- Angle Measurement Systems
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Angle Measurement Systems

Angles can be measured using three different systems: Degree, Radian, and Centesimal.

Degree Measure

A right angle is divided into 90 equal parts called degrees (°). Each degree is further divided into 60 minutes (') and each minute into 60 sec

- 1° = 60'
- 1' = 60"

Radian Measure

In a circle of radius r, an arc of length l subtends an angle θ radians at the center, given by:

$$l = r\theta \implies \theta = \frac{l}{r}$$

Centesimal System

A right angle is divided into 100 grades, each grade into 100 minutes, and each minute into 100 seconds:

- 1 right angle = 100 grades
- 1 grade = 100 minutes
- 1 minute = 100 seconds

Relation Between Systems

The relation between degree (D), grade (G), and radian (R) measures is:

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Since a full circle subtends 2π radians or 360°, we have:

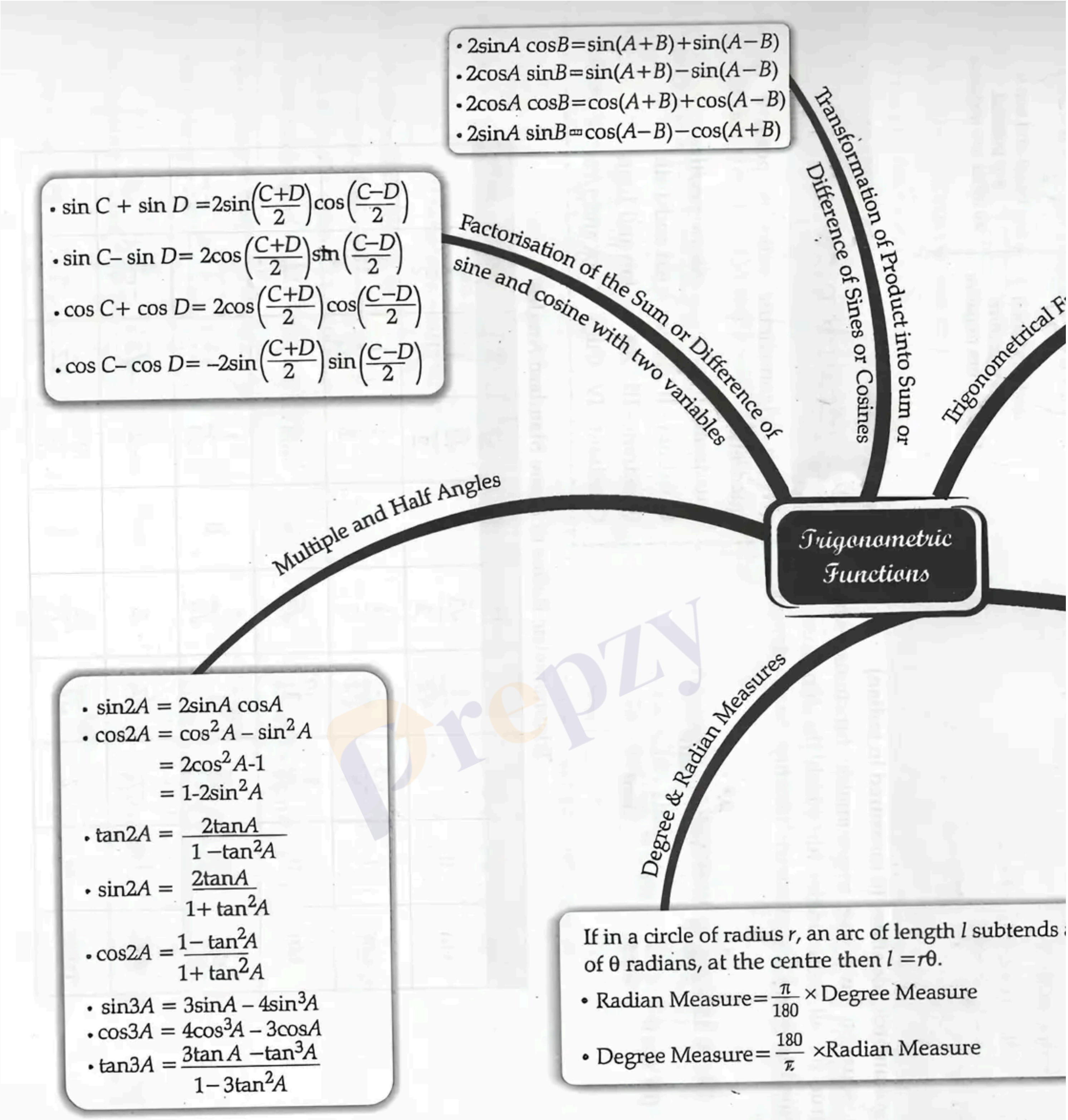
$$2\pi \text{ radians} = 360^\circ \implies \pi \text{ radians} = 180^\circ$$

Therefore,

- $1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57^\circ 16' 22''$
- $\text{Radian} = \frac{\pi}{180} \times \text{degree measure}$
- $\text{Degree} = \frac{180}{\pi} \times \text{radian measure}$

Note:

- The angle between two consecutive digits on a clock is 30° ($= \frac{\pi}{6}$ radians).
- The minute hand rotates through 6° in one minute.
- Radian is a constant angle.
- $1^\circ = \frac{\pi}{180} \text{ rad} = 0.0176 \text{ rad}$



Basic Trigonometric Identities

Trigonometric identities are equations involving trigonometric functions that hold true for all angles where the functions are defined.

