Introduction to Linear Equations in One Variable

Linear equations in one variable are algebraic equations where the highest power of the variable is 1, and the equation contains only one expressions such as $2x$, $3y - 7$, or $\frac{5}{4}(x-4)+10$. Expressions like x^2+1 or $y+y^2$ are not linear because the variable's power is greater

Concept Explanation

An algebraic equation is an equality involving variables and an equality sign $=$. The expression on the left side is called the Left Hand Sid Right Hand Side (RHS). For example, in the equation $2x-3=7$, $2x-3$ is the LHS and 7 is the RHS.

Formula Derivation

There is no specific formula for linear equations, but the key property is that the variable's highest power is 1, and the equation can be so operations.

Worked Illustration

Consider the equation $2x-3=7$. To find x, add 3 to both sides:

$$2x-3+3=7+32x=10$$

Then divide both sides by 2:

$$\frac{2x}{2}=\frac{10}{2}x=5$$

Solved Example

Example: Solve $2x-3=7$.

Solution:

- Add 3 to both sides to keep the equation balanced:

$$2x-3+3=7+32x=10$$

- Divide both sides by 2:

$$x=\frac{10}{2}=5$$

Thus, $x=5$ is the solution.

Practice Set

- **Level 1 – Easy**
- Solve $3x+4=10$.
- Solve $5x-7=8$.
- **Level 2 – Moderate**
- Solve $4x-5=3x+2$.
- Solve $7x+3=2x+18$.
- **Level 3 – Challenging**
- Solve $\frac{3x+2}{2}=\frac{5x-4}{3}$.
- Solve $2(3x-1)+4=3(x+2)-5$.

Answer Key

- **Level 1**

- $3x + 4 = 10 \Rightarrow 3x = 6 \Rightarrow x = 2$
- $5x - 7 = 8 \Rightarrow 5x = 15 \Rightarrow x = 3$
- **Level 2**
- $4x - 5 = 3x + 2 \Rightarrow 4x - 3x = 2 + 5 \Rightarrow x = 7$
- $7x + 3 = 2x + 18 \Rightarrow 7x - 2x = 18 - 3 \Rightarrow 5x = 15 \Rightarrow x = 3$
- **Level 3**
- $\frac{3x+2}{2} = \frac{5x-4}{3} \Rightarrow 3(3x + 2) = 2(5x - 4) \Rightarrow 9x + 6 = 10x - 8 \Rightarrow 9x - 10x = -8 - 6 \Rightarrow -x = -14 \Rightarrow x = 14$
- $2(3x - 1) + 4 = 3(x + 2) - 5 \Rightarrow 6x - 2 + 4 = 3x + 6 - 5 \Rightarrow 6x + 2 = 3x + 1 \Rightarrow 6x - 3x = 1 - 2 \Rightarrow 3x = -1 \Rightarrow x = -\frac{1}{3}$

Quick Reference

Term	Meaning
Variable	Unknown quantity represented by a letter (e.g., x)
Equation	Equality between two expressions (e.g., $2x - 3 = 7$)
LHS	Left Hand Side of the equation
RHS	Right Hand Side of the equation
Solution	Value of the variable that satisfies the equation

Glossary

- **Algebraic Expression:** A combination of variables, numbers, and operations without an equality sign.
- **Linear Expression:** An expression where the variable has the highest power of 1.
- **Equation:** A statement that two expressions are equal.
- **Variable:** A symbol representing an unknown number.
- **Solution of Equation:** The value of the variable that makes the equation true.

Solving Equations with Variable on Both Sides

Equations can have variables on both sides, such as $2x - 3 = x + 2$. To solve these, we perform algebraic operations to isolate the vari

Concept Explanation

To solve equations with variables on both sides, we use the principle of performing the same operation on both sides to maintain equalit
constants from one side to the other by adding or subtracting them.

Formula Derivation

No specific formula, but the key steps involve collecting like terms and isolating the variable.

Worked Illustration

Given $2x - 3 = x + 2$, add 3 to both sides:

$$2x = x + 5$$

Subtract x from both sides:

$$2x - x = 5x = 5$$

Solved Examples

Example 1: Solve $2x - 3 = x + 2$.

