Elastic Behaviour of Solids

Stress and its Types

Stress is defined as the restoring force acting per unit area of a deformed body. Mathematically, stress = restoring force / area = F / A. The SI unit of stress is newton per square meter (N m^{-2}) and its dimensional formula is $[ML^{-1}T^{-2}]$. Stress is a tensor quantity and normal stress has three types:

- **Longitudinal stress:** Stress normal to the surface causing change in length. It can be tensile (increase in length) or compressive (decrease in length).
- **Volumetric stress:** Stress causing change in volume under normal force acting on all surfaces.
- **Tangential stress (Shearing stress):** Stress due to force applied tangentially, causing change in shape without volume change.

Strain and its Types

Strain is the ratio of change in configuration of a body due to deforming force to its original configuration. It is dimensionless and has no units.

- **Longitudinal strain:** Change in length / Original length = Δl / l
- **Volumetric strain:** Change in volume / Original volume = ΔV / V
- **Shearing strain:** When force is applied parallel to surface, causing shape change without volume change. Defined as the angle θ through which a vertical line rotates under tangential force, θ = ΔL / L, where ΔL is displacement and L is perpendicular distance from fixed surface.

Hooke's Law

Within the elastic limit, stress is directly proportional to strain, i.e., Stress ∝ Strain.

Key Terms

- **Deforming force:** Force that changes the configuration of a body.
- **Elasticity:** Property of a body to regain original shape after removal of deforming force.
- **Perfectly elastic body:** Regains original form completely, e.g., quartz.
- **Plastic body:** Does not regain original form after deforming force removal, e.g., putty.
- **Elastic limit:** Maximum deforming force up to which body regains original form completely.

Key Formulae

- Normal stress, S = F / A
- Breaking force = Breaking stress × Area
- Longitudinal strain = Δl / l
- Volumetric strain = ΔV / V
- Shearing strain, θ = ΔL / L

Solved Examples

Example 1: A wire of length 2 m and cross-sectional area 1 mm^2 is stretched by a force of 10 N. Calculate the longitudinal stress and strain if the extension produced is 0.5 mm.

Solution:

Given: Length, l = 2 m = 2000 mm; Area, A = 1 mm^2; Force, F = 10 N; Extension, Δl = 0.5 mm

Longitudinal stress, S = F / A = 10 N / 1 mm^2 = 10 N/mm^2

Longitudinal strain = Δl / l = 0.5 mm / 2000 mm = 0.00025

Thus, stress = 10 N/mm^2, strain = 0.00025 (dimensionless).

Practice Set

- **Level 1:** Define stress and strain. What are their SI units?
- **Level 2:** Explain the difference between longitudinal stress and tangential stress.
- **Level 3:** A wire of length 1.5 m and cross-sectional area 2 mm^2 is stretched by a force of 20 N. If the extension produced is 0.3 mm, calculate the longitudinal stress and strain.

Answer Key

- **Level 1:** Stress is force per unit area (N/m^2), strain is change in length/original length (dimensionless).
- **Level 2:** Longitudinal stress acts normal to surface causing length change; tangential stress acts parallel causing shape change without volume change.
- **Level 3:** Stress = 20 N / 2 mm^2 = 10 N/mm^2; Strain = 0.3 mm / 1500 mm = 0.0002.

Modulus of Elasticity

Definition and Types

Modulus of elasticity (E) is the ratio of stress to corresponding strain within the elastic limit, E = Stress / Strain.

Types of modulus of elasticity:

- **Young's modulus (Y):** Ratio of longitudinal stress to longitudinal strain, $Y = (F / A) / (\Delta l / l) = (F / A) \times (l / \Delta l)$. It measures stiffness of solid materials.
- **Bulk modulus (K):** Ratio of volume stress to volumetric strain, $K = -(P V) / \Delta V$. It applies to solids, liquids, and gases.
- **Modulus of rigidity (η):** Ratio of tangential stress to shearing strain, $\eta = (F / A) / \theta = F / (A\theta)$. It applies to solids only.

Poisson's Ratio

Poisson's ratio (σ) is the ratio of lateral strain to longitudinal strain, $\sigma = (\Delta D / D) / (\Delta l / l) = -(\Delta D \cdot l) / (D \cdot \Delta l)$. Its practical value lies between 0 and 0.5.

Key Terms

- **Compressibility:** Reciprocal of bulk modulus, $c = 1 / K = -\Delta V / (P V)$.
- **Elastic fatigue:** Loss of strength due to repeated alternating strains.
- **Yield strength:** Maximum stress a material can sustain without permanent deformation.

Key Formulae

- Young's modulus, $Y = Fl / (A\Delta l)$
- Bulk modulus, $K = -(F / A) / (\Delta V / V) = -P V / \Delta V$
- Modulus of rigidity, $\eta = Fl / (A\Delta l)$
- Poisson's ratio, $\sigma = -(\Delta D / D) / (\Delta l / l)$

- Relations among elastic constants:
  $Y = 3K(1 - 2\sigma)$
  $Y = 2\eta(1 + \sigma)$
  $\sigma = (3K - 2\eta) / (2\eta + 6K)$
  $9 / Y = 1 / K + 3 / \eta$
- Elastic potential energy, U = ½ × Stress × Strain × Volume
- Elastic potential energy per unit volume, u = ½ × Stress × Strain = ½ Y × $(\text{Strain})^2$
- Work done in stretching wire, W = ½ × Load × Extension

Solved Examples

Example 1: A wire of length 2 m and cross-sectional area 1 mm^2 is stretched by a force of 10 N producing an extension of 0.5 mm. Calculate Young's modulus.

Solution:

Given: l = 2 m = 2000 mm, A = 1 mm^2, F = 10 N, Δl = 0.5 mm

Young's modulus, $Y = (F / A) \times (l / \Delta l) = (10 / 1) \times (2000 / 0.5) = 10 \times 4000 = 40000\ N/mm^2 = 4 \times 10^{10}\ N/m^2$

Thus, Young's modulus is 4×10^{10} pascal.

Practice Set

- **Level 1:** Define Young's modulus and bulk modulus.
- **Level 2:** Explain Poisson's ratio and its practical range.
- **Level 3:** A wire of length 1 m and cross-sectional area 0.5 mm^2 is stretched by 5 N force producing an extension of 0.2 mm. Calculate Young's modulus.

Answer Key

- **Level 1:** Young's modulus is ratio of longitudinal stress to strain; bulk modulus is ratio of volume stress to volumetric strain.
- **Level 2:** Poisson's ratio is lateral strain divided by longitudinal strain; practical values lie between 0 and 0.5.
- **Level 3:** $Y = (F / A) \times (l / \Delta l) = (5 / 0.5) \times (1000 / 0.2) = 10 \times 5000 = 50000\ N/mm^2 = 5 \times 10^{10}$ Pa.

Quick Reference Table

Common Mistakes and Misconceptions

Glossary