

- Linear Inequalities

Linear Inequalities

Linear inequalities are mathematical expressions involving variables of degree one, connected by inequality symbols such as $<$, $>$, \leq , and \geq .

Concept Explanation

A linear inequality in one variable has the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$, or $ax + b \geq 0$, where $a \neq 0$. For two variables, it takes the form $ax + by + c < 0$, $ax + by + c \leq 0$, $ax + by + c > 0$, or $ax + by + c \geq 0$.

These inequalities define regions on the number line (one variable) or the coordinate plane (two variables).

Formula Derivation

Rules for solving linear inequalities:

- If $a \geq b$, then $a \pm k \geq b \pm k$ for any real number k .
- If $a \geq b$ and $k > 0$, then $ka \geq kb$.
- If $a \geq b$ and $k < 0$, then $ka \leq kb$ (inequality sign reverses).

These rules allow manipulation of inequalities similarly to equations, with caution when multiplying or dividing by negative numbers.

Worked Illustrations

Example 1: Solve $3x - 5 < 7$.

Step 1: Add 5 to both sides:

$$3x - 5 + 5 < 7 + 5 \quad 3x < 12$$

Step 2: Divide both sides by 3 (positive number, so inequality sign remains):

$$x < \frac{12}{3} \quad x < 4$$

Example 2: Solve $-2x + 3 \geq 7$.

Step 1: Subtract 3 from both sides:

$$-2x + 3 - 3 \geq 7 - 3 - 2x \geq 4$$

Step 2: Divide both sides by -2 (negative number, reverse inequality):

$$x \leq \frac{4}{-2} x \leq -2$$

Solved Examples

Example 3: Solve $5 - 2x > 1$.

Step 1: Subtract 5 from both sides:

$$5 - 2x - 5 > 1 - 5 - 2x > -4$$

Step 2: Divide both sides by -2 (negative number, reverse inequality):

$$x < \frac{-4}{-2} x < 2$$

Practice Set

Level 1 – Easy

- Solve $x + 3 > 7$.
- Solve $4x - 1 \leq 11$.
- Solve $2x + 5 < 15$.

Level 2 – Moderate

- Solve $-3x + 4 \geq 10$.
- Solve $5 - 2x < 9$.
- Solve $7x + 2 \leq 3x + 10$.

Level 3 – Challenging

- Solve $\frac{2x-3}{4} > 1$.
- Solve $3(2x - 1) \leq 2(x + 5)$.
- Solve $-4x + 7 > 3x - 8$.

Answer Key

- Level 1
 - $x > 4$
 - $x \leq 3$
 - $x < 5$
- Level 2
 - $x \leq -2$
 - $x > -2$
 - Subtract $3x$ both sides: $4x - 3x + 2 \leq 10 \Rightarrow x + 2 \leq 10 \Rightarrow x \leq 8$
- Level 3
 - Multiply both sides by 4: $2x - 3 > 4 \Rightarrow 2x > 7 \Rightarrow x > \frac{7}{2}$
 - Expand: $6x - 3 \leq 2x + 10 \Rightarrow 6x - 2x \leq 10 + 3 \Rightarrow 4x \leq 13 \Rightarrow x \leq \frac{13}{4}$
 - Bring terms: $-4x - 3x > -8 - 7 \Rightarrow -7x > -15$ Divide by -7 (reverse inequality): $x < \frac{15}{7}$

Quick Reference

	Operation	
Add or subtract same number		Inequality sig
Multiply or divide by positive number		Inequality sig
Multiply or divide by negative number		Inequality sig

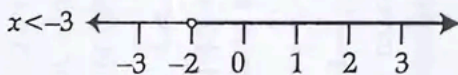
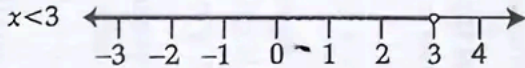
Glossary

- **Inequality:** A relation between two expressions using $<$, $>$, \leq , \geq .
- **Linear Inequality:** Inequality involving variables of degree one.
- **Solution Set:** All values satisfying the inequality.
- **Replacement Set:** Set of values from which variable values are chosen.
- **Strict Inequality:** Inequalities with $<$ or $>$.
- **Slack Inequality:** Inequalities with \leq or \geq .

- The notation $a < b$ means that 'a' is less than 'b'.
- The notation $a > b$ means that 'a' is greater than 'b'.
- The notation $a \geq b$ means that 'a' is greater than or equal to 'b'.
- The notation $a \leq b$ means that 'a' is less than or equal to 'b'.
- The notation $a \neq b$ means that 'a' is not equal to 'b'.

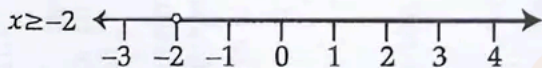
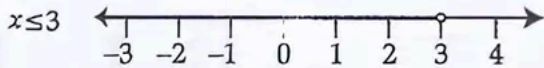
To represent $x < a$ or $x > -a$ on a number line, put a circle on the number 'a' and dark line to the left (or right) of the number 'a'.

e.g:



To represent $x \leq a$ (or $x \geq -a$) on a number line, put a dark circle on the number 'a' and dark the line to the left (or right) of the number 'a'.

e.g:



• Equal numbers may be added to (or subtracted from) both the sides of an inequality without affecting the sign of inequality:

e.g: $x < 7$ is same as $x + 2 < 7 + 2$

• Both sides of an inequality can be multiplied (or divided) by the same positive number without affecting the sign of inequality.

e.g: $x + y < 7$ is same as $(x + y) \times 3 < 7 \times 3$

• When both sides are multiplied or divided by a -ve number, then sign of inequality is reversed.

e.g: $x + y < 7$ is same as $(x + 7) \times (-3) > 7 \times (-3)$

Solution and Solution set

Solution: Values of x , which make inequality a true statement.

e.g.: '3' is the solution for $x < 7$.

Solution Set:

The set of values of x is called its solution set. e.g.: $\{1, 2, 3, 4, 5, 6\}$ is solution set for $x < 7$, where x is a natural number.

Linear Inequalities

Notations

Solution of Linear Inequality in One Variable

Representation

Rules of Inequality

Inequalities in two Variables
Graphical Solution of Linear

Typ

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Condit

• A line divides the cartesian plane into two or will in either of half planes. The region

region.
• In order to identify the half plane represent (not on line) and check whether it satisfies the half plane and shade the region which plane which does not contain the point w

• If an inequality is of the type $ax + by \geq c$ or the solution region, so draw a dark line in
• If an inequality is the form of $ax + by > c$ or included in the solution region, so draw a

Trace the Mind Map

► First Level