Cartesian Plane and Coordinates

The Cartesian plane is formed by two perpendicular number lines intersecting at the origin (0,0). The horizontal line is called the x-axis and the vertical line is called the y-axis. These axes divide the plane into four quadrants numbered I, II, III, and IV in an anti-clockwise direction starting from the positive x-axis.

Every point on the Cartesian plane is represented by an ordered pair (x, y), where x is the abscissa (distance from the y-axis) and y is the ordinate (distance from the x-axis). The coordinates indicate the position of the point relative to the origin.

Key Properties

- The coordinate of a point on the x-axis is of the form $(x, 0)$.
- The coordinate of a point on the y-axis is of the form $(0, y)$.
- The sign of the coordinates depends on the quadrant:

Quadrant	Sign of x	Sign of y
I	+	+
II	-	+
III	-	-

IV	+	-

Worked Illustration

Plot the point $P(3, 4)$ on the Cartesian plane.

Solution:

1. Start at the origin $(0, 0)$.
2. Move 3 units along the positive x-axis.
3. From there, move 4 units up along the positive y-axis.
4. Mark the point P at this position.

Practice Set

Level 1 – Easy

- Identify the coordinates of the origin.
- State the coordinates of a point on the x-axis 5 units to the right of the origin.
- State the coordinates of a point on the y-axis 7 units above the origin.

Level 2 – Moderate

- Determine the quadrant of the point $(-4, 6)$.
- Plot the points $(-3, -2)$ and $(5, -1)$ and state their quadrants.

Level 3 – Challenging

- Given a point $Q(x, y)$ lies in the second quadrant and $x + y = 0$, find the possible coordinates of Q.

Answer Key

- Origin coordinates: $(0, 0)$.
- Point on x-axis 5 units right: $(5, 0)$.
- Point on y-axis 7 units above: $(0, 7)$.
- Quadrant of $(-4, 6)$: II.
- Point $(-3, -2)$ lies in III; point $(5, -1)$ lies in IV.
- For $x + y = 0$ in quadrant II, $x < 0$ and $y > 0$, so $Q = (-a, a)$ where $a > 0$.

Quick Reference

- Origin: $(0, 0)$
- Quadrants: I (+,+), II (-,+), III (-,-), IV (+,-)
- Coordinates: (x, y) where x is abscissa and y is ordinate

Glossary

- **Abscissa:** The x-coordinate of a point.
- **Ordinate:** The y-coordinate of a point.
- **Origin:** The point $(0, 0)$ where the axes intersect.
- **Quadrant:** One of the four regions into which the plane is divided by the axes.

Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the Cartesian plane is the length of the line segment PQ. It is calculated using the Pythagorean theorem.

Formula Derivation

Consider the points $P(x_1, y_1)$ and $Q(x_2, y_2)$. Construct a right triangle by drawing perpendiculars from these points to the axes.

The horizontal side length is $|x_2 - x_1|$ and the vertical side length is $|y_2 - y_1|$.

By the Pythagorean theorem, the distance d is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Worked Illustration

Find the distance between points $A(3, 2)$ and $B(7, 6)$.

Solution:

Using the distance formula,

$$AB = \sqrt{(7 - 3)^2 + (6 - 2)^2} = \sqrt{4^2 + 4^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$$

Practice Set

Level 1 – Easy

- Calculate the distance between $(0, 0)$ and $(3, 4)$.
- Find the distance between $(1, 2)$ and $(1, 5)$.

Level 2 – Moderate

- Find the distance between $(-2, -3)$ and $(4, 1)$.
- Calculate the distance of the point $(5, 12)$ from the origin.

Level 3 – Challenging

- Prove that the points $(1, 2)$, $(4, 6)$, and $(7, 10)$ are collinear using the distance formula.

Answer Key

- Distance between $(0, 0)$ and $(3, 4)$ is 5 units.
- Distance between $(1, 2)$ and $(1, 5)$ is 3 units.
- Distance between $(-2, -3)$ and $(4, 1)$ is $\sqrt{(4+2)^2 + (1+3)^2} = \sqrt{36 + 16} = \sqrt{52} = 2\sqrt{13}$.
- Distance of $(5, 12)$ from origin is $\sqrt{5^2 + 12^2} = 13$.
- Check collinearity: $AB = \sqrt{(4-1)^2 + (6-2)^2} = 5$, $BC = \sqrt{(7-4)^2 + (10-6)^2} = 5$, $AC = \sqrt{(7-1)^2 + (10-2)^2} = 10$. Since $AB + BC = AC$, points are collinear.