Solution:

- Add 3 to both sides:

$$2x = x + 5$$

- Subtract x from both sides:

$$x = 5$$

Example 2: Solve $\frac{5x+7}{2} = \frac{3}{2}x - 14$.

Solution:

- Multiply both sides by 2 to eliminate denominators:

$$2 \times \frac{5x+7}{2} = 2 \times \left(\frac{3}{2}x - 14\right) 5x + 7 = 3x - 28$$

- Transpose $3x$ to LHS and 7 to RHS:

$$5x - 3x = -28 - 72x = -35$$

- Divide both sides by 2:

$$x = -\frac{35}{2} = -17.5$$

Practice Set

- **Level 1 – Easy**
- Solve $3x + 2 = x + 6$.
- Solve $4x - 5 = 2x + 3$.
- **Level 2 – Moderate**
- Solve $\frac{x+3}{2} = \frac{3x-1}{4}$.
- Solve $5x + 7 = 3x - 1$.
- **Level 3 – Challenging**
- Solve $2(3x - 4) + 5 = 3(x + 2) - 1$.
- Solve $\frac{4x-3}{3} = \frac{2x+5}{2}$.

Answer Key

- **Level 1**
- $3x + 2 = x + 6 \Rightarrow 3x - x = 6 - 2 \Rightarrow 2x = 4 \Rightarrow x = 2$
- $4x - 5 = 2x + 3 \Rightarrow 4x - 2x = 3 + 5 \Rightarrow 2x = 8 \Rightarrow x = 4$
- **Level 2**
- $\frac{x+3}{2} = \frac{3x-1}{4} \Rightarrow 4(x+3) = 2(3x-1) \Rightarrow 4x + 12 = 6x - 2 \Rightarrow 4x - 6x = -2 - 12 \Rightarrow -2x = -14 \Rightarrow x = 7$
- $5x + 7 = 3x - 1 \Rightarrow 5x - 3x = -1 - 7 \Rightarrow 2x = -8 \Rightarrow x = -4$
- **Level 3**
- $2(3x - 4) + 5 = 3(x + 2) - 1 \Rightarrow 6x - 8 + 5 = 3x + 6 - 1 \Rightarrow 6x - 3x = 6 - 1 + 8 - 5 \Rightarrow 3x = 8 \Rightarrow x = \frac{8}{3}$
- $\frac{4x-3}{3} = \frac{2x+5}{2} \Rightarrow 2(4x - 3) = 3(2x + 5) \Rightarrow 8x - 6 = 6x + 15 \Rightarrow 8x - 6x = 15 + 6 \Rightarrow 2x = 21 \Rightarrow x = \frac{21}{2} = 10.5$

Quick Reference

Operation	
Adding/Subtracting same term on both sides	Maintains equality and helps isolate varia
Multiplying/Dividing both sides by same non-zero number	Eliminates fractions or coefficients
Transposing terms	Moving terms from one side to another by

Glossary

- **Transposition:** Moving a term from one side of the equation to the other by changing its sign.
- **Like Terms:** Terms with the same variable and power.
- **Coefficient:** Numerical factor of a variable.
- **LCM (Least Common Multiple):** Smallest number divisible by given denominators, used to clear fractions.

Reducing Equations to Simpler Form

Some equations may contain fractions or complex expressions. To solve them, we first reduce the equation to a simpler form by eliminat

Concept Explanation

To simplify equations with fractions, multiply both sides by the Least Common Multiple (LCM) of the denominators to clear fractions. The the equation to a linear form.

Formula Derivation

Multiplying both sides by LCM of denominators:

$$\text{If } \frac{A}{m} = \frac{B}{n}, \text{ multiply both sides by LCM}(m, n) \text{ to get } \frac{\text{LCM}}{m}A = \frac{\text{LCM}}{n}B$$

Worked Illustration

Given $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$, multiply both sides by 6 (LCM of 3 and 6):

$$6 \times \left(\frac{6x+1}{3}+1\right) = 6 \times \frac{x-3}{6}$$

Opening brackets:

$$6 \times \frac{6x+1}{3} + 6 \times 1 = x - 32(6x+1) + 6 = x - 312x + 2 + 6 = x - 312x + 8 = x$$

Transpose x to LHS and 8 to RHS:

$$12x - x = -3 - 811x = -11x = -1$$

Solved Examples

Example 1: Solve $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$.

