Units and Measurement

Definition of Units

A unit is the chosen standard of measurement of a quantity which has essentially the same nature as that of the quantity.

Fundamental Units

Fundamental physical quantities are independent of each other and can represent other physical quantities. Their units are called fundame

Seven Fundamental Physical Quantities in SI System

- Mass - kilogram (kg)
- Length - meter (m)
- Time - second (s)
- Temperature - kelvin (K)
- Electric current - ampere (A)
- Luminous intensity - candela (cd)
- Amount of substance - mole (mol)

Derived Units

Derived units are obtained from fundamental units. Examples include velocity (m/s), acceleration (m/s^2), pressure (pascal, Pa), and force (n

Systems of Units

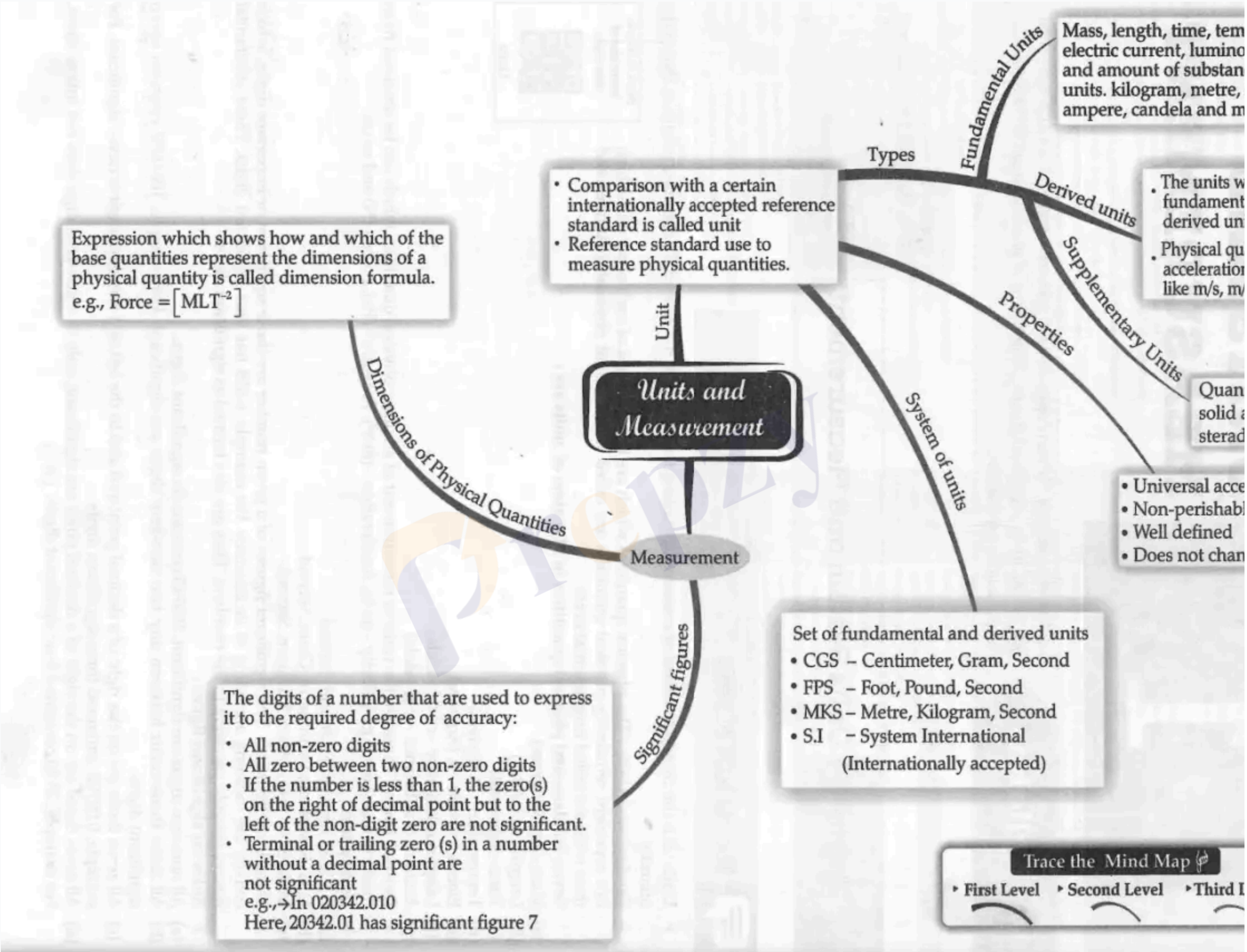
- FPS system: Foot, Pound, Second
- CGS system: Centimeter, Gram, Second
- MKS system: Meter, Kilogram, Second

Significant Figures

Significant figures are digits in a number that convey its accuracy. They include all non-zero digits, zeros between non-zero digits, and certa on their position relative to the decimal point.

