Electric Potential

Definition and Formula

Electric potential at a point is the work done by an external force in moving a unit positive charge from infinity to that point in an electrostatic field without acceleration. It is a scalar quantity.

Mathematically, electric potential V is given by:

$$V = \frac{W}{q}$$

where W is the work done and q is the charge.

The SI unit of electric potential is volt (V), equivalent to joule per coulomb (J/C).

Potential Difference

Electric potential difference between two points is the work done in moving a unit charge from one point to another in an electric field.

It is expressed as:

$$\Delta V = V_B - V_A = \frac{W}{q}$$

The work done by the electric field in moving a charge q over a small displacement $\vec{dl}$ is:

$$dW = q\vec{E} \cdot \vec{dl}$$

where $\vec{E}$ is the electric field vector.

Electric Potential Due to Point Charge

The electric potential V_E at a distance r from a point charge q is:

$$V_E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

where ε_0 is the permittivity of free space.

Electric Dipole and System of Charges

An electric dipole consists of two equal and opposite charges separated by a distance. The potential due to a dipole at a point at distance r and angle θ with the dipole moment p is:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

For a system of charges, the total potential at a point is the algebraic sum of potentials due to individual charges.

Equipotential Surfaces

Equipotential surfaces are surfaces where the electric potential is constant. No work is required to move a charge along such surfaces.

Electric field lines are always perpendicular to equipotential surfaces.

Equipotential surfaces never intersect.

Electric Potential Energy

Potential energy of two point charges q_1 and q_2 separated by distance r_{12} is:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

For three charges, total potential energy is the sum of energies of all pairs.

Potential energy of a charge q in an external potential $V(\vec{r})$ is:

$$U = qV(\vec{r})$$

Potential energy of a dipole in an external electric field $\vec{E}$ making angle θ with dipole moment p is:

$$U(\theta) = -pE\cos\theta$$

Solved Examples

Example 1: Two point charges $q_1 = 10 \times 10^{-8}\mathrm{C}$ and $q_2 = -2 \times 10^{-8}\mathrm{C}$ are separated by 60 cm in air.

(i) Find the point where the electric potential is zero.

(ii) Calculate the electrostatic potential energy of the system.

Solution:

Let the point be at distance x from q_1. Then distance from q_2 is $0.6 - x$ m.

Potential at point due to q_1: $V_1 = \frac{kq_1}{x}$

Potential at point due to q_2: $V_2 = \frac{kq_2}{0.6-x}$

At zero potential, $V_1 + V_2 = 0$:

$\frac{10\times10^{-8}}{x} + \frac{-2\times10^{-8}}{0.6-x} = 0$

Solving, $10/(x) = 2/(0.6 - x)$ gives $x = 0.5$ m.

Distance from first charge is 0.5 m.

Electrostatic potential energy:

$$U = \frac{kq_1q_2}{r} = \frac{9\times10^9\times10\times10^{-8}\times(-2)\times10^{-8}}{0.6} = -3\times10^{-5}\text{J}$$

Practice Set

- **Level 1:** Define electric potential and state its SI unit.
- **Level 2:** Calculate the electric potential at a point 0.2 m away from a charge of $5\times10^{-6}\text{C}$.
- **Level 3:** Two charges $+3\times10^{-6}\text{C}$ and $-3\times10^{-6}\text{C}$ are 0.5 m apart. Find the potential at the midpoint.

Answer Key

- **Level 1:** Electric potential is the work done in moving a unit positive charge from infinity to a point in an electric field. SI unit is volt (V).
- **Level 2:** $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 9\times10^9\times\frac{5\times10^{-6}}{0.2} = 225000\text{V}$
- **Level 3:** Potential at midpoint is sum of potentials due to both charges. Distance from each charge is 0.25 m.

  $$V = 9\times10^9\times\left(\frac{3\times10^{-6}}{0.25} - \frac{3\times10^{-6}}{0.25}\right) = 0\text{V}$$

Capacitance

Conductors, Insulators and Dielectrics

Conductors allow free movement of charge (e.g., metals, graphite). Insulators do not allow free charge movement (e.g., plastics, glass).