Fundamental Identities

- $\sin\theta = \frac{1}{\csc\theta} \quad \text{and} \quad \csc\theta = \frac{1}{\sin\theta}$
- $\cos\theta = \frac{1}{\sec\theta} \quad \text{and} \quad \sec\theta = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} \quad \text{and} \quad \tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta}$
- $\cos^2\theta + \sin^2\theta = 1 \quad \Rightarrow \quad 1 - \cos^2\theta = \sin^2\theta \quad \text{and} \quad 1 - \sin^2\theta = \cos^2\theta$
- $1 + \tan^2\theta = \sec^2\theta \quad \Rightarrow \quad \sec^2\theta - \tan^2\theta = 1$
- $1 + \cot^2\theta = \csc^2\theta \quad \Rightarrow \quad \csc^2\theta - \cot^2\theta = 1$

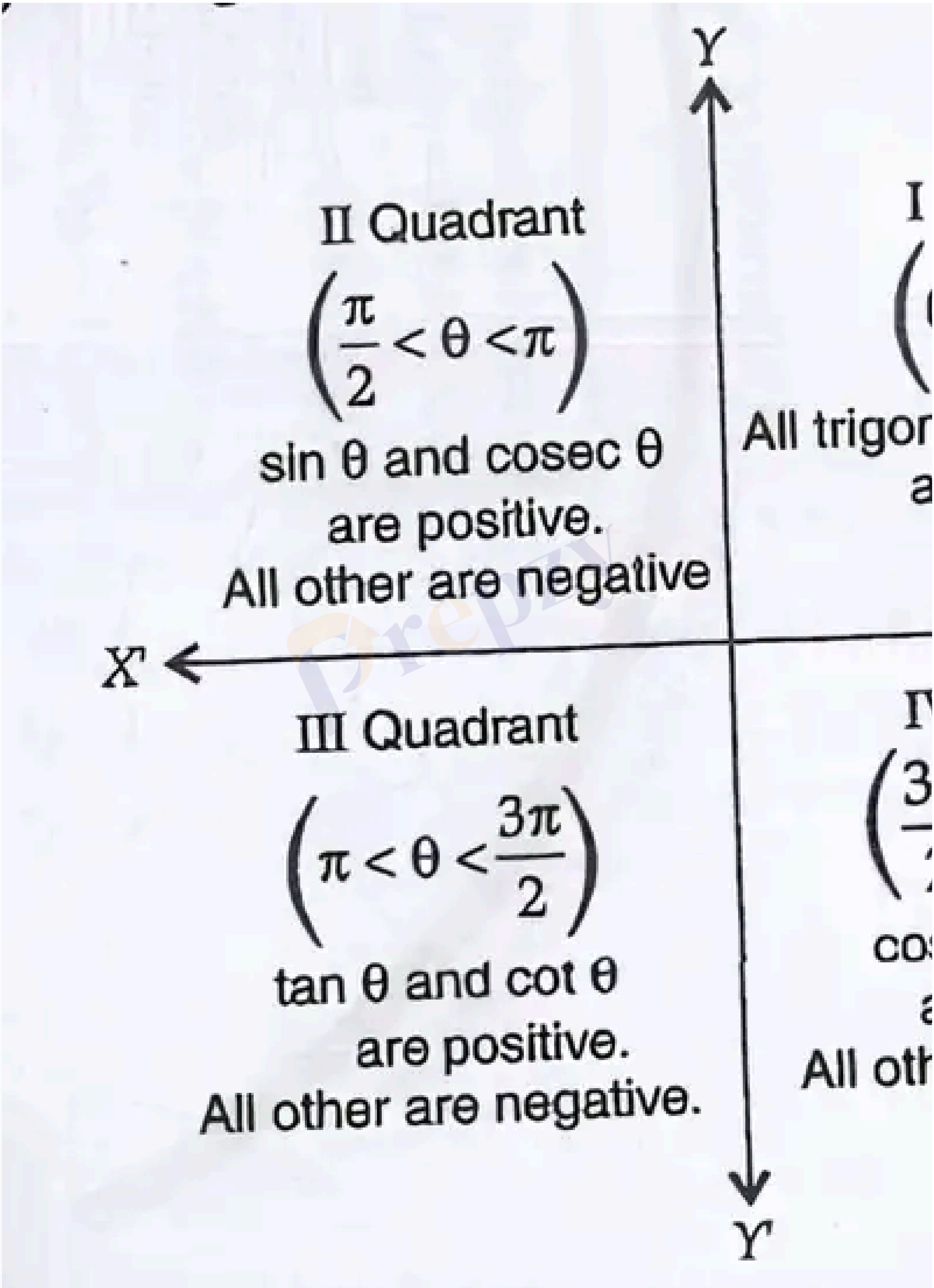
Sign of Trigonometric Functions in Different Quadrants