Quick Reference

- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Distance from origin: $\sqrt{x^2 + y^2}$

Glossary

- **Distance:** The length between two points in the plane.
- **Collinear points:** Points lying on the same straight line.

Midpoint Formula

The midpoint of a line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is the point M that divides the segment into two equal parts.

Formula Derivation

The coordinates of the midpoint M are the averages of the corresponding coordinates of A and B:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Worked Illustration

Find the midpoint of the segment joining $P(2, 3)$ and $Q(6, 7)$.

Solution:

$$M = \left(\frac{2 + 6}{2}, \frac{3 + 7}{2} \right) = (4, 5)$$

Practice Set

Level 1 – Easy

- Find the midpoint of $(0, 0)$ and $(4, 4)$.
- Find the midpoint of $(1, 5)$ and $(3, 9)$.

Level 2 – Moderate

- Find the midpoint of $(-2, 7)$ and $(6, -1)$.
- Find the midpoint of (x, y) and $(2x, 3y)$.

Level 3 – Challenging

- Given the midpoint $M(3, 4)$ of segment AB, and point $A(1, 2)$, find the coordinates of B.

Answer Key

- Midpoint of $(0, 0)$ and $(4, 4)$ is $(2, 2)$.
- Midpoint of $(1, 5)$ and $(3, 9)$ is $(2, 7)$.
- Midpoint of $(-2, 7)$ and $(6, -1)$ is $(2, 3)$.
- Midpoint of (x, y) and $(2x, 3y)$ is $\left(\frac{3x}{2}, 2y\right)$.
- Coordinates of B are $(5, 6)$ since $3 = \frac{1+x_B}{2}$ and $4 = \frac{2+y_B}{2}$.

Quick Reference

- Midpoint formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Glossary

- **Midpoint:** The point dividing a line segment into two equal parts.

Section Formula

The section formula gives the coordinates of a point P dividing the line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$.

Formula Derivation

If P divides AB internally in the ratio $m : n$, then the coordinates of P are:

$$x = \frac{mx_2 + nx_1}{m + n}, \quad y = \frac{my_2 + ny_1}{m + n}$$

Worked Illustration

Find the coordinates of the point dividing the segment joining $A(2, 3)$ and $B(8, 7)$ in the ratio $3 : 2$.

Solution:

$$x = \frac{3 \times 8 + 2 \times 2}{3 + 2} = \frac{24 + 4}{5} = \frac{28}{5} = 5.6$$

$$y = \frac{3 \times 7 + 2 \times 3}{3 + 2} = \frac{21 + 6}{5} = \frac{27}{5} = 5.4$$

Coordinates of the point are $(5.6, 5.4)$.

Practice Set

Level 1 – Easy

- Find the point dividing $(0, 0)$ and $(4, 4)$ in the ratio $1 : 1$.
- Find the point dividing $(1, 2)$ and $(3, 6)$ in the ratio $2 : 3$.

Level 2 – Moderate

- Find the point dividing $(-2, 5)$ and $(4, -1)$ in the ratio $3 : 1$.
- Find the ratio in which $(2, 3)$ divides the segment joining $(1, 1)$ and $(5, 7)$.

Level 3 – Challenging

- Find the coordinates of the point dividing the segment joining $A(-6, 10)$ and $B(3, -8)$ in the ratio $2 : 7$.

Answer Key

- Point dividing $(0, 0)$ and $(4, 4)$ in ratio $1 : 1$ is $(2, 2)$.
- Point dividing $(1, 2)$ and $(3, 6)$ in ratio $2 : 3$ is $\left(\frac{2\times3+3\times1}{5}, \frac{2\times6+3\times2}{5}\right) = (1.8, 3.6)$.
- Point dividing $(-2, 5)$ and $(4, -1)$ in ratio $3 : 1$ is $\left(\frac{3\times4+1\times(-2)}{4}, \frac{3\times(-1)+1\times5}{4}\right) = (2.5, 0.5)$.
- Ratio in which $(2, 3)$ divides $(1, 1)$ and $(5, 7)$ is $1 : 2$.
- Coordinates dividing $A(-6, 10)$ and $B(3, -8)$ in ratio $2 : 7$ are $(-4, 6)$.

Quick Reference

- Section formula (internal division): $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Glossary

- **Section formula:** Formula to find a point dividing a segment in a given ratio.
- **Internal division:** When the point lies between the two points.