Solution:

- Multiply both sides by 6:

$$6 \times \left(\frac{6x+1}{3}+1\right) = 6 \times \frac{x-3}{6} 2(6x+1) + 6 = x - 3$$

- Expand and simplify:

$$12x + 2 + 6 = x - 312x + 8 = x - 3$$

- Transpose terms:

$$12x - x = -3 - 811x = -11x = -1$$

Example 2: Solve $5x - 2(2x-7) = 2(3x-1) + \frac{7}{2}$.

Solution:

- Open brackets:

$$5x - 4x + 14 = 6x - 2 + \frac{7}{2} x + 14 = 6x - 2 + \frac{7}{2}$$

- Convert constants to common denominator and simplify RHS:

$$-2 + \frac{7}{2} = -\frac{4}{2} + \frac{7}{2} = \frac{3}{2}$$

So, equation becomes:

$$x + 14 = 6x + \frac{3}{2}$$

- Transpose terms:

$$14 - \frac{3}{2} = 6x - x \frac{28}{2} - \frac{3}{2} = 5x \frac{25}{2} = 5x x = \frac{25}{2} \times \frac{1}{5} = \frac{5}{2}$$

Practice Set

- **Level 1 – Easy**
- Solve $\frac{3x+2}{4} = 2$.
- Solve $\frac{x-1}{3} + 2 = 5$.
- **Level 2 – Moderate**
- Solve $\frac{2x+3}{5} = \frac{x-2}{3}$.
- Solve $3(2x-1) = 2(x+4) + 5$.
- **Level 3 – Challenging**
- Solve $\frac{4x-5}{2} + \frac{3x+1}{3} = 7$.
- Solve $2(3x-4) - \frac{5x-2}{2} = 3x + 1$.

Answer Key

- **Level 1**
- $\frac{3x+2}{4} = 2 \Rightarrow 3x + 2 = 8 \Rightarrow 3x = 6 \Rightarrow x = 2$
- $\frac{x-1}{3} + 2 = 5 \Rightarrow \frac{x-1}{3} = 3 \Rightarrow x - 1 = 9 \Rightarrow x = 10$
- **Level 2**
- $\frac{2x+3}{5} = \frac{x-2}{3} \Rightarrow 3(2x+3) = 5(x-2) \Rightarrow 6x + 9 = 5x - 10 \Rightarrow 6x - 5x = -10 - 9 \Rightarrow x = -19$

- $3(2x-1)=2(x+4)+5 \Rightarrow 6x-3=2x+8+5 \Rightarrow 6x-2x=13+3 \Rightarrow 4x=16 \Rightarrow x=4$
- **Level 3**
- $\frac{4x-5}{2}+\frac{3x+1}{3}=7 \Rightarrow 3(4x-5)+2(3x+1)=42 \Rightarrow 12x-15+6x+2=42 \Rightarrow 18x-13=42 \Rightarrow 18x=55 \Rightarrow x=\frac{55}{18}$
- $2(3x-4)-\frac{5x-2}{2}=3x+1 \Rightarrow 6x-8-\frac{5x-2}{2}=3x+1 \Rightarrow 12x-16-(5x-2)=6x+2 \Rightarrow 12x-16-5x+2=6x+2 \Rightarrow$

Quick Reference

Step	P
Multiply both sides by LCM	Eliminate fractions
Open brackets	Simplify expressions
Transpose terms	Collect like terms on one side
Divide by coefficient	Isolate variable

Glossary

- **LCM (Least Common Multiple):** The smallest number that is a multiple of given denominators.
- **Opening Brackets:** Expanding expressions like $a(b+c)=ab+ac$.
- **Transposing:** Moving terms from one side of the equation to the other by changing their signs.
- **Coefficient:** The numerical factor multiplying the variable.