The coordinate plane is divided into four quadrants, each with specific signs for trigonometric functions:

- **Quadrant I:** $0 < \theta < \frac{\pi}{2}$ – All functions are positive.
- **Quadrant II:** $\frac{\pi}{2} < \theta < \pi$ – Only $\sin\theta$ and $\csc\theta$ are positive.
- **Quadrant III:** $\pi < \theta < \frac{3\pi}{2}$ – Only $\tan\theta$ and $\cot\theta$ are positive.
- **Quadrant IV:** $\frac{3\pi}{2} < \theta < 2\pi$ – Only $\cos\theta$ and $\sec\theta$ are positive.

Mnemonic to remember positive functions:

- Quadrant I: **All**
- Quadrant II: **Silver** (sin and cosec)
- Quadrant III: **Tea** (tan and cot)
- Quadrant IV: **Cups** (cos and sec)



Standard Trigonometric Ratios

Values of trigonometric functions for standard angles are tabulated below:

Trigonometric Ratios of Some S

Angle	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Domain and Range of Trigonometric Functions

Function	Domain
$\sin x$	$\mathbb{R}$
$\cos x$	$\mathbb{R}$
$\tan x$	$\mathbb{R} - \{(2n+1)\left(\frac{\pi}{2}\right) ;n \in$
$\operatorname{cosec} x$	$\mathbb{R} - \{n\pi ; n \in \mathbb{Z}\}$
$\sec x$	$\mathbb{R} - \{(2n+1)\left(\frac{\pi}{2}\right); n$
$\cot x$	$\mathbb{R} - \{n\pi ; n \in \mathbb{Z}\}$

Compound Angle Formulas

Compound angle formulas express trigonometric functions of sums or differences of angles in terms of functions of individual angles.

Formulas

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$
- $\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$
- $\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y \tan z - \tan z \tan x}$

Note

- $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$
- $\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$

Transformation Formulas

Transformation formulas convert sums or differences of sines and cosines into products, and vice versa.

- $2\sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2\cos x \sin y = \sin(x+y) - \sin(x-y)$
- $2\cos x \cos y = \cos(x+y) + \cos(x-y) \quad [x > y]$
- $2\sin x \sin y = \cos(x-y) - \cos(x+y)$
- $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
- $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$
- $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$
- $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

Multiple and Half Angle Formulas

These formulas express trigonometric functions of multiple or half angles in terms of functions of the original angle.

Multiple Angle Formulas

- $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1-\tan^2 x}{1+\tan^2 x}$
- $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

Half Angle Formulas

- $\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$
- $\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$
- $\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$

The sign depends on the quadrant in which $\frac{A}{2}$ lies.

Trigonometric Functions of 18 Degrees

Let $\theta = 18^\circ$. Using multiple angle formulas, we derive $\sin 18^\circ$ and $\cos 18^\circ$.

Since $2\theta = 90^\circ - 3\theta$,

$$\sin 2\theta = \sin(90^\circ - 3\theta) = \cos 3\theta$$

Using the triple angle formula for cosine,

$$\sin 2\theta = 4\cos^3\theta - 3\cos\theta$$

Express $\sin 2\theta$ in terms of $\sin\theta$:

$$2\sin\theta = 4\cos^2\theta - 3 = 4(1 - \sin^2\theta) - 3 = 4 - 4\sin^2\theta - 3 =$$

Rearranged as a quadratic in $\sin\theta$:

$$4\sin^2\theta + 2\sin\theta - 1 = 0$$

Solving for $\sin\theta$:

$$\sin\theta = \frac{-2 \pm \sqrt{4 + 16}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

Since $\sin 18^\circ > 0$,

$$\sin 18^\circ = \frac{\sqrt{5} - 1}{4}$$

Calculate $\cos 18^\circ$:

$$\cos 18^\circ = \sqrt{1 - \sin^2 18^\circ} = \sqrt{1 - \left(\frac{\sqrt{5} - 1}{4}\right)^2} = \sqrt{\frac{10 + 2\sqrt{5}}{16}} =$$

Using $\cos 36^\circ = 1 - 2\sin^2 18^\circ$,

$$\cos 36^\circ = 1 - 2\left(\frac{\sqrt{5} - 1}{4}\right)^2 = 1 - \frac{6 - 2\sqrt{5}}{8} = \frac{2 + 2\sqrt{5}}{8} =$$

Calculate $\sin 36^\circ$:

$$\sin 36^\circ = \sqrt{1 - \cos^2 36^\circ} = \sqrt{1 - \left(\frac{\sqrt{5} + 1}{4}\right)^2} = \sqrt{\frac{10 \cdots}{}}$$