Harmonic Oscillations

Definition and Mathematical Representation

Harmonic oscillations are oscillations that can be expressed using a single harmonic function such as sine or cosine. The displacement of a particle undergoing harmonic oscillation can be written as:

y = a \sin \omega t or y = a \cos \omega t

Periodic Nature

These oscillations repeat themselves after a fixed interval of time, making them periodic functions. The sine and cosine functions used to describe harmonic oscillations are periodic with period T.

Phase of Oscillation

The phase of a vibrating particle at any instant is a quantity that fully describes its position and direction of motion relative to the mean position. For a displacement given by $y = a \sin (\omega t + \phi)$, the term $(\omega t + \phi)$ is called the phase, where ϕ is the initial phase.

Solved Examples

Example 1: A particle executes simple harmonic motion with displacement $y = 5 \sin (10t + \pi/6)$ cm. Find the displacement at $t = 0.2$ s.

Solution:

Given: $a = 5$ cm, $\omega = 10$ rad/s, $\phi = \pi/6$, $t = 0.2$ s

Displacement, $y = 5 \sin (10 \times 0.2 + \pi/6) = 5 \sin (2 + \pi/6)$

Calculate the angle in radians: $2 + \pi/6 \approx 2 + 0.5236 = 2.5236$ rad

$y = 5 \times \sin 2.5236 \approx 5 \times 0.579 = 2.895$ cm

Therefore, the displacement at $t = 0.2$ s is approximately 2.895 cm.

Practice Set

- **Level 1:** Define harmonic oscillation and write its general mathematical expression.
- **Level 2:** Explain the significance of phase and initial phase in harmonic oscillations.
- **Level 3:** A particle executes harmonic motion with displacement $y = 3 \cos (5t + \pi/4)$. Calculate the displacement at $t = 0.1$ s.

Answer Key

- **Level 1:** Harmonic oscillations are oscillations that can be expressed as a single sine or cosine function. The general expression is $y = a \sin \omega t$ or $y = a \cos \omega t$.
- **Level 2:** The phase $(\omega t + \phi)$ determines the position and direction of the particle at any instant. The initial phase ϕ is the phase at time $t = 0$, indicating the starting position of the oscillation.
- **Level 3:** $y = 3 \cos (5 \times 0.1 + \pi/4) = 3 \cos (0.5 + 0.7854) = 3 \cos 1.2854 \approx 3 \times 0.28 = 0.84$ units.

Simple Harmonic Motion

Definition and Restoring Force

Simple Harmonic Motion (S.H.M.) is a special type of periodic motion where a particle moves back and forth about a mean (equilibrium) position under a restoring force. This force is always directed towards the mean position and is proportional to the displacement from it:

$$F = -ky$$

Here, k is the force constant, and the negative sign indicates the force acts opposite to displacement.

Geometrical Interpretation

S.H.M. can be viewed as the projection of uniform circular motion on a diameter of the circle. It can be linear or angular:

- Linear S.H.M. occurs along a straight line about a fixed point.

- Angular S.H.M. occurs along an arc of a circle about a fixed point.

Characteristics of S.H.M.

- **Displacement:** Distance from mean position at any instant.
- **Velocity:** Rate of change of displacement.
- **Amplitude:** Maximum displacement from mean position.
- **Acceleration:** Rate of change of velocity.
- **Time Period:** Time taken to complete one oscillation.

Solved Examples

Example 2: A particle of mass 0.5 kg executes S.H.M. with amplitude 0.1 m and force constant k = 20 N/m. Find the time period of oscillation.

Solution:

Time period, $T = 2\pi \sqrt{m/k} = 2\pi \sqrt{0.5/20} = 2\pi \sqrt{0.025} = 2\pi \times 0.1581 = 0.994$ s

Therefore, the time period is approximately 0.994 seconds.

Practice Set

- **Level 1:** State the force law governing simple harmonic motion.
- **Level 2:** Describe the difference between linear and angular S.H.M.
- **Level 3:** Calculate the force constant for a particle of mass 2 kg oscillating with angular frequency 5 rad/s.

Answer Key

- **Level 1:** The restoring force in S.H.M. is $F = -ky$, where k is the force constant.
- **Level 2:** Linear S.H.M. occurs along a straight line, while angular S.H.M. occurs along an arc of a circle.
- **Level 3:** $k = m \omega^2 = 2 \times 5^2 = 2 \times 25 = 50$ N/m.

Simple Pendulum

Definition and Description

A simple pendulum consists of a heavy point mass suspended by a weightless, inextensible, and flexible string from a rigid support. It oscillates freely about the point of suspension.

Time Period Formula

The time period of a simple pendulum is given by:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity.

Solved Examples

Example 3: Calculate the time period of a simple pendulum of length 1 m. Take $g = 9.8$ m/s².

Solution:

$T = 2\pi \sqrt{1/9.8} = 2\pi \times 0.319 = 2.006$ s

The time period is approximately 2.006 seconds.

Practice Set

- **Level 1:** What is a simple pendulum?
- **Level 2:** Derive the formula for the time period of a simple pendulum.
- **Level 3:** If the length of a pendulum is quadrupled, how does its time period change?

Answer Key

- **Level 1:** A simple pendulum is a heavy point mass suspended by a weightless string that oscillates about a fixed point.
- **Level 2:** The time period $T = 2\pi \sqrt{l/g}$ is derived by equating restoring torque and angular acceleration and solving the differential equation of motion.
- **Level 3:** Time period T is proportional to $\sqrt{l}$, so if length is quadrupled, T becomes 2 times the original.

Energy in Simple Harmonic Motion

Types of Energy

A particle executing S.H.M. possesses two types of energy:

- **Potential Energy (U):** Energy due to displacement from mean position, given by $U = \frac{1}{2} m \omega^2 y^2$.

- **Kinetic Energy (K):** Energy due to velocity, given by $K = \frac{1}{2} m \omega^2 (a^2 - y^2)$.

Total Energy

The total mechanical energy E remains constant and is the sum of kinetic and potential energies:

$$E = \frac{1}{2} m \omega^2 a^2$$

Solved Examples

Example 4: A particle of mass 0.2 kg oscillates with amplitude 0.05 m and angular frequency 10 rad/s. Calculate its total energy.

Solution:

$$E = \frac{1}{2} m \omega^2 a^2 = 0.5 \times 0.2 \times 10^2 \times 0.05^2 = 0.1 \times 100 \times 0.0025 = 0.025 J$$

The total energy of the particle is 0.025 joules.

Practice Set

- **Level 1:** What are the two types of energy in S.H.M.?
- **Level 2:** Write the expression for potential energy in S.H.M.
- **Level 3:** Calculate the kinetic energy of a particle in S.H.M. at displacement $y = 0.02$ m, given $m = 0.1$ kg, $\omega = 20$ rad/s, and amplitude $a = 0.05$ m.

Answer Key

- **Level 1:** Potential energy and kinetic energy.
- **Level 2:** $U = \frac{1}{2} m \omega^2 y^2$.
- **Level 3:** $K = \frac{1}{2} m \omega^2 (a^2 - y^2) = 0.5 \times 0.1 \times 20^2 \times (0.05^2 - 0.02^2) = 0.05 \times 400 \times (0.0025 - 0.0004) = 20 \times 0.0021 = 0.042$ J.

Quick Reference Table

Common Mistakes and Misconceptions

Glossary