Coordinate Axes and Coordinate Planes in Three Dimensional Space

In three-dimensional geometry, space is defined by three mutually perpendicular lines called coordinate axes: XOX', YOY', and ZOZ'. These int
named X-axis, Y-axis, and Z-axis respectively.

These axes define three mutually perpendicular coordinate planes: XOY (or XY plane), YOZ (or YZ plane), and ZOX (or ZX plane). These planes
octants.

The octants are labeled and characterized by the signs of the coordinates (x, y, z) of points lying within them. For example, in the first octant,

octants	I	II	III	IV	V	VI
coordinates	OXYZ	OX'YZ	OX'Y'Z	OXY'Z	OXYZ'	OX'YZ'
x	+	−	−	+	+	−
y	+	+	−	−	+	+
z	+	+	+	+	−	−

Coordinates of points on axes and planes are as follows:

- On X-axis: (x, 0, 0)
- On Y-axis: (0, y, 0)
- On Z-axis: (0, 0, z)
- On XY plane: (x, y, 0)
- On YZ plane: (0, y, z)
- On ZX plane: (x, 0, z)

Practice Set

- Level 1 – Identify the octant of the point (3, -2, 5).
- Level 2 – Find the coordinates of a point lying on the YZ plane with y = 4 and z = -3.
- Level 3 – Determine the octant of the point (-5, -6, -7) and explain the signs of its coordinates.

Answer Key

- Level 1 – Point (3, -2, 5) lies in the fourth octant where x is positive, y is negative, and z is positive.
- Level 2 – Coordinates are (0, 4, -3) as it lies on the YZ plane.
- Level 3 – Point (-5, -6, -7) lies in the seventh octant where all coordinates are negative.

Quick Reference

Octant	x	y
I	+	+
II	-	+

III	-	-
IV	+	-
V	+	+
VI	-	+
VII	-	-
VIII	+	-

Glossary

- **Origin:** The point (0, 0, 0) where the three coordinate axes intersect.
- **Coordinate Axes:** The three mutually perpendicular lines X, Y, and Z used to define position in space.
- **Coordinate Planes:** The planes formed by pairs of coordinate axes: XY, YZ, ZX.
- **Octant:** One of the eight divisions of three-dimensional space determined by the signs of coordinates.

Coordinates of a Point in Space

A point P in three-dimensional space is represented by an ordered triple of numbers (x, y, z), where:

- x is the signed distance from the YZ-plane (the plane where x=0).
- y is the signed distance from the ZX-plane (the plane where y=0).
- z is the signed distance from the XY-plane (the plane where z=0).

These coordinates specify the exact location of the point in space relative to the origin.

Worked Illustration

Find the coordinates of a point P which is 3 units along the X-axis, 4 units along the Y-axis, and 5 units above the XY-plane.

Solution: The coordinates of P are (3, 4, 5).

Practice Set

- Level 1 – Write the coordinates of a point on the Z-axis 7 units above the origin.
- Level 2 – Find the coordinates of a point 5 units along the negative X-axis and 2 units along the positive Y-axis on the XY-plane.
- Level 3 – Determine the coordinates of a point 3 units from the YZ-plane, -4 units from the ZX-plane, and 6 units below the XY-plane.

Answer Key

- Level 1 – (0, 0, 7)
- Level 2 – (-5, 2, 0)
- Level 3 – (3, -4, -6)

Quick Reference

Axis/Plane	Coordinate
X-axis	(x, 0, 0)
Y-axis	(0, y, 0)
Z-axis	(0, 0, z)
XY-plane	(x, y, 0)
YZ-plane	(0, y, z)
ZX-plane	(x, 0, z)

Glossary

- **Coordinates:** Ordered triple (x, y, z) representing a point's position in space.
- **Signed Distance:** Distance measured with positive or negative sign depending on direction from a reference plane.

Distance Formula in Three Dimensional Geometry

The distance between two points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) in space is given by the formula derived from the Pythagorean theorem

First, find the differences in each coordinate:

$$\Delta x = x_2 - x_1, \quad \Delta y = y_2 - y_1, \quad \Delta z = z_2 - z_1$$

The distance PQ is then:

$$PQ = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Worked Example

Find the distance between points P(2, -1, 3) and Q(5, 3, 7).