Rules for Significant Figures

- All non-zero digits are significant. Example: 198745 has six significant digits.
- Zeros between non-zero digits are significant. Example: 108.0097 has seven significant digits.
- Zeros to the left of the first non-zero digit are not significant. Example: 0.00798 has three significant digits.
- Zeros to the right of a decimal point and after a non-zero digit are significant. Example: 20.00 has four significant digits.
- Zeros to the right of the last non-zero digit after the decimal point are significant. Example: 0.0079800 has five significant digits.
- Zeros to the right of the last non-zero digit in a measurement are significant. Example: 1090 m has four significant digits.



Key Terms

- **Fundamental Units:** Units of physical quantities independent of each other.
- **Derived Units:** Units obtained from fundamental units.

Key Formulae

- 1 AU = 1.496×10^{11} m
- 1 ly = 9.46×10^{15} m
- 1 parsec = 3.1×10^{16} m
- 1 Å = 10^{-10} m; 1 nm = 10^{-9} m
- 1 μm = 10^{-6} m; 1 mm = 10^{-3} m
- 60 seconds (of arc) = 1 minute (of arc)

- 60 minutes (of arc) = 1 degree (of arc)
- 180 degrees (of arc) = π radians
- Angular diameter (θ) = d / D, where d = diameter, D = distance
- Linear magnification = Size of image / Size of object
- Linear magnification = √(Areal magnification)

Mnemonics for Fundamental Quantities

Mnemonic: At the last moment she luckily caught the train.

- A – Amount of substance
- t – Temperature (thermodynamic)
- l – Length
- m – Mass
- lu – Luminous intensity
- c – Current
- t – Time

Dimensional Analysis

Definition

Dimensional analysis studies the relationship between physical quantities and fundamental quantities.

Dimensional Equation

An equation expressing the relationship between physical quantities and fundamental quantities.

Principle of Homogeneity

Dimensions of each term on both sides of a dimensional equation must be the same.

Conversion of Units

If a physical quantity has magnitude n_1 in one system and n_2 in another, with dimensions $[M^a L^b T^c]$, and units u_1 and u_2 respectively, then:

$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

Uses of Dimensional Analysis

- Deriving formulas of physical quantities.
- Checking correctness of formulas using homogeneity.

Limitations

- Cannot provide information about dimensional constants.
- Cannot derive formulas involving trigonometric, logarithmic, or exponential functions.
- Cannot determine if a quantity is scalar or vector.
- Difficult when physical quantity depends on more than three quantities.
- Cannot find formulas when quantities are not related by multiplication.

Key Terms

- **Dimensions of physical quantity:** Powers to which fundamental quantities are raised to represent derived units.
- **Dimensional formula:** Expression showing how fundamental quantities represent dimensions of a physical quantity.
- **Dimensional constants:** Quantities with constant values and dimensions (e.g., gravitational constant G).
- **Dimensional variables:** Quantities with variable values and dimensions (e.g., area, volume).
- **Dimensionless constants:** Constants without dimensions (e.g., π, e).
- **Dimensionless variables:** Variables without dimensions (e.g., angle, strain).

Key Formula

Conversion of units between systems:

$n_2 = n_1 (M_1 / M_2)^a (L_1 / L_2)^b (T_1 / T_2)^c$

Solved Examples

Practice Set

- **Level 1:** Define fundamental and derived units with examples.
- **Level 2:** Explain the principle of homogeneity in dimensional analysis.
- **Level 3:** A physical quantity has dimensions $[M^1 L^2 T^{-3}]$. If its magnitude is 50 in one system where mass unit is 2 kg, length unit is 3 m, and its magnitude in another system where mass unit is 1 kg, length unit is 1 m, and time unit is 2 s.

Answer Key

- **Level 1:** Fundamental units are basic units independent of each other, e.g., meter, kilogram, second. Derived units are obtained from fun velocity (m/s), force (N).
- **Level 2:** Principle of homogeneity states that all terms in a physical equation must have the same dimensions.
- **Level 3:** Given dimensions: $[M^1 L^2 T^{-3}]$, magnitude $n_1 = 50$, units in system 1: $M_1=2$ kg, $L_1=3$ m, $T_1=4$ s; units in system 2: $M_2=1$ kg, $L_2=1$ m, $T_2=2$

  Using formula: $n_2 = n_1 (M_1 / M_2)^a (L_1 / L_2)^b (T_1 / T_2)^c$

  $n_2 = 50 \times (2/1)^1 \times (3/1)^2 \times (4/2)^{-3} = 50 \times 2 \times 9 \times (2)^{-3} = 50 \times 2 \times 9 \times 1/8 = 50 \times 18 / 8 = 50 \times 2.25 = 112.5$

  Therefore, magnitude in system 2 is 112.5.

Quick Reference Table

Common Mistakes and Misconceptions