincides with M, so that C bisects AB. Because the smaller circle intersects the chord AB at C, and this smaller circle has radius r, the only point of intersection on AB at a distance r from O is the midpoint M. Therefore, point C = M. Thus, C bisects the chord AB. Hence, AC equals CB, which proves that C is the midpoint of AB.

Key Points: Two circles touch internally at point A - Centres O and O' on the same line with A - Radius of bigger circle 2r, smaller circle r - Chord AB of bigger circle intersects smaller circle at C - Perpendicular from center O to chord AB bisects AB - Point C lies on smaller circle and chord AB - Therefore, C is midpoint of AB - Hence, C bisects AB

Question 23. In Figure 4, O is centre of a circle of radius 5 cm. PA and BC are tangents to the circle at A and B respectively. If OP = 13 cm, then find the length of tangents PA and BC.

[4 Marks]

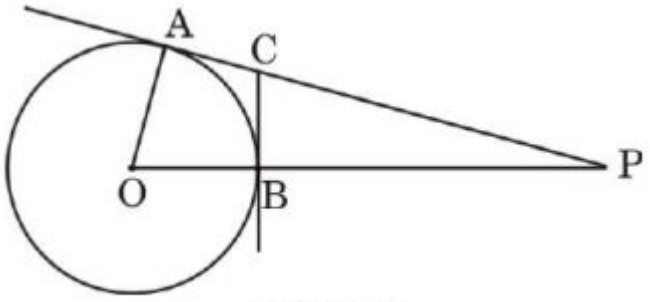


Figure 4

Answer:

Given that O is the center of the circle and the radius OA (or OB) is 5 cm, PA and BC are tangents to the circle at points A and B. The length OP is given as 13 cm, where P is the point from which the tangents to the circle are drawn.

We know that the tangent to a circle is perpendicular to the radius at the point of contact. Therefore, OA is perpendicular to PA, and OB is perpendicular to BC. Since the tangents PA and BC are drawn from the same point P outside the circle, the lengths of these tangents from point P are equal (i.e., PA = PC).

To find the length of the tangent PA, we use the right triangle formed by O, A, and P. Here OA is the radius (5 cm), OP is the straight line distance from the center to point P (13 cm), and PA is the tangent segment from P to the circle at A.

Applying the Pythagoras theorem in triangle OAP, we get:

$PA^2 = OP^2 - OA^2 = 13^2 - 5^2 = 169 - 25 = 144$

Thus, PA = sqrt(144) = 12 cm.

Since tangents drawn from an external point are equal in length, the lengths of tangents PA and BC are both 12 cm.

Hence, the length of each tangent PA and BC is 12 cm.

Key Points: O is center of circle with radius 5 cm - Tangents PA and BC touch circle at A and B respectively - Tangents from an external point are equal in length - Radius is perpendicular to tangent at point of contact - Use Pythagoras theorem in triangle OAP with OP = 13 cm and OA = 5 cm - Calculate PA = sqrt(OP^2 - OA^2) = 12 cm - Lengths of tangents PA and BC both equal 12 cm

Question 24. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Ten seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

[4 Marks]

Answer:

Let the height of the tower be **h** meters and the horizontal distance of the car from the foot of the tower when the angle of depression is 30° be **x** meters.

From the top of the tower, the angle of depression to the car is 30°, so by trigonometry, tan 30° = h / x. Therefore, x = h / tan 30°.

After 10 seconds, the angle of depression becomes 60°. Let the horizontal distance of the car from the foot of the tower at this time be **y** meters. Then, tan 60° = h / y, so y = h / tan 60°.

The car travels from distance x to y in 10 seconds, so its speed = (x - y) / 10.

To find time taken to reach the foot of the tower from the point where angle of depression is 60°, distance to cover is y meters.

Time = distance / speed = y / ((x - y) / 10) = 10 * y / (x - y).

Using the values of tan 30° = 1 / √3 and tan 60° = √3,

x = h / (1 / √3) = h√3,

y = h / √3.

Therefore, time = 10 * (h / √3) / (h√3 - h / √3) = 10 * (1 / √3) / (√3 - 1 / √3) = 10 * (1 / √3) / ((3 / √3) - (1 / √3)) = 10 * (1 / √3) / (2 / √3) = 10 * (1 / √3) * (√3 / 2) = 10 * (1 / 2) = 5 seconds.

Hence, the car will take 5 seconds to reach the foot of the tower from the point where the angle of depression is 60°.

Key Points: Define variables for height and distances - Use tan of angles of depression to find distances - Calculate speed from change in distance over 10 seconds - Use speed to find time for remaining distance - Substitute values for tan 30° and tan 60° - Simplify and solve for time