Solution:

Calculate the differences:

$$\Delta x = 5 - 2 = 3, \quad \Delta y = 3 - (-1) = 4, \quad \Delta z = 7 - 3 = 4$$

Distance PQ:

$$PQ = \sqrt{3^2 + 4^2 + 4^2} = \sqrt{9 + 16 + 16} = \sqrt{41}$$

Practice Set

- Level 1 – Find the distance between points A(1, 2, 3) and B(4, 6, 3).
- Level 2 – Calculate the distance between points C(-2, 0, 5) and D(1, 3, 9).
- Level 3 – Find the distance between points E(3, -1, 4) and F(-1, 2, 0).

Answer Key

- Level 1 – $\sqrt{(4-1)^2 + (6-2)^2 + (3-3)^2} = \sqrt{3^2 + 4^2 + 0} = 5$
- Level 2 – $\sqrt{(1+2)^2 + (3-0)^2 + (9-5)^2} = \sqrt{3^2 + 3^2 + 4^2} = \sqrt{34}$
- Level 3 – $\sqrt{(-1-3)^2 + (2+1)^2 + (0-4)^2} = \sqrt{(-4)^2 + 3^2 + (-4)^2} = \sqrt{16 + 9 + 16} = \sqrt{41}$

Quick Reference

Step	Formula
Difference in x	$x_2 - x_1$
Difference in y	$y_2 - y_1$
Difference in z	$z_2 - z_1$
Distance	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

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Glossary

- **Distance:** The length of the straight line segment joining two points in space.
- **Pythagorean Theorem:** A fundamental relation in Euclidean geometry among the three sides of a right triangle.

Midpoint Formula in Three Dimensional Geometry

The midpoint M of a line segment joining two points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) is the point that divides the segment into two equal

The coordinates of M are the averages of the corresponding coordinates of P and Q:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Worked Example

Find the midpoint of the segment joining points A(2, 3, 4) and B(6, 7, 8).

Solution:

$$M = \left(\frac{2+6}{2}, \frac{3+7}{2}, \frac{4+8}{2} \right) = (4, 5, 6)$$

Practice Set

- Level 1 – Find the midpoint of points (1, 2, 3) and (3, 4, 5).
- Level 2 – Calculate the midpoint of points (-2, 0, 4) and (2, 6, 8).
- Level 3 – Find the midpoint of points (5, -3, 7) and (-1, 9, 3).

Answer Key

- Level 1 – $\left(\frac{1+3}{2}, \frac{2+4}{2}, \frac{3+5}{2}\right) = (2, 3, 4)$
- Level 2 – $\left(\frac{-2+2}{2}, \frac{0+6}{2}, \frac{4+8}{2}\right) = (0, 3, 6)$
- Level 3 – $\left(\frac{5-1}{2}, \frac{-3+9}{2}, \frac{7+3}{2}\right) = (2, 3, 5)$

Quick Reference

Formula
$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Glossary

- **Midpoint:** The point that divides a line segment into two equal parts.

Centroid of a Triangle in Three Dimensional Geometry

The centroid G of a triangle with vertices A(x_1, y_1, z_1), B(x_2, y_2, z_2), and C(x_3, y_3, z_3) is the point where the three medians intersect

The coordinates of the centroid are the averages of the coordinates of the vertices:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3} \right)$$

Worked Example

Find the centroid of the triangle with vertices A(1, 2, 3), B(4, 0, 5), and C(7, 6, 9).

Solution:

$$G = \left(\frac{1+4+7}{3}, \frac{2+0+6}{3}, \frac{3+5+9}{3}\right) = (4, 8/3, 17/3)$$

Practice Set

- Level 1 – Find the centroid of triangle with vertices (0, 0, 0), (3, 0, 0), and (0, 4, 0).
- Level 2 – Calculate the centroid of triangle with vertices (1, 2, 3), (4, 5, 6), and (7, 8, 9).
- Level 3 – Find the centroid of triangle with vertices (-1, 0, 2), (3, -2, 4), and (5, 6, -1).

Answer Key

- Level 1 – $\left(\frac{0+3+0}{3}, \frac{0+0+4}{3}, \frac{0+0+0}{3}\right) = (1, \frac{4}{3}, 0)$
- Level 2 – $\left(\frac{1+4+7}{3}, \frac{2+5+8}{3}, \frac{3+6+9}{3}\right) = (4, 5, 6)$
- Level 3 – $\left(\frac{-1+3+5}{3}, \frac{0-2+6}{3}, \frac{2+4-1}{3}\right) = (\frac{7}{3}, \frac{4}{3}, \frac{5}{3})$

Quick Reference

Formula
$G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Glossary

- **Centroid:** The point of concurrency of the medians of a triangle; the center of mass.
- **Median:** A line segment joining a vertex of a triangle to the midpoint of the opposite side.

