

- Recall of Two Dimensional Geometry and Slope of a line
- Various forms of equations of a straight line

Prepzy

Recall of Two Dimensional Geometry and Slope of a line

Coordinate geometry involves locating points on a plane using two perpendicular number lines called axes: the X-axis (horizontal) and the Y-axis (vertical). The intersection point of these axes is the origin, denoted as (0,0). The plane is divided into four quadrants:

- Quadrant I: $x > 0, y > 0$
- Quadrant II: $x < 0, y > 0$
- Quadrant III: $x < 0, y < 0$
- Quadrant IV: $x > 0, y < 0$

Points are represented as ordered pairs (x, y) indicating their position relative to the axes.

Section Formula

To find the coordinates of a point R dividing the segment joining points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m:n$, use:

$$R \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n} \right)$$

For external division in the ratio $m:n$, the coordinates are:

$$R \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n} \right)$$

Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of a point $P(x_1, y_1)$ from the origin is:

$$OP = \sqrt{x_1^2 + y_1^2}$$

Area of Triangle

For triangle ABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$, the area is:

$$\Delta = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

If $\Delta = 0$, points A, B, and C are collinear.

Area of Quadrilateral

For quadrilateral ABCD with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, and $D(x_4, y_4)$, the area is:

$$\text{Area} = \frac{1}{2} |(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)|$$

Area of Trapezium

For trapezium with vertices $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, and $S(x_4, y_4)$, the area is:

$$\text{Area} = \frac{1}{2} [(y_1 + y_2)(x_1 - x_2) + (y_3 + y_1)(x_3 - x_1) + (y_2 + y_3)(x_3 - x_2)]$$

Slope of a Line

The slope m of a line joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ is the tangent of the angle θ it makes with the positive X-axis:

$$m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Angle Between Two Lines

For two lines with slopes m_1 and m_2 , the angle θ between them is:

$$\theta = \tan^{-1} \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

Lines are parallel if $m_1 = m_2$ and perpendicular if $m_1 m_2 = -1$.

Collinearity of Three Points

Points A, B, and C are collinear if the slope of AB equals the slope of BC.

Additional Concepts

- Midpoint of segment PQ: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- Centroid of triangle ABC: $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$
- Centroid divides each median in ratio 2:1.

- Coordinate translation: If origin shifts from $(0,0)$ to (α, β) , new coordinates of $P(x,y)$ are $P'(x - \alpha, y - \beta)$.
- Slope of vertical line is undefined (∞).
- Positive $\tan \theta$ indicates acute angle; negative indicates obtuse angle.

Worked Example

Find the slope of the line joining points A(2, 3) and B(5, 11).

Step 1: Identify coordinates: $x_1 = 2, y_1 = 3, x_2 = 5, y_2 = 11$.

Step 2: Use slope formula:

$$m = \frac{11 - 3}{5 - 2} = \frac{8}{3}$$

Answer: Slope $m = \frac{8}{3}$.

Practice Set

- **Level 1 (Easy):** Find the distance between points (1,2) and (4,6).
- **Level 2 (Moderate):** Find the coordinates of the point dividing the segment joining (3,4) and (7,8) internally in ratio 2:3.
- **Level 3 (Challenging):** Determine if points (1,2), (3,6), and (5,10) are collinear.

Answer Key

- Level 1: Distance = $\sqrt{(4 - 1)^2 + (6 - 2)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.
- Level 2: Coordinates = $\left(\frac{2 \times 7 + 3 \times 3}{2 + 3}, \frac{2 \times 8 + 3 \times 4}{2 + 3}\right) = \left(\frac{14 + 9}{5}, \frac{16 + 12}{5}\right) = (4.6, 5.6)$.
- Level 3: Slope AB = $\frac{6 - 2}{3 - 1} = 2$, Slope BC = $\frac{10 - 6}{5 - 3} = 2$, slopes equal, so points are collinear.

Quick Reference

Concept	Formula
Distance between points	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Section formula (internal)	$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$
Slope of line	$\frac{y_2 - y_1}{x_2 - x_1}$
Area of triangle	$\frac{1}{2} x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) $

Glossary

- **Coordinate Plane:** A plane with two perpendicular axes used to locate points.
- **Origin:** The point (0,0) where axes intersect.
- **Slope:** Measure of steepness of a line.
- **Collinear:** Points lying on the same straight line.
- **Section Formula:** Formula to find a point dividing a segment in a given ratio.

Various forms of equations of a straight line

Equations of straight lines describe the set of points lying on the line in the coordinate plane. Different forms are used depending on the given information.

Horizontal and Vertical Lines

- Horizontal line (parallel to X-axis):
 $y = a$ where a is constant.
- Vertical line (parallel to Y-axis):
 $x = b$ where b is constant.

Slope-Intercept Form

If a line has slope m and y-intercept c , its equation is:

$$y = mx + c$$

If the line passes through the origin, $c = 0$, so equation is $y = mx$.

Point-Slope Form

For a line with slope m passing through point $P_0(x_0, y_0)$, the equation is:

$$y - y_0 = m(x - x_0)$$

Two-Point Form

For a line passing through points (x_1, y_1) and (x_2, y_2) , the equation is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Intercept Form

If a line cuts intercepts a and b on the X-axis and Y-axis respectively, its equation is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Additional Information

- Equation of X-axis: $y = 0$
- Equation of Y-axis: $x = 0$
- If a line has slope m and X-intercept d , equation is $y = m(x - d)$.

Worked Example

Find the equation of the line passing through points (1,2) and (3,6).

Step 1: Calculate slope m :

$$m = \frac{6 - 2}{3 - 1} = \frac{4}{2} = 2$$

Step 2: Use point-slope form with point (1,2):

$$y - 2 = 2(x - 1)$$

Step 3: Simplify:

$$y - 2 = 2x - 2 \Rightarrow y = 2x$$

Practice Set

- **Level 1 (Easy):** Write the equation of a horizontal line passing through (0, 5).
- **Level 2 (Moderate):** Find the equation of the line with slope 3 passing through (2, 4).
- **Level 3 (Challenging):** Find the equation of the line passing through points (1, 2) and (4, 8) in intercept form.

Answer Key

- Level 1: $y = 5$
- Level 2: Using point-slope form, $y - 4 = 3(x - 2) \Rightarrow y = 3x - 2$
- Level 3: Slope $m = \frac{8-2}{4-1} = 2$. Equation in slope-intercept form:
 $y - 2 = 2(x - 1) \Rightarrow y = 2x$. Intercepts: X-intercept a when $y = 0$: $0 = 2x \Rightarrow x = 0$,
Y-intercept b when $x = 0$: $y = 0$. Since both intercepts are zero, the line passes through origin, so intercept form is not defined here.

Quick Reference

Form	Equation
Horizontal line	$y = a$
Vertical line	$x = b$
Slope-intercept	$y = mx + c$
Point-slope	$y - y_0 = m(x - x_0)$
Two-point	$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
Intercept	$\frac{x}{a} + \frac{y}{b} = 1$

Glossary

- **Slope:** Ratio of vertical change to horizontal change between two points on a line.
- **Intercept:** Point where a line crosses an axis.
- **Point-slope form:** Equation of a line using slope and a point.
- **Two-point form:** Equation of a line using two points.
- **Intercept form:** Equation of a line using intercepts on axes.