Dielectrics are insulating materials that become polarized in an electric field, inducing dipole moments proportional to the field.

Electric Polarisation

Polarisation P in dielectrics is related to electric susceptibility X_e and electric field E by:

$$P = \varepsilon_0 X_e E$$

Dielectric constant $K = 1 + X_e$, always greater than 1.

Capacitor and Capacitance

A capacitor stores electric charge. It consists of two conducting plates separated by a distance.

Charge stored Q is proportional to potential difference V:

$$Q = CV$$

where C is capacitance, measured in farads (F).

Capacitors in Series and Parallel

In series, charge on each capacitor is the same, and total voltage is sum of voltages across each capacitor.

Equivalent capacitance in series:

$$\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$$

In parallel, voltage across each capacitor is the same, and total charge is sum of charges on each capacitor.

Equivalent capacitance in parallel:

$$C_{eq} = \sum C_i$$

Capacitance of Parallel Plate Capacitor

Capacitance with dielectric constant ε_r:

$$C = \frac{\varepsilon_r \varepsilon_0 A}{d}$$

where A is plate area and d is separation.

For multiple dielectric slabs of thickness t_i and dielectric constants k_i:

$$C = \frac{\varepsilon_0 A}{\sum \frac{t_i}{k_i}}$$

Energy Stored in Capacitor

Energy stored U is:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Solved Examples

Example 2: A dielectric slab of thickness $\frac{3}{4}d$ and dielectric constant K is inserted between plates of a capacitor separated by distance d. Find the new capacitance.

Solution:

Electric field without dielectric: $E_0 = \frac{V_0}{d}$

Electric field in dielectric: $E = \frac{E_0}{K}$

Potential difference:

$$V = E_0 \times \frac{d}{4} + E \times \frac{3d}{4} = E_0 d \left(\frac{1}{4} + \frac{3}{4K} \right) = V_0 \frac{K+3}{4K}$$

Capacitance increases by factor:

$$C = \frac{4K}{K+3} C_0$$

Practice Set

- **Level 1:** Define dielectric and give two examples.
- **Level 2:** Calculate the equivalent capacitance of three capacitors 2 μF, 3 μF, and 6 μF connected in series.
- **Level 3:** A parallel plate capacitor has plate area 0.02 m² and plate separation 1 mm. Calculate its capacitance if the space between plates is filled with a dielectric of constant 5.

Answer Key

- **Level 1:** Dielectric is an insulating material that increases capacitance when placed between capacitor plates. Examples: glass, mica.
- **Level 2:** $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1 \Rightarrow C_{eq} = 1\mu F$
- **Level 3:** $C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{5 \times 8.85 \times 10^{-12} \times 0.02}{1 \times 10^{-3}} = 8.85 \times 10^{-10}\text{F} = 0.885\text{nF}$

Quick Reference Table

Electric Potential: $V = \frac{W}{q}$, unit: volt (V)

Potential due to point charge: $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

Capacitance of parallel plate capacitor: $C = \frac{\varepsilon_r \varepsilon_0 A}{d}$

Capacitors in series: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$

Capacitors in parallel: $C_{eq} = \sum C_i$

Energy stored in capacitor: $U = \frac{1}{2} C V^2$

Common Mistakes and Misconceptions

- Confusing electric potential (scalar) with electric field (vector).
- Incorrect unit conversions, especially for capacitance (e.g., microfarads to farads).
- Misapplying capacitance formulas without considering geometry and dielectric properties.
- Assuming potential difference is always positive; it depends on direction.
- For capacitors in series, forgetting that charge is the same on all capacitors.
- For capacitors in parallel, forgetting that voltage across each capacitor is the same.

Glossary

- **Electric Potential:** Work done per unit charge in bringing a charge from infinity to a point.
- **Capacitance:** Ability of a capacitor to store charge per unit voltage.
- **Dielectric:** Insulating material that increases capacitance when placed between capacitor plates.
- **Equipotential Surface:** Surface where electric potential is constant.
- **Electric Dipole:** Two equal and opposite charges separated by a distance.
- **Polarisation:** Alignment of dipoles in a dielectric under an electric field.