Section Formula in Three Dimensional Geometry

The section formula is used to find the coordinates of a point P which divides the line segment joining two points A(x_1, y_1, z_1) and B(x_2, y

If P divides AB internally, the coordinates of P are given by:

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

If P divides AB externally, the coordinates of P are:

$$P = \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Worked Example

Find the coordinates of the point dividing the segment joining A(2, 3, 4) and B(8, 7, 6) in the ratio 2:1 internally.

Solution:

$$P = \left(\frac{2\times 8 + 1\times 2}{2+1}, \frac{2\times 7 + 1\times 3}{2+1}, \frac{2\times 6 + 1\times 4}{2+1}\right) = \left(\frac{16+2}{3}, \frac{14+3}{3}, \frac{12+4}{3}\right) = (6, 17/3,$$

Practice Set

- Level 1 – Find the point dividing the segment joining (1, 2, 3) and (4, 5, 6) in the ratio 1:1 internally.
- Level 2 – Find the point dividing the segment joining (2, 3, 4) and (8, 7, 6) in the ratio 3:2 externally.
- Level 3 – Find the point dividing the segment joining (-1, 0, 2) and (3, 4, 6) in the ratio 2:3 internally.

Answer Key

- Level 1 – $\left(\frac{1+4}{2}, \frac{2+5}{2}, \frac{3+6}{2}\right) = (2.5, 3.5, 4.5)$
- Level 2 – $\left(\frac{3\times8-2\times2}{3-2}, \frac{3\times7-2\times3}{3-2}, \frac{3\times6-2\times4}{3-2}\right) = (20, 15, 10)$
- Level 3 – $\left(\frac{2\times3+3\times(-1)}{2+3}, \frac{2\times4+3\times0}{2+3}, \frac{2\times6+3\times2}{2+3}\right) = (0.6, 1.6, 3.6)$

Quick Reference

Type	Formula
Internal Division	$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$
External Division	$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

Glossary

- **Section Formula:** Formula to find coordinates of a point dividing a line segment in a given ratio.
- **Internal Division:** Point lies between the two points on the segment.
- **External Division:** Point lies outside the segment, extending the line.

Properties of Geometrical Figures

Triangles:

- **Scalene Triangle:** All three sides are unequal.
- **Right Angled Triangle:** The sum of squares of any two sides equals the square of the third side (Pythagoras theorem).
- **Isosceles Triangle:** Any two sides are equal.
- **Equilateral Triangle:** All three sides are equal.

Quadrilaterals:

- **Rectangle:** Opposite sides are equal and diagonals are equal.

- **Parallelogram:** Opposite sides are equal, diagonals are unequal, and diagonals bisect each other.
- **Rhombus:** All four sides are equal and diagonals are unequal.
- **Square:** All four sides are equal and diagonals are equal.

Practice Set

- Level 1 – Identify the type of triangle with sides 3 cm, 4 cm, and 5 cm.
- Level 2 – Determine if a quadrilateral with sides 5 cm, 5 cm, 8 cm, 8 cm and equal diagonals is a rectangle or a rhombus.
- Level 3 – Prove that a triangle with sides 7 cm, 24 cm, and 25 cm is right angled.

Answer Key

- Level 1 – Right angled triangle (since $3^2 + 4^2 = 5^2$).
- Level 2 – Rectangle (opposite sides equal and diagonals equal).
- Level 3 – Check Pythagoras theorem: $7^2 + 24^2 = 49 + 576 = 625 = 25^2$, so it is right angled.

Quick Reference

Figure	Properties
Scalene Triangle	All sides unequal
Right Angled Triangle	$a^2 + b^2 = c^2$
Isosceles Triangle	Two sides equal
Equilateral Triangle	All sides equal
Rectangle	Opposite sides equal, diagonals equal
Parallelogram	Opposite sides equal, diagonals unequal, bisect each other
Rhombus	All sides equal, diagonals unequal
Square	All sides equal, diagonals equal

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Glossary

- **Right Angled Triangle:** Triangle with one 90° angle.
- **Diagonal:** A line segment joining two non-adjacent vertices of a polygon.
- **Bisect:** To divide into two